The University of the State of New York

305TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Monday, January 24, 1949 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer three questions from part II.

26 Prove that the sum of the angles of a triangle is a straight angle. [10]

27 Prove that if from a point outside a circle a tangent and a secant are drawn to the circle, the tangent is the mean proportional between the secant and its external segment. [10]

28 In triangle ABC, $AB = AC$. AC is extended through C to point E and AB is extended through B to point D so that $BD > CE$. Line DE is drawn. Prove that angle $E >$ angle D. [10]

29 Prove that if two equal chords of a circle intersect, the longer segment of one chord equals the longer segment of the other. [10]

Part III

Answer one question from part III.

30 Angle D in quadrilateral $ABCD$ is a right angle and diagonal AC is perpendicular to BC. $BC = 20$, angle $B = 35°$ and angle $DAC = 65°$.

a Find AC to the *nearest integer*. [6]

b Using the result obtained in answer to *a*, find DC to the *nearest integer*. [4]

31 The diameter AB of circle O is a base of the inscribed trapezoid $ABCD$. Angle $A = 60°$ and radius $OD = 12$.

a Find the altitude of the trapezoid. [4]

b Find the area of the trapezoid. [6]

[Answers may be left in radical form.]

[OVER]

Part IV

Answer one question from part IV.

32 $ABCD$ is a rectangle in which $AB = 32$ and $AD = 24$. $AE = CF$ and $BE = ED$.

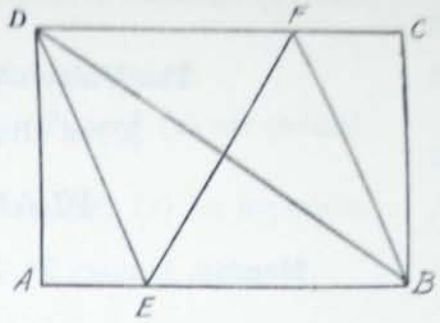
a Prove $EDFB$ is a rhombus. [3]

b Find DB. [1]

c If AE is represented by x, express ED in terms of x. [Suggestion: Use the relationship $ED = EB$.] [1]

d Find x. [2]

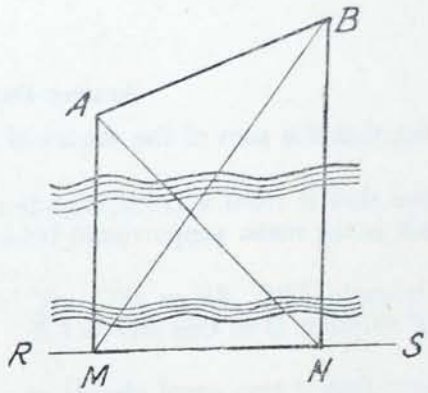
e Find EF. [3]



33 *a* One method of laying out a corner B for a rectangular field is as follows: Stakes are set at points B and F, 50 feet apart. A third stake is then set at D, 50 feet from both B and F. A fourth stake is set at E, in line with F and D and 50 feet from D. Explain why the line through B and E is perpendicular to the line through B and F. [3]

b A boy has a map whose scale is 1 inch = 50 miles and which just fits on a sheet of his notebook paper. He wishes to enlarge the map so the scale will be 2 inches = 50 miles. What is the smallest number of sheets of his notebook paper that must be taped together in order to contain the larger map? [3]

c The distance AB between two points on one side of a river is to be found by measurements made on the opposite side. On line RS, points M and N are taken so that MA and NB are perpendicular to RS. Angle MNA, angle NMB and distance MN are measured. Explain briefly how these measurements may be used to find AB. [Suggestion: Draw a perpendicular from A to NB.] [4]



Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments 5 and 20. Find the altitude. 1.....

2 In circle O, chords AB and CD intersect at E. $AE = 4$, $EB = 10$ and $CE = 8$. Find ED. 2.....

3 Find the area of a circle whose radius is 7. [Answer may be left in terms of π.] 3.....

4 In a circle whose radius is 9, find the length of an arc of 40°. [Answer may be left in terms of π.] 4.....

5 An angle formed by a tangent and a chord drawn from the point of contact intercepts an arc of 50°. How many degrees are there in the angle? 5.....

6 From a point outside a circle two secants are drawn intercepting arcs of 110° and 30°. Find the angle formed by the secants. 6.....

7 One angle of a quadrilateral inscribed in a circle is 70°. Find the number of degrees in the opposite angle. 7.....

8 How many degrees are there in the sum of the interior angles of a seven-sided polygon? 8.....

9 Two sides of a parallelogram are 2 and 5 and the included angle is 45 degrees. Find the altitude to side 5. [Answer may be left in radical form.] 9.....

10 Find the area of an equilateral triangle whose side is 9. [Answer may be left in radical form.] 10.....

11 Find the area of a rhombus whose diagonals are 10 and 24. 11.....

12 Corresponding sides of two similar polygons are 8 and 9. Find the ratio of their perimeters. 12.....

13 The line joining the mid-points of two adjacent sides of a rectangle is 12. Find a diagonal. 13.....

14 In triangle ABC, D is a point on AB and E is a point on AC. $AD = 4$, $BD = 3$, $AE = 6$, $EC = 8$. Is DE parallel to BC? [Answer *yes* or *no*.] 14.....

15 If angle t is the complement of angle s and angle r is the supplement of s, which of the three angles is the largest? 15.....

Directions (questions 16–20) — Indicate whether *each* statement is true or false by writing the word *true* or *false* on the line at the right.

16 Regular polygons of the same number of sides are similar. 16.....

17 Every trapezoid has at least one acute angle. 17.....

18 If an exterior angle of a triangle is double a nonadjacent interior angle, the triangle is isosceles. 18.....

19 If two circles are concentric, chords of the larger circle which are tangent to the smaller circle are equal. 19.....

20 A diagonal divides a trapezoid into two triangles whose areas have the same ratio as the bases of the trapezoid. 20.....

[OVER]

Directions (questions 21–23) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a*, *b* or *c*.

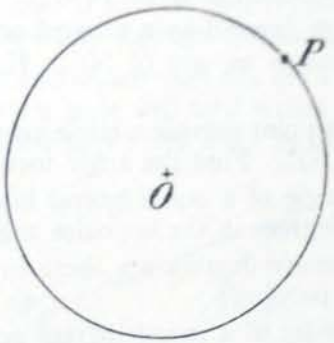
21 A circle can always be circumscribed about a (*a*)parallelogram (*b*)rectangle (*c*)rhombus 21.....

22 The center of a circle which circumscribes a triangle is always the intersection of (*a*)the altitudes of the triangle (*b*)the bisectors of the angles of the triangle (*c*)the perpendicular bisectors of the sides of the triangle 22.....

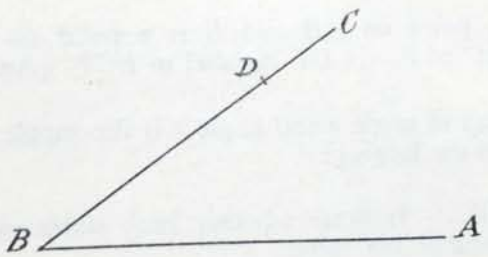
23 The locus of the vertices of the right angles of all right triangles which have a given line segment as hypotenuse is (*a*)a straight line (*b*)two parallel lines (*c*)a circle 23.....

Directions (questions 24–25) — Leave all construction lines on your paper.

24 At P construct a tangent to circle O.



25 Through D construct a line which will be parallel to BA.



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