The University of the State of New York

PLANE GEOMETRY

Monday, January 23, 1950 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in plane geometry, (c) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer three questions from part II.

26 Prove that the diagonals of a parallelogram bisect each other. [10]

27 Line ABC is tangent to circle O at B. Line ADE is drawn through the center of the circle cutting it in points D and E. Chord BF is drawn parallel to AE and radii OF and OB are drawn. Prove that angle CBF is complementary to angle FOE. [10]

28 AB and DC are the bases of trapezoid $ABCD$. Diagonals AC and BD meet in E.
    Prove: a Triangle ABE is similar to triangle DCE. [5]
            b Triangle ADE is equal in area to triangle BCE. [5]

29 Prove that the area of a regular polygon is equal to one half the product of its perimeter and its apothem. [10]

Part III

Answer one question from part III.

30 In parallelogram $ABCD$, BE is an altitude to base AD. Angle $A = 41°$, $AB = 12$ and diagonal $BD = 10$.
    a Find BE to the *nearest integer*. [4]
    b Find AE to the *nearest integer*. [3]
    c Using the values found in answer to parts a and b, find DE and the area of $ABCD$. [1, 2]

31 In trapezoid $ABCD$, base BC is to base AD as 3 is to 8. Legs AB and DC are extended to meet at E and the altitude EF of triangle AED intersects BC at G. The area of triangle AED is 192 and $EF = 24$.
    a Find AD. [2]
    b Find BC and EG. [2, 2]
   c Find the area of $ABCD$. [4]

Part IV

Answer one question from part IV.

32 If the blank in each of the following statements is filled by one of the words, *always, sometimes,* or *never,* the resulting statement will be true. Write on your answer paper the letters *a, b, c, d, e* and opposite *each* write the word that will correctly complete the corresponding statement.

a A median of a triangle ...... divides it into two congruent triangles.   [2]

b If two chords of a circle bisect each other, the opposite intercepted arcs are ...... equal.   [2]

c In triangle *ABC,* if *AB* is greater than *AC,* the altitude to *AB* is ...... greater than the altitude to *AC.*   [2]

d If the diagonals of a quadrilateral are perpendicular to each other and one diagonal bisects the angles through which it is drawn, the quadrilateral is ...... a rhombus.   [2]

e If to each of two unequal concentric circles a tangent is drawn from point *P* outside the circles, the tangents are ...... equal.   [2]

33 It is required to construct a square equal in area to a rhombus whose diagonals are the given line segments *d* and *d'*.

d	*d'*
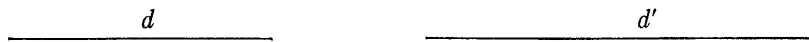
a Representing the side of the square by *x,* write an equation showing the relationship between *x, d* and *d'*.   [3]

b Construct *x*.   [5]

c Construct the required square.   [2]

[2]

Fill in the following lines:

Name of pupil..Name of school...

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 In parallelogram $ABCD$, angle $BAD = 70°$. Find the number of degrees in angle ABC.

1................

2 How many degrees are there in each *exterior* angle of an equiangular polygon of 10 sides?

2................

3 Two chords intersecting within a circle intercept opposite arcs of $60°$ and $100°$. Find the number of degrees in an acute angle formed by the chords.

3................

4 In triangle ABC, angle $C = 90°$ and AB is twice BC. Find the number of degrees in angle A.

4................

5 A chord 16 inches long is 5 inches from the center of the circle. Find the radius of the circle. [Answer may be left in radical form.]

5................

6 The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments one of which is 4. If the altitude is 10, find the other segment of the hypotenuse.

6................

7 Chords AB and CD of a circle intersect at E. $AE = 8$, $EB = 9$ and $DE = 6$. Find EC.

7................

8 From a point outside a circle, a tangent and a secant are drawn to the circle. If the tangent is 6 and the secant is 12, find the external segment of the secant.

8................

9 From point P, lines are drawn tangent to a circle at points A and B. Chord AB is drawn. If angle P is $50°$, find the number of degrees in angle PAB.

9................

10 The corresponding sides of two similar polygons are in the ratio $1:3$. Find the ratio of the perimeters of the polygons.

10................

11 The line segment joining the midpoints of the legs of a right triangle is 10. Find the hypotenuse.

11................

12 Find the diagonal of a square whose side is 10. [Answer may be left in radical form.]

12................

13 Find the area of an equilateral triangle whose side is 10. [Answer may be left in radical form.]

13................

14 The altitude of an equilateral triangle is 6. Find the radius of the inscribed circle.

14................

15 In a circle whose radius is 24 inches, find the length of an arc of $30°$. [Answer may be left in terms of π.]

15................

16 Find the radius of a circle whose area is $64\,\pi$.

16................

17 Find to the *nearest degree* the angle of elevation of the sun when a 25-foot vertical flagpole casts a shadow 10 feet long.

17................

18 Point P is on line m. How many points are there which are 2 inches from m and 3 inches from P?

18................

19 Is statement B the converse of statement A? [Answer *yes* or *no*.]

A If two triangles are congruent, then they are similar.

B If two triangles are similar, then they are congruent.

19................

[OVER]

Directions (questions 20–23) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a, b* or *c.*

20 In triangle ABC, angle $A = 60°$ and AB is greater than AC. The smallest angle of the triangle is   (*a*)angle A (*b*)angle B (*c*)angle C 20......

21 Two sides of a triangle are 5 and 8. The third side is   (*a*)less than 3 (*b*)equal to 3   (*c*)greater than 3   21......

22 According to your textbook, the definition of a parallelogram is: (*a*) a parallelogram is a quadrilateral whose opposite sides are equal   (*b*)a parallelogram is a quadrilateral whose opposite sides are parallel   (*c*)a parallelogram is a quadrilateral two of whose sides are both equal and parallel   22......

23 The center of a circle inscribed in a triangle is the intersection of the   (*a*)altitudes   (*b*)angle bisectors   (*c*)perpendicular bisectors of the sides   of the triangle.   23......

Directions (questions 24–25) — Leave all construction lines on the paper.

24 Construct the median to side AC of the given triangle ABC.
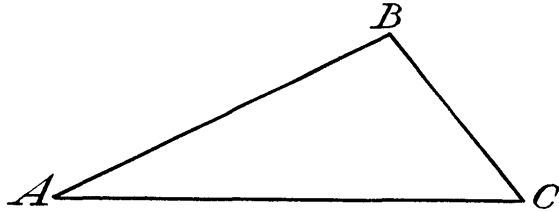
25 Using the given line segments r, s and t, construct x so that $r:s = t:x$.

r

s

t

[4]