The University of the State of New York

320TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Monday, January 25, 1954 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer three questions from part II.

26 Prove: Two right triangles are congruent if the hypotenuse and a leg of one are equal to the hypotenuse and a leg of the other. [10]

27 Diagonal AC of parallelogram $ABCD$ is extended through A to point E and through C to point F, making CF equal to AE. Lines ED, DF, FB and BE are drawn. Prove that $EDFB$ is a parallelogram. [10]

28 Prove: If two chords intersect within a circle, the product of the segments of one is equal to the product of the segments of the other. [10]

29 In triangle ABC, sides AB and AC are equal. A line through B intersects AC at D. BD is extended through D to point E, and CE is drawn. Prove that BE is greater than CE. [10]

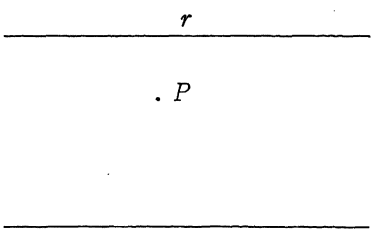
30 Parallel lines r and s are d distance apart and point P is any point between the two lines.

a What is the length of the radius of a circle that is tangent to both r and s? [1]

b What is the locus of the center of a circle that is tangent to both r and s? [2]

c What is the locus of the center of a circle whose radius is $\frac{d}{2}$ and which passes through P? [2]

d On your answer paper draw the figure at the right. [*Construction* of r parallel to s is not required.] Now construct a circle tangent to the two parallel lines and passing through the given point P. [4]

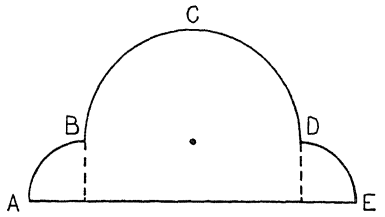


e How many different circles are there that satisfy the requirements specified in part *d*? [1]

Part III

Answer two questions from part III.

31 In the accompanying figure, BCD is a semicircle and AB and ED are equal quadrants. (A quadrant is a quarter of a circle.) AE is 42 inches and the diameter of the semicircle is 28 inches. Find the area of the figure. [Use $\pi = \frac{22}{7}$] [10]



32 The longer base of an isosceles trapezoid is 40, one leg is 20 and one of the base angles is 63°.

a Find to the *nearest integer:*

(1) the altitude of the trapezoid [3]

(2) the shorter base of the trapezoid [4]

b Using the results found in answer to part *a,* find the area of the trapezoid. [3]

33 Quadrilateral $ABCD$ is inscribed in a circle and arcs AB, BC, CD and DA are in the ratio 3 : 3 : 4 : 2.

a Find the number of degrees in each of the four arcs. [2]

b If AC is drawn, find the number of degrees in angles CAB and ACD. [2]

c If chord AD is 10, find the

(1) circumference of the circle. [Answer may be left in terms of π.] [3]

(2) area of triangle ABC. [3]

34 For each item listed in column I the numerical value, correct to the *nearest tenth,* is given in column II. List the numbers 1–5 on your answer paper and after each number write the *letter* indicating the numerical value of the corresponding item. [10]

Column I	*Column II*
(1) The side of a square whose diagonal is 10	*a* 7.1
(2) The area of a rhombus whose diagonals are 4.8 and 4	*b* 8.3
(3) The hypotenuse of a right triangle whose legs are 6 and 14	*c* 9.6
(4) The area of a regular hexagon whose side is 2	*d* 10.4
(5) The length of an arc of 80° in a circle whose circumference is $37\frac{1}{2}$	*e* 13.4
	f 15.2
	g 15.3

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 In parallelogram $ABCD$, angle A is twice angle B. Find the number of degrees in angle B. 1.....

2 Find the sum of the interior angles of a polygon of 12 sides. 2.....

3 Is there a regular polygon such that each exterior angle is 50°? [Answer *yes* or *no*.] 3.....

4 The hypotenuse of a right triangle is 2.5 and one leg is 1.5. Find the other leg. 4.....

5 If a point is equidistant from the three sides of a triangle, it must be the intersection of the three (*a*) medians, (*b*) angle bisectors, (*c*) perpendicular bisectors of the sides. Which is correct *a*, *b*, or *c*? 5.....

6 In triangle ABC a line parallel to AC intersects AB at D and CB at E. If $AB = 12$, $DB = 8$, and $AC = 15$, find DE. 6.....

7 The areas of two similar triangles are 40 and 90. Find the ratio of a side of the smaller polygon to the corresponding side of the larger. 7.....

8 A tangent and a secant drawn to a circle from the same point are 6 and 18 respectively. Find the external segment of the secant. 8.....

9 The segments made by the altitude on the hypotenuse of a right triangle are 3 and 7. Find the altitude on the hypotenuse. [Answer may be left in radical form.] 9.....

10 An angle of a rhombus is 60° and its shorter diagonal is 4. Find the altitude of the rhombus. [Answer may be left in radical form.] 10.....

11 The locus of points equidistant from two concentric circles whose radii are 8 and 14 is a circle concentric with the given circles. Find its radius. 11.....

12 Two unequal circles are tangent externally to the same line at the same point. From any point in this line one tangent is drawn to the larger circle and one tangent is drawn to the smaller circle. The tangent to the larger circle is (*a*) longer than, (*b*) equal to, (*c*) shorter than, the tangent to the smaller circle. Which is correct *a*, *b* or *c*? 12.....

13 Express the area of a regular polygon in terms of the number of sides (n), the length of one side (s) and the apothem (a). 13.....

[OVER]

14 In triangle ABC, angle $C = 90°$, $AC = 20$ and $BC = 36$. Find angle A to the *nearest degree*. 14.....

Directions (15–18): Write on the line at the right of *each* question the expression that, when inserted in the blank, will make the statement true.

15 If line a is perpendicular to line b and line c is parallel to a, then c is ... to b. 15.....

16 An angle formed by two chords intersecting within the circle is measured by one half the ... of the intercepted arcs. 16.....

17 If the center of the circle that is circumscribed about a triangle is outside the triangle, the triangle is .... 17.....

18 If two parallelograms have equal bases, their areas are to each other as their .... 18.....

Directions (19–24): For *each* of the following, if the statement is *always* true, write the word *true* on the line at the right; if it is *not always* true, write the word *false*.

19 It is always possible to construct a right triangle if the given parts are the hypotenuse and one of the acute angles. 19.....

20 An equilateral polygon inscribed in a circle is regular. 20.....

21 If the diagonals of a parallelogram are equal, the parallelogram is a square. 21.....

22 AB and $A'B'$ are bases of isosceles triangles ABC and $A'B'C'$. If $AB : A'B' = AC : A'C'$, the triangles are similar. 22.....

23 If the sides of a triangle are a, b and c, the perimeter of the triangle formed by joining the mid-points of the sides of the given triangle is $\frac{a+b+c}{2}$ 23.....

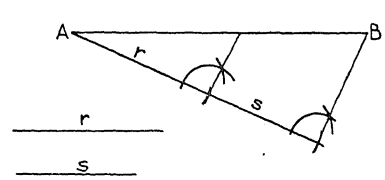
24 The median drawn to the hypotenuse of a right triangle divides the triangle into two isosceles triangles. 24.....

25 The accompanying diagram shows the division of given line segment AB into two parts which are in the ratio $r : s$.

Which statement, a or b, is used to prove that the construction is correct?

a If a line is drawn through two sides of a triangle parallel to the third side, it divides those sides proportionally.

b A line that divides two sides of a triangle proportionally is parallel to the third side.



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