PLANE GEOMETRY

Tuesday, January 18, 1944 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used. The minimum time requirement is five recitations a week for a school year.
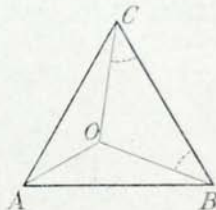
Part II

Answer two questions from part II.

26 *a* Prove that the sum of the interior angles of a triangle is a straight angle. [6]
  b Prove that the two acute angles of a right triangle are complementary. [4]

27 *a* For the following exercise, draw a figure, letter it and in terms of the figure state what is given and what is to be proved: [No proof is required in this part.] [3]

From any point in the base of an isosceles triangle perpendiculars are drawn to the equal sides. Prove that the sum of these perpendiculars is equal to the altitude drawn to one of the equal sides.

b Given triangle ABC with $AB = BC = CA$ and angle $OCB =$ angle OBC
Prove that AO bisects angle BAC. [7]

28 Prove that if from a point outside a circle a tangent and a secant are drawn to the circle, the tangent is the mean proportional between the secant and its external segment. [10]

Part III

Answer two questions from part III.

29 Nine circular disks each 3 inches in diameter were cut from a square of aluminum alloy 9 inches on a side. How much of the metal was wasted? Give your answer correct to the *nearest square inch.* [Use $\pi = \frac{22}{7}$] [10]

30 Tangent PC and secant PBA are drawn from point P to a circle and chords BC and AC are drawn. Angle $APC = 37°$ and arc $ABC = 236°$. Find the number of degrees in arc BC, angle BCP and angle ACB. [5, 2, 3]

31 The base of a parallelogram is 16 and one base angle is 36°. If the area of the parallelogram is 80, find, correct to the *nearest tenth,* the side adjacent to the base. [10]
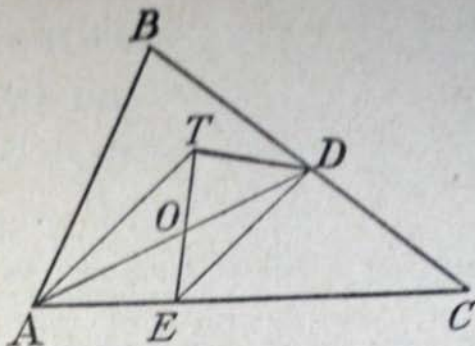
[1]

Part IV
Answer one question from part IV.

32 T is any point in triangle *ABC* and *D* is the mid-point of *BC*. Points *T* and *A* are joined and *DE* is drawn parallel to *TA*. Lines *TD*, *TE* and *AD* are drawn.

Prove:

   a Triangle *ADC* = triangle *ADB* [3]
   b Triangle *AED* = triangle *ETD* [4]
   c Triangle *AOE* = triangle *TOD* [3]

33 In each of the following statements either too much or too little information has been given to justify the conclusion. Rewrite *each* statement so that the information given is just enough to justify the conclusion.

   a If the altitude and one side of an equilateral triangle are given, the triangle can be constructed. [2]
   b If the mid-points of the sides of a parallelogram are joined in order, the quadrilateral formed is a rhombus. [2]
   c If the angles of a triangle are in the ratio 1:2:3 and one of the angles is 30° more than another, the angles can be determined. [2]
   d If a teeterboard (seesaw) is 16 feet long and is supported at its center by a block 3 feet high, the height to which each end of the board can rise can be determined. [2]
   e If two circular disks each 1 inch in diameter and $\frac{1}{16}$ inch thick are melted and made into a single circular piece of metal of the same thickness, the ratio of the diameter of the new piece to the diameter of one of the given disks is $\sqrt{2}:1$ [2]

Fill in the following lines:

Name of school...Name of pupil...

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 In parallelogram $ABCD$, angle B is twice as large as angle A. How many degrees are there in angle A?

1................

2 In right triangle ABC, altitude CD is drawn to hypotenuse AB. If angle A contains $32°$, how many degrees are there in angle BCD?

2................

3 How many degrees are there in each angle of a regular polygon of 8 sides?

3................

4 Angle ABC, formed by diameter AB and chord BC of a circle, contains $30°$. If the diameter of the circle is 10, find chord AC.

4................

5 A line parallel to base AB of triangle ABC intersects the sides AC and BC at points D and E respectively. If $CD = 4$, $DA = 6$ and $CE = 8$, find EB.

5................

6 The area of a triangle is 24. If one side of the triangle is 12, find the altitude drawn to that side.

6................

7 Find the ratio of the areas of two similar polygons whose corresponding sides are in the ratio $2:3$.

7................

8 The area of a circle is 49π. Find its circumference. [Answer may be left in terms of π.]

8................

9 Find the area of a square circumscribed about a circle whose radius is 10.

9................

10 In triangle ABC, angle $C = 90°$, $BC = 3$, $AC = 4$; find angle A correct to the *nearest degree*.

10................

Directions (questions 11–17) — If the blank in each statement is filled by one of the words *always, sometimes* or *never,* the resulting statement will be true. Select the word that will correctly complete *each* statement and write the word on the line at the right.

11 Triangles that have equal bases and equal altitudes are ... congruent.

11................

12 If angle B of triangle ABC contains $95°$, then it is ... possible to determine which is the longest side of the triangle.

12................

13 The altitude and the median are drawn from the same vertex of a triangle. The altitude is ... greater than the median.

13................

14 The center of the circle which is circumscribed about a triangle ... lies inside the triangle.

14................

15 If the perpendiculars drawn from the center of a circle upon two chords are equal, then the minor arcs subtended by these chords are ... equal.

15................

16 Tangents drawn from an external point to a circle ... make equal angles with the chord joining the points of tangency.

16................

17 If the dimensions of a rectangle are doubled, then its area is ... doubled.

17................

Directions (questions 18–23) — Indicate the correct answer to each question by writing on the line at the right the letter a, b, c or d.

18 Two angles that are both equal and supplementary are (a) adjacent, (b) acute, (c) right, (d) obtuse.

18..................

19 The mid-point of the hypotenuse of a right triangle is the point of intersection of (a) the three medians, (b) the bisectors of the three angles, (c) the perpendicular bisectors of the three sides, (d) the three altitudes.

19..................

20 If the bases of a trapezoid are b and b', then the median of the trapezoid, in terms of b and b', is (a) $\frac{1}{2} bb'$, (b) $\frac{1}{2}(b + b')$, (c) $b + b'$, (d) $2(b + b')$

20..................

21 If acute triangle ABC is inscribed in a circle and angle B is greater than angle A, then arc BC is (a) greater than arc AC, (b) equal to arc AC, (c) less than arc AC.

21..................

22 The locus of the center of a circle having a given line segment as a chord is (a) a line parallel to the chord, (b) the mid-point of the chord, (c) the perpendicular bisector of the chord, (d) a circle with the chord as diameter.

22..................

23 The area of a sector of n degrees in a circle of radius r is

(a) $\frac{n}{360} \times \pi r^2$, (b) $\frac{n}{180} \times \pi r^2$, (c) $\frac{n}{180} \times \pi r$, (d) $\frac{n}{360} \times \pi d^2$

23..................

Directions (questions 24–25) — Leave all construction lines on the paper.

24 Construct a circle of radius r that will be tangent to line m at point P.

r

m P
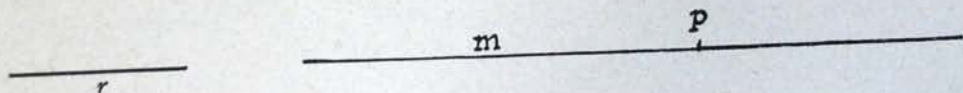
25 Find the point within angle CBA, equidistant from sides BC and BA and at the given distance d from vertex B.

d

A

B C

[4]