The University of the State of New York

280TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Tuesday, January 21, 1941 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Part I

This part is to be done first and the maximum time allowed for it is one and one half hours.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Parts II, III and IV

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for a school year.

157

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–13) — Write on the dotted line at the right of each question the expression which, when inserted in the corresponding blank, will make the statement true.

1 The area K of a regular polygon whose apothem is a and whose perimeter is p is given by the formula $K =$ .... 1.....

2 Chords AB and CD intersect within a circle at point E. If $AE =$ 4 inches, $EB = 6$ inches and $CE = 3$ inches, then the length of ED is ... inches. 2.....

3 An exterior angle at the base of an isosceles triangle contains 130°. The vertex angle of the triangle contains ... degrees. 3.....

4 PA and PB are tangents to circle O from point P and chord AB is drawn. If angle $APB = 60°$ and $PA = 8$, then chord $AB =$ .... 4.....

5 A line parallel to base AB of triangle ABC intersects AC at D and BC at E. If $CD = 4$, $DA = 2$ and $BE = 3$, then $CE =$ .... 5.....

6 From point P outside circle O tangent PD and secant PCA are drawn. If secant $PA = 16$ and chord $AC = 12$, then tangent $PD =$ .... 6.....

7 If the area of a trapezoid is 98 and the two parallel sides are 12 and 16, then the altitude is .... 7.....

8 If a and b are the sides of a parallelogram and h is the altitude on side b, the formula for the area K of the parallelogram is $K =$ .... 8.....

9 An inscribed angle and a central angle intercept the same arc of a circle. The ratio of the inscribed angle to the central angle is .... 9.....

10 If the hypotenuse of a right triangle is 25 and one leg is 24, the other leg is .... 10.....

11 A vertical pole standing on a horizontal plane casts a shadow 20 feet long at a time when the angle of elevation of the sun is 43°. The height of the pole correct to the *nearest foot* is .... 11.....

12 If two corresponding sides of similar polygons are in the ratio 1 : 3, then the area of the larger polygon is ... times the area of the smaller polygon. 12.....

13 The area K of an equilateral triangle whose side is s is given by the formula $K =$ .... 13.....

Directions (questions 14–18) — Indicate the correct answer to each question by writing on the dotted line at the right the letter a, b or c.

14 Two triangles are always congruent if (a) three angles of one are equal respectively to three angles of the other, (b) two sides and an angle of one are equal respectively to two sides and an angle of the other or (c) three sides of one are equal respectively to three sides of the other. 14.....

15 Every triangle is divided into two triangles that are equal in area by (*a*) a median, (*b*) a bisector of one of its angles or (*c*) an altitude. 15.....

16 The sum of the exterior angles of a quadrilateral made by extending each of its sides in order is (*a*) equal to, (*b*) less than or (*c*) greater than the sum of its interior angles. 16.....

17 The locus of the mid-points of all chords drawn from a given point on a circle is (*a*) a diameter of the circle, (*b*) a circle concentric with the given circle or (*c*) a circle whose diameter is the radius of the given circle. 17.....

18 All residents of Albany are residents of New York State. Mr Brown is a resident of New York State and therefore is a resident of Albany. This reasoning (*a*) is sound and is an example of indirect reasoning, (*b*) is unsound and is an example of indirect reasoning or (*c*) is unsound and is an example of reasoning from a converse. 18.....

Directions (questions 19–23) — Indicate whether each statement is *always* true, *sometimes* true or *never* true by writing on the line at the right the word *always*, *sometimes* or *never*.

19 The bisectors of the angles of a triangle meet in a point which is equidistant from the three sides of the triangle. 19.....

20 If two arcs of a circle are equal, the chords of these arcs are parallel. 20.....

21 Two right triangles inscribed in the same or equal circles are congruent. 21.....

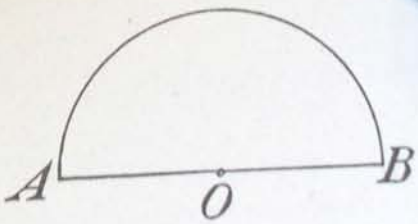
22 If the perpendicular bisectors of the sides of a triangle intersect on a side of the triangle, the triangle is a right triangle. 22.....

23 A parallelogram inscribed in a circle has two acute angles. 23.....

Directions (questions 24–25) — Leave all construction lines on the paper.

24 Locate by construction a point within the angle ABC which is equidistant from the sides of the angle and at a given distance d from the vertex of the angle.

25 Using the diameter AB of the semicircle as a base, inscribe in the semicircle a triangle whose area shall be greater than the area of any other triangle that can be inscribed in the semicircle.



160

See instructions for parts II, III and IV on page 1.

Part II

Answer two questions from this part.

26 Prove:

a A diagonal divides a parallelogram into two congruent triangles. [6]

b The opposite sides and the opposite angles of a parallelogram are equal. [4]

27 Prove that if the median AD of triangle ABC is greater than BD, angle BAC is less than the sum of the angles B and C. [10]

28 Prove:

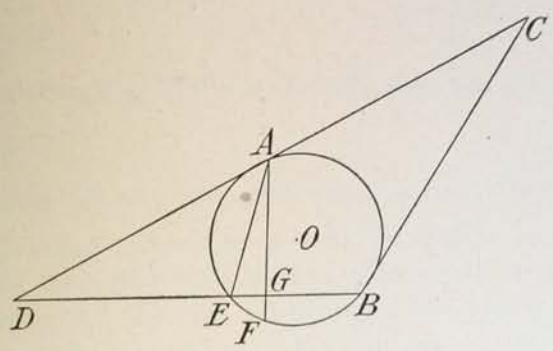
a The altitude on the hypotenuse of a right triangle divides the triangle into two similar triangles. [6]

b The altitude on the hypotenuse is a mean proportional between the segments of the hypotenuse. [4]

Part III

Answer two questions from this part.

29 In the figure at the right CD and CB are tangent to circle O at points A and B respectively. Secant DB intersects the circle at E, chord AE is drawn and chord AF intersects DB at G. Arc $EF = 30°$, arc $FB = 70°$ and arc $AE = 118°$. Find the number of degrees in arc AB and in angles EAF, FGB, DAF and ACB. [10]



30 Angle A of the rhombus $ABCD$ contains 60° and the diagonals intersect at O. The radius of the circle inscribed in the rhombus is 5 inches. Find

a The circumference of the circle [2]

b The area of the circle [2]

c The length of AO [6]

[Answers to *a* and *b* may be left in terms of π.]

31 In circle O chord AC forms an angle of 37° with diameter AB. If the length of AB is 20 inches, find, correct to the *nearest inch*

a The length of the chord BC [7]

b The distance from the center of the circle to chord AC [3]

Part IV

Answer one question from this part.

32 *a* A pupil proved that a point was not outside a circle and hence concluded that it was inside the circle. Was his conclusion sound? Justify your answer. [4]

b Explain briefly what is meant by *indirect proof*. [4]

c Quote *one* proposition from your text for which an indirect proof is used. [2]

33 In parallelogram $ABCD$, E is the mid-point of AD and F is the mid-point of BC. BE, DF and AC are drawn. Prove that BE and DF divide AC into three equal parts. [10]

158