The University of the State of New York

296TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Tuesday, January 29, 1946 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for a school year.

Part II

Answer three questions from part II.

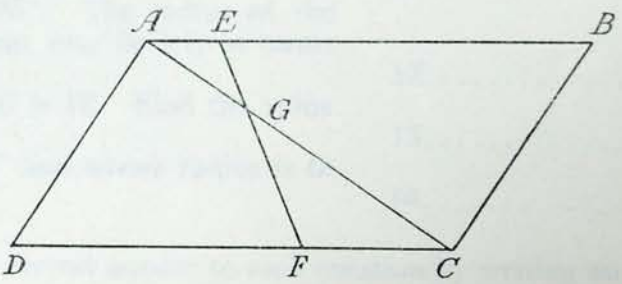
26 *a* Prove that tangents drawn to a circle from an external point are equal. [8]

b Using the proof given in answer to *a*, show that the line from the external point to the center of the circle bisects the angle formed by the tangents. [2]

27 *a* For the following statement, draw a figure, letter it and state, *in terms of the letters of the figure*, what is given and what is to be proved: [The proof is not required.]

If the bisector of an exterior angle of a triangle is parallel to a side of the triangle, the triangle is isosceles. [3]

b In the figure at the right, $ABCD$ is a parallelogram; EF is any line intersecting diagonal AC in G. Prove: $AG \times GF = GC \times GE$ [7]



28 Quadrilateral $ABCD$ is inscribed in a circle. Diagonals AC and BD intersect at E and $BE = CE$. Prove that chord AB = chord CD. [10]

29 Prove that if one angle of a triangle is greater than a second angle, the side opposite the first angle is greater than the side opposite the second angle. [10]

Part III

Answer one question from part III.

30 In trapezoid $ABCD$, angle A is 63° and bases AB and DC are perpendicular to leg BC. Base DC is 14 and leg AD is 12.

a Find the altitude of the trapezoid correct to the *nearest tenth*. [3]

b Find the area of the trapezoid correct to the *nearest integer*. [7]

31 A man 6 feet tall stands on level ground 10 feet from a point A, which is directly below a light.

a If the man's shadow is 8 feet long, find the height of the light above the ground. [6]

b If the man takes a position 3 feet nearer A, find the length of his shadow. [4]

Part IV

Answer one question from part IV.

32 Each of the statements below contains in the hypothesis either more information than is necessary or too little information to prove the conclusion. Rewrite *each* statement so that the hypothesis will contain *just enough* information.

a The areas of two triangles are to each other as the squares of any two corresponding sides. [2]

b If a line is perpendicular to a chord of a circle, it passes through the center of the circle. [2]

c If two parallel chords of a circle are equal, they are equally distant from the center of the circle. [2]

d If from a point outside a circle two equal secants are drawn, the product of one secant and its external segment is equal to the product of the other secant and its external segment. [2]

e In any quadrilateral whose opposite sides are equal, the figure formed by joining in order the mid-points of the sides of the quadrilateral is a parallelogram. [2]

33 Prove that two lines drawn from the ends of one side of a triangle to the opposite sides can not bisect each other. [Suggestion: Use the indirect method.] [10]

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 A regular hexagon is inscribed in a circle. If the radius of the circle is 2, find the perimeter of the hexagon. 1.....

2 Find the area of an equilateral triangle whose side is 6. [Answer may be left in radical form.] 2.....

3 The perimeter of a rectangle is 20 and one of its sides is 6. Find the area of the rectangle. 3.....

4 Find the area of the circle inscribed in a square whose side is 8. [Answer may be left in terms of π.] 4.....

5 In triangle ABC, angle C equals 60° and an exterior angle at A is equal to 121°. Which is the longest side of the triangle? 5.....

6 In right triangle ABC, the hypotenuse AB equals 12. Find the length of the median drawn from C to AB. 6.....

7 Side BC of inscribed angle ABC is a diameter of the circle. If arc AB contains 100°, find the number of degrees in angle ABC. 7.....

8 A tangent is drawn to a circle at a vertex of an inscribed square. Find the number of degrees in one of the acute angles formed by the tangent and a side of the square. 8.....

9 D is the mid-point of side AC of triangle ABC and a line from D parallel to AB cuts BC at E. If $DE = 9$, find AB. 9.....

10 In triangle ABC, a line parallel to AC cuts AB at D and BC at E. $BD = 6$, $DA = 2$ and $BC = 12$; find BE. 10.....

11 The bases of an isosceles trapezoid are 6 and 14 and one of the base angles equals 45°. Find the altitude of the trapezoid. 11.....

12 An arc of a circle has a central angle of 45°. The radius of the circle is 12". Find the length of the arc. [Answer may be left in terms of π.] 12.....

13 Diameter AB of a circle is 13 and chord AC is 12. Find the value of cos A. 13.....

14 Find the area of a sector whose angle is 30° and whose radius is 6. [Answer may be left in terms of π.] 14.....

Directions (questions 15–19) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a*, *b* or *c*.

15 Which of the following sets may be used as the lengths of the sides of a triangle? (*a*) 5, 4, 9 (*b*) 5, 4, 6 (*c*) 5, 4, 10 15.....

16 If a point is equidistant from the sides of a triangle, it must be the intersection of the three (*a*) altitudes (*b*) medians (*c*) angle bisectors 16.....

17 If altitude CD is drawn upon the hypotenuse AB of right triangle ABC, then $(AC)^2$ equals (*a*) $CB \times DB$ (*b*) $AB \times AD$ (*c*) $DB \times AD$ 17.....

18 Which conclusion makes the following statement *false*? If the diagonals of a quadrilateral are perpendicular to each other, (*a*) the quadrilateral may be a rhombus (*b*) the quadrilateral may be a square (*c*) the quadrilateral must be a square 18.....

19 Which conclusion makes the following statement *false*? If the number of sides of a regular polygon inscribed in a circle is increased (*a*) the apothem decreases (*b*) each interior angle increases (*c*) the perimeter increases 19.....

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Directions (questions 20–23) — If the blank in each statement is replaced by one of the words *always*, *sometimes* or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

20 The diagonals of a trapezoid ... bisect each other. 20.....

21 If from point A on a circle, chord AB and tangent AP are drawn, then angle PAB is ... less than a right angle. 21.....

22 If two chords of a circle are perpendicular to a third chord, at its extremities, the two chords are ... equal. 22.....

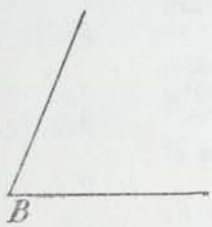
23 If two lines intersect, a line perpendicular to one of these lines will ... intersect the other line. 23.....

Directions (questions 24–25) — Leave all construction lines on your paper.

24 Given sides m and n and angle B, construct a parallelogram in the space at the right.

m

n



25 Construct the locus of all points outside circle O and at the distance d from the circle.

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