The University of the State of New York

311TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Monday, January 22, 1951 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer three questions from part II.

26 *a* Prove that the diagonals of a parallelogram bisect each other. [8]

b State the converse of the theorem given in *a*. [2]

27 Prove that an angle formed by two secants intersecting outside the circle is measured by one half the difference of the intercepted arcs. [10]

28 BM and $B'M'$ are bisectors of corresponding angles of similar triangles ABC and $A'B'C'$ respectively. Prove that

a triangle ABM is similar to triangle $A'B'M'$ [7]

b $BM : B'M' = AC : A'C'$ [3]

29 In triangle ABC, in which AB is the base, medians BN and CM intersect in E. Prove:

a area of $\triangle AMC = \frac{1}{2}$ area of $\triangle ABC$ [3]

b area of $\triangle AMC =$ area of $\triangle BNC$ [3]

c area of quad. $ANEM =$ area of $\triangle BEC$ [4]

Part III

Answer one question from part III.

30 Chord AB of circle O is 10 inches long and 5 inches from the center of the circle. Radii OA and OB are drawn. Find

a the radius of the circle [Answer may be left in radical form.] [2]

b the angle AOB [2]

c the area of minor sector AOB to the *nearest integer* [Use $\pi = \frac{22}{7}$] [3]

d the area of the minor segment of the circle [3]

31 A triangle whose base is b and whose altitude is h is equal in area to a rhombus whose diagonals are d and d'.

a If $b = 24$, $h = 8$ and $d = 16$, find d'. [5]

b Find, to the *nearest degree*, one of the acute angles of the rhombus. [5]

Part IV

Answer one question from part IV.

32 Given circle O, whose radius is r, and two intersecting lines l and l'.

a Describe fully the locus of points which are

(1) outside circle O and at a given distance d from the circle [2]

(2) equidistant from l and l' [2]

b What is the number of points which will satisfy both conditions given in *a* if l and l' intersect

(1) inside circle O [1]

(2) on circle O [1]

c If l and l' intersect outside circle O, which of the following choices represents the least number of points that will satisfy the conditions given in *a*: 0, 1, 2, 3, 4? [4]

33 Given a parallelogram with adjacent sides c and d and an angle of 30°, and a trapezoid whose bases are b and b'.

a Express the area of the parallelogram in terms of c and d. [2]

b If x represents the altitude of the trapezoid, express its area in terms of b, b' and x. [2]

c If the area of the trapezoid is equal to the area of the parallelogram, write a proportion which shows the relation between b, b', x, c and d. [2]

d Using the line segments given below, find by construction line segment x. [4]

Fill in the following lines:

Name of pupil .....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 The legs of a right triangle are 15 and 20. Find the hypotenuse. 1.....

2 Find the circumference of a circle whose radius is 20. [Answer may be left in terms of π.] 2.....

3 Angle ABC is inscribed in circle O, and radii OA and OC are drawn. If angle $ABC = 35°$, how many degrees are there in angle AOC? 3.....

4 From a point outside a circle, a tangent and a secant are drawn. The tangent is 15 and the external segment of the secant is 9. Find the secant. 4.....

5 Two chords intersect within a circle. The segments of one chord are a and b. If one segment of the other chord is r, express the other segment in terms of a, b and r. 5.....

6 The areas of two similar triangles are in the ratio 16:25, and a side of the smaller triangle is 8. Find the corresponding side of the larger triangle. 6.....

7 The perimeter of a triangle is 12. Find the perimeter of the triangle formed by joining the mid-points of the sides of the given triangle. 7.....

8 Find the apothem of a regular hexagon whose side is 6. [Answer may be left in radical form.] 8.....

9 The area of an equilateral triangle is $9\sqrt{3}$. Find a side of the triangle. 9.....

10 The altitude on the hypotenuse of a right triangle is 10 and it divides the hypotenuse into two segments, one of which is 5. Find the other segment. 10.....

11 The angles of a triangle are in the ratio 3:4:5. Find the number of degrees in the smallest angle of the triangle. 11.....

12 M is the mid-point of line segment AB. How many points are there which are equidistant from A and B and also at a given distance from M? 12.....

13 In triangle ABC, angle $C = 90°$, $AB = 8$ and angle $A = 22°$. Find CB to the *nearest integer*. 13.....

Directions (questions 14–19) — In *each* of the following, if the statement is *always* true, write the word *true* on the line at the right; if it is *not always* true, write the word *false*.

14 If one angle of a parallelogram is a right angle, the parallelogram is a rectangle. 14.....

15 A line intersects sides AB and BC of triangle ABC in points D and E respectively. If $BA:BD = BC:BE$, then DE is parallel to AC. 15.....

16 If two circles intersect, their common chord bisects the line segment joining their centers. 16.....

17 The bisectors of angles B and C of triangle ABC intersect in O. If AB is greater than AC, then OB is greater than OC. 17.....

18 If the diagonals of a trapezoid are drawn, a pair of similar triangles is formed. 18.....

19 Two circles are tangent to a line at point T. If P is any other point on this line, then tangents from P to the two circles are equal. 19.....

[OVER]

Directions (questions 20–24) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a*, *b* or *c*.

20 If the altitudes of a triangle intersect inside the triangle, the triangle is (*a*)acute (*b*)right (*c*)obtuse 20.....

21 A quadrilateral must be a parallelogram if (*a*)its diagonals are equal (*b*)two sides are equal and the other two sides are parallel (*c*)its opposite angles are equal 21.....

22 A regular pentagon has a side s and an apothem a. The area of this pentagon is (*a*) $10as$ (*b*) $5as$ (*c*) $\frac{5as}{2}$ 22.....

23 An exterior angle of a regular polygon may be (*a*)140° (*b*)120° (*c*)75° 23.....

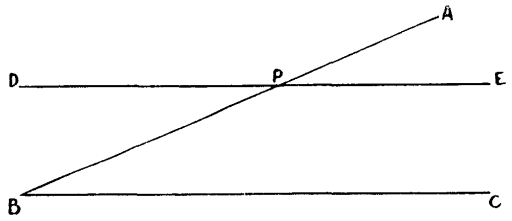
24 In the diagram, P is a point on side AB of angle ABC. To construct a line through P parallel to BC, line DE is drawn through P making angle APE equal to angle ABC.

Which of the following statements is the theorem used to prove that DE is parallel to BC?

a One and only one line can be drawn through a given point parallel to a given line.

b If two parallel lines are cut by a transversal, the corresponding angles are equal.

c If two lines are cut by a transversal so that a pair of corresponding angles are equal, the lines are parallel.



24.....

25 Find by construction the center of the circle at the right. [Leave all construction lines on the paper.]