PLANE GEOMETRY

Tuesday, January 23, 1945 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in plane geometry, (c) author of textbook used.

The minimum time requirement is five recitations a week for a school year.

Part II

Answer two questions from part II.

26 Prove that if two chords intersect within a circle, the product of the segments of one is equal to the product of the segments of the other.   [10]

27 *ABCDE* is a regular pentagon with diagonals *AC, AD* and *BD* drawn. Prove that triangles *ABD* and *CDA* are congruent.   [10]

28 Prove that the area of a regular polygon is equal to one half the product of its perimeter and its apothem.   [10]

Part III

Answer two questions from part III.

29 The equal sides of an isosceles trapezoid are each 5 and its altitude is 4. If the area of the trapezoid is 48, find the bases.   [10]

30 Diagonals *AC* and *BD* of rhombus *ABCD* intersect at *O, BD* being the shorter diagonal. A line is drawn from *O* perpendicular to *AB*, meeting it at *X*. *AB* = *BD* = 12.
  a Find *OB, BX* and *OX*.   [1, 4, 2]
  b Find in terms of π the area of the circle that can be inscribed in the rhombus.   [3]

31 A belt moves over two equal wheels and crosses itself at right angles as shown in the figure. *A, B, C* and *D* are points of tangency, *E* is the intersection of the tangents and *O* is the center of one of the circles. The radius of each wheel is 7 inches.
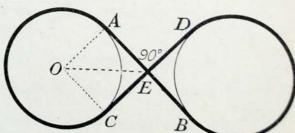
  a Show that angle *AOE* = 45°   [3]

  b Find
    (1) The length of *AE*   [2]
    (2) The length of major arc *AC*
        [Use $\pi = \frac{22}{7}$]   [3]
    (3) The length of the entire belt   [2]

[1]

Part IV
Answer one question from part IV.

32 Each of the following "definitions" is unsatisfactory because it contains either too little or too much information about the figure defined. In *each* case indicate whether too little or too much information is given and rewrite the definition in an acceptable form:

 a A parallelogram is a quadrilateral whose opposite sides are equal and parallel. [2½]

 b An isosceles trapezoid is a trapezoid in which two sides are equal. [2½]

 c An obtuse angle is an angle greater than a right angle. [2½]

 d An angle inscribed in a circle is one formed by two chords. [2½]

33 A straight road crosses a straight railroad at an angle of 60°. On the road, 40 miles from the crossing, a gun with a range of 37 miles is located. A train moving along the railroad track has just passed the crossing.

 a Find, correct to the *nearest mile,* the distance of the gun from the railroad. [3]

 b Show that the train will come within range of the gun. [2]

 c How far from the crossing will the train first come within range of the gun? [4]

 d At what point will the train pass out of range of the gun? [1]

[2]

Fill in the following lines:

Name of school.................................Name of pupil.................................

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 In triangle ABC, angle $C = 90°$ and angle $B = 35°$. Name the shortest side of this triangle.

1.....................

2 A side of an equilateral triangle is 10. Find its area. [Answer may be left in radical form.]

2.....................

3 A leg of an isosceles right triangle is 4. Find the hypotenuse. [Answer may be left in radical form.]

3.....................

4 In triangle ABC, a line joins D and E, the mid-points of AB and CB respectively. If angle $A = 40°$, how many degrees are there in angle BDE?

4.....................

5 The sides of one angle are parallel to the sides of another. If the angles are unequal and one angle is $50°$, how many degrees are there in the other angle?

5.....................

6 The altitude upon the hypotenuse of a right triangle divides the hypotenuse into segments 3 and 12. Find the altitude.

6.....................

7 The sides of a triangle are 8, 10 and 12. If the shortest side of a similar triangle is 6, find its longest side.

7.....................

8 From an external point P, tangent PD and secant PCA are drawn to a circle. If $PD = 8$ and $PCA = 16$, find PC.

8.....................

9 The circumferences of two circles are to each other as $5:4$ and the larger circle has a radius of 25. Find the radius of the smaller circle.

9.....................

10 Find the number of degrees in an interior angle of a regular octagon.

10.....................

11 If the ratio of two corresponding sides of two similar polygons is $1:2$, express the ratio of their areas.

11.....................

12 From point P outside a circle, two secants PAC and PBD are drawn. Angle $P = 26°$ and minor arc $DC = 144°$. Find the number of degrees in minor arc AB.

12.....................

13 The area of a rhombus is 54 and one of its diagonals is 12. Find the other diagonal.

13.....................

14 The angle of a sector of a circle is $72°$ and the area of the sector is 5π. Find the radius of the circle.

14.....................

15 In triangle ABC, angle $C = 90°$. $AB = 40$ and $BC = 25$. Find angle A correct to the *nearest degree*.

15.....................

Directions (questions 16–19) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a*, *b* or *c*.

16 The medians to sides AC and BC of scalene triangle ABC meet at point F. If CF is drawn and extended to meet AB at D, then CD (*a*)bisects angle C (*b*)bisects AB (*c*)is perpendicular to AB

16.......

17 The locus of points at a given distance from a given straight line is   (*a*)a circle   (*b*)one straight line   (*c*)two parallel lines

17.......

18 If the sides of a triangle are 6, 8 and 11, the triangle is   (*a*)right   (*b*)obtuse   (*c*)acute

18......

19 Two circles are externally tangent. The greatest number of common tangents which these circles can have is   (*a*)one   (*b*)three   (*c*)four

19......

[3]

Directions (questions 20–23) — Indicate whether *each* statement is true or false by writing the word *true* or the word *false* on the line at the right.

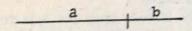**20** The area of a parallelogram whose angles are oblique is equal to the product of two of its adjacent sides.   20................

21 If parallelogram *ABCD* is inscribed in a circle, the sides *AB* and *DC* are equally distant from the center of the circle.   21................

22 There are two and only two circles of radius *r* which are tangent to each of two intersecting lines.   22................

23 The diagonals of an isosceles trapezoid bisect each other.   23................

Directions (questions 24–25) — Leave all construction lines on the paper.

24 Construct the mean proportional between line segments *a* and *b*.

a _____ | b _____

25 Construct a line through point *P* parallel to line *AB*.

.P

A _____ B

[4]