The University of the State of New York

277TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Tuesday, January 23, 1940 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II, III and IV

Write at top of first page of answer paper to groups II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for a school year.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–13) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 An inscribed angle of 60° intercepts an arc of ... degrees. 1.....

2 If the hypotenuse of a right triangle is twice the shorter leg, the smallest angle of the triangle contains ... degrees. 2.....

3 In an isosceles right triangle the number of degrees in either acute angle is .... 3.....

4 If the areas of two similar polygons are in the ratio 4:9, then any two corresponding sides of the polygons are in the ratio .... 4.....

5 The area K of a regular polygon whose apothem is a and whose perimeter is p is given by the formula $K =$ .... 5.....

6 The diagonal of a square whose side is 2 is .... [Answer may be left in radical form.] 6.....

7 If the circumference of a circle is 10π, its radius is .... 7.....

8 The sum of the interior angles of a polygon of *five* sides is ... degrees. 8.....

9 If the altitude upon the hypotenuse of a right triangle divides the hypotenuse into two segments whose lengths are 4 and 16, then the length of the altitude is .... 9.....

10 If the side of an equilateral triangle is 6 feet, the area of the triangle is ... square feet. [Answer may be left in radical form.] 10.....

11 In a circle whose circumference is 24 inches, the length of an arc of 30° is ... inches. 11.....

12 Given triangle ABC with a line drawn parallel to AC intersecting AB at D and CB at E; if $AB = 8$, $BC = 12$ and $BD = 6$, then $BE =$ .... 12.....

13 If in right triangle ACB angle $C = 90°$, angle $A = 66°$ and $AC = 100$, then BC, correct to the *nearest integer*, is .... 13.....

Directions (questions 14–17) — Indicate whether each statement is true or false by writing the word *true* or *false* on the dotted line at the right.

14 When two straight lines are cut by a transversal, if the interior angles on the same side of the transversal are supplementary, the two lines are parallel. 14.....

15 If two sides of a triangle are equal, one median of this triangle is perpendicular to the side to which it is drawn. 15.....

16 Two parallelograms are always congruent if they have equal bases and equal altitudes. 16.....

17 If the diagonals of a parallelogram are equal and perpendicular to each other, the parallelogram is always a square. 17.....

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Directions (questions 18–22) — Indicate the correct answer to each question by writing on the dotted line at the right the letter *a*, *b* or *c*.

18 If angles *A* and *B* of triangle *ABC* contain 59° and 61° respectively, then the longest side of the triangle is (*a*) *AB*, (*b*) *BC* or (*c*) *AC*. 18.....

19 A polygon is always regular if it is (*a*) equilateral, (*b*) equiangular or (*c*) equilateral and equiangular. 19.....

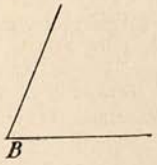
20 If the sum of two angles is an obtuse angle, one of the two angles must be (*a*) acute, (*b*) right or (*c*) obtuse. 20.....

21 The center of a circle circumscribed about a triangle lies within the triangle if the triangle is (*a*) acute, (*b*) right or (*c*) obtuse. 21.....

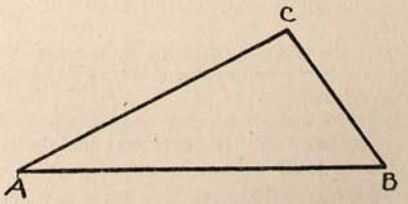
22 The locus of the vertex of all triangles which have the same base and whose medians to that base are all equal is (*a*) a straight line, (*b*) a pair of straight lines or (*c*) a circle. 22.....

Directions (questions 23–25) — Leave all construction lines on the paper.

23 Construct an angle equal to angle *B*.



24 Find by construction the center of the inscribed circle of triangle *ABC*.



25 Construct a circle of radius *r* which shall pass through the two given points *A* and *B*.

r

.*A*

.*B*

See instructions for groups II, III and IV on page 1.

Group II

Answer two questions from this group.

26 Prove that an angle formed by two chords intersecting within a circle is measured by one half the sum of the intercepted arcs. [10]

27 On diagonal AC of parallelogram $ABCD$ equal line segments AE and CF are laid off and DE, EB, BF and FD are drawn. Prove that $DEBF$ is a parallelogram. [10]

28 Prove that two triangles are similar if their sides are respectively proportional. [10]

Group III

Answer two questions from this group.

29 A chord AB of circle O is 10 inches long and is 5 inches from the center of the circle. Radii OA and OB are drawn. Find:

a The number of degrees in angle AOB [2]

b The radius of the circle [Answer may be left in radical form.] [2]

c The area of the triangle AOB [2]

d The area of the minor sector of the circle [Answer may be left in terms of π.] [2]

e The area of the minor segment of the circle [Answer may be left in terms of π.] [2]

30 The sides of a parallelogram are 10 and 14 and the area is 84.

a Find, correct to the *nearest degree*, an acute angle of the parallelogram. [8]

b If the sides of the parallelogram remain the same but the area decreases, do the acute angles decrease or increase? [2]

31 P is one of the two points of intersection of two circles O and O'. A tangent to O' at P intersects O at R and a tangent to O at P intersects O' at Q. Radii OP, OR, $O'P$ and $O'Q$ are drawn.

a Prove that angle OPR equals angle $O'PQ$. [4]

b Prove that triangles OPR and $O'PQ$ are similar. [3]

c If $PR = 24$, $PQ = 42$, $OP = 8$, find $O'P$. [3]

Group IV

Answer one question from this group.

32 A triangle whose base is b and whose altitude is h is equal to a trapezoid whose bases are r and s and whose altitude is x.

a Express x in terms of b, h, r, and s. [6]

b If b equals r, does x equal $\frac{h}{s}$? [2]

c If b equals $\frac{1}{2}(r + s)$, is h less than, equal to, or greater than x? [2]

33 *a* Explain in detail why the following statement should not be used as the definition of a parallelogram: A *parallelogram* is a quadrilateral whose opposite sides are equal and parallel. [3]

b Is it necessary to leave some terms in geometry undefined? [1]

c Explain in detail what is meant by "sequence in definition," and illustrate by arranging in proper sequence the following terms: quadrilateral, rectangle, polygon, parallelogram. [2, 2]

d Explain in detail why *both* of the following statements are not used as definitions in the same text book in geometry: [2]

A *right angle* is an angle whose sides are perpendicular.

Perpendicular lines are lines which intersect at right angles.