The University of the State of New York

286TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Tuesday, January 19, 1943—9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for a school year.

Part II

Answer two questions from part II.

26 Prove that if the opposite sides of a quadrilateral are equal the figure is a parallelogram. [10]

27 In triangle ABC, angle C is obtuse. AE is the altitude on BC and CD is the altitude on AB.

a Prove that $BC : AB = CD : AE$ [8]

b Would this proportion hold true if angle C were acute? [2]

28 Prove that the area of a trapezoid is equal to one half the product of its altitude and the sum of its bases. [10]

Part III

Answer two questions from part III.

29 Sides AB and BC of parallelogram $ABCD$ are 10 and 6 respectively. The bisector of angle A meets DC at point E and BC extended at point F.

a Prove that triangle ADE is isosceles. [4]

b Prove that triangle ADE is similar to triangle ECF. [3]

c Find the length of CF. [3]

30 PA and PB are tangents to circle O and intersect at an angle of 60 degrees. Radius OA is 8 inches long.

a Find, correct to the *nearest inch*, the length of AP. [6]

b Find the length of minor arc AB. [Answer may be left in terms of π.] [4]

31 A regular pentagon is circumscribed about a circle whose radius is 10.

a Find a side of the pentagon correct to the *nearest tenth*. [6]

b Using the result obtained in answer to *a*, find the area of the pentagon correct to the *nearest integer*. [4]

/5-51

Part IV

Answer one question from part IV.

32 For each of the following statements indicate whether you have been given too little information, just enough information or more information than is needed to justify the conclusion.

a The areas of two isosceles triangles are to each other as the squares of any two corresponding sides. [2]

b If two chords of a circle are equal and parallel, then the chord passing through their mid-points is a diameter. [2]

c If the drive wheels on a locomotive are each 60 inches in diameter and each makes 168 revolutions per minute, then the speed of the locomotive can be determined. [2]

d A boy said he had just 45 cents in his pocket, consisting of 5 coins one of which was a nickel. His friend concluded that one of the coins must be a quarter. [2]

e The entrance requirements of a certain college are: an average grade of not less than 80% in science, mathematics and English, and a grade of not less than 90% in one of them. Therefore, a candidate for admission who has an average of 80% in the required subjects and whose grade in English is 60% meets these requirements. [2]

33 Tangents PA and PB are drawn to a circle. Diameter AD and chords AB and DB are drawn. Let the number of degrees in minor arc AB be represented by x.

a Express in terms of x the number of degrees in

(1) Major arc AB [1]

(2) Minor arc DB [1]

(3) Angle ADB [1]

(4) Angle APB [2]

b As x decreases,

(1) Does angle APB increase, decrease or remain the same? [2]

(2) Does the ratio of angle DAB to angle APB increase, decrease or remain the same? [3]

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–10) — Write on the line at the right of *each* question the expression which when inserted in the blank will make the statement true.

1 If two chords of a circle bisect each other, the chords must be ... of the circle. 1.....

2 Chords AB and CD intersect within a circle at point E. If $AE = 3$, $EB = 12$ and $CE = 4$, then ED equals .... 2.....

3 In a circle a chord 8 inches long is 3 inches from the center. The number of inches in the length of the radius is .... 3.....

4 From a point P outside a circle, a secant PCA is drawn. If $PC = 5$ inches and $PA = 20$ inches, then the length of a tangent drawn to the circle from P is ... inches. 4.....

5 The sides of a triangle are 5, 6 and 7 and the perimeter of a similar triangle is 36. The side of the second triangle corresponding to side 5 of the first is .... 5.....

6 The area of a rhombus in terms of its diagonals d and d' is .... 6.....

7 The area of an equilateral triangle whose perimeter is 12 is .... [Answer may be left in radical form.] 7.....

8 Triangle ABC is inscribed in a circle. Point P bisects minor arc AB and chord BP is drawn. If angle ABP is 40 degrees, the number of degrees in angle C is .... 8.....

9 An architect uses a scale of $\frac{1}{4}$ inch to a foot. If the dimensions of a room as represented on the plans of a house are $4\frac{1}{4}$ inches by $3\frac{3}{4}$ inches, the actual dimensions in feet are .... 9.....

10 If the angle of a sector is 30 degrees and the radius of the circle is 12, the area of the sector is .... [Answer may be left in terms of π.] 10.....

Directions (questions 11–15) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a*, *b* or *c*.

11 Two figures must be similar if they are (*a*) rectangles, (*b*) equilateral triangles or (*c*) rhombuses. 11.....

12 If two circles have a common chord, then the sum of their radii is (*a*) greater than, (*b*) equal to or (*c*) less than, the distance between their centers. 12.....

13 In triangle ABC, side CA is greater than side CB and angle B equals 35 degrees. Angle C is (*a*) an acute angle, (*b*) a right angle or (*c*) an obtuse angle. 13.....

14 If a median of a triangle is equal to one half the side to which it is drawn, the triangle is (*a*) acute, (*b*) obtuse or (*c*) right. 14.....

15 Below are given a proposition and one of its converses.

Proposition: The line which bisects the vertex angle of an isosceles triangle is perpendicular to the base and bisects the base.

Converse: The line from the vertex of an isosceles triangle which is perpendicular to the base bisects the base and also the vertex angle.

a Both the proposition and the converse are true.
b The proposition is true and the converse is false.
c Both the proposition and the converse are false. 15.....

/57

Directions (questions 16–23) — Indicate whether *each* statement is true or false by writing the word *true* or *false* on the line at the right.

16 The altitude upon any side of a triangle is greater than the median drawn to that side. 16.....

17 A parallelogram is defined as a polygon whose opposite sides are parallel. 17.....

18 Sectors of unequal circles may be congruent. 18.....

19 The perpendicular bisector of line segment AB is the locus of the centers of all circles passing through A and B. 19.....

20 If a secant and a tangent to a circle are parallel, the diameter drawn to the point of tangency is perpendicular to the secant. 20.....

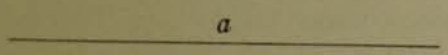
21 If the altitude CD is drawn upon the hypotenuse AB of right triangle ABC, the angle ACD equals the angle CBD. 21.....

22 A diagonal of an isosceles trapezoid divides the trapezoid into two congruent triangles. 22.....

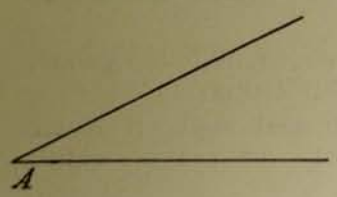
23 If the mid-points of the sides of a rhombus are joined in order, the figure thus formed is a rectangle. 23.....

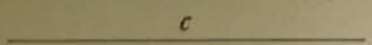
Directions (questions 24–25) — Leave all construction lines on the paper.

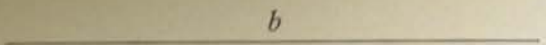
24 Given line segment a and fixed point P. Construct the locus of points whose distance from P is $\frac{1}{2}a$.



25 Construct triangle RST in which angle R = angle A, $RS = c$ and $RT = b$







158