The University of the State of New York

268TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Tuesday, January 19, 1937 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for a school year.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–9) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 If two parallel lines are cut by a transversal, the interior angles on the same side of the transversal must be ... *Ans*.....

2 One angle of a right triangle is 60°. If the shortest side of the triangle is 8 inches, the hypotenuse is ... inches. *Ans*.....

3 The sum of the interior angles of a polygon of five sides is ... degrees. *Ans*.....

4 In a right triangle, the segments of the hypotenuse made by drawing the altitude upon the hypotenuse are 2 inches and 8 inches. The length of this altitude is ... inches. *Ans*.....

5 A point is 6 inches from a circle whose radius is 9 inches. The length of the tangent from this point to the circle is ... inches. *Ans*.....

6 The area of a square inscribed in a circle whose diameter is 10 inches is ... square inches. *Ans*.....

7 A cross section of an irrigation ditch has the form of a trapezoid. The upper base of the trapezoid is 30 feet, the lower base is 12 feet and the height is 8 feet. The area of the cross section is ... square feet. *Ans*.....

8 Two circles have radii of 5 feet and 12 feet respectively. The radius of a circle whose area is equal to the sum of the areas of these two circles is ... feet. *Ans*.....

9 Point P lies between two parallel lines a and b, which are 3 inches apart. The number of points that are equidistant from a and b and 2 inches from P is ... *Ans*.....

Directions (questions 10–13) — Indicate the correct answer to each of the following questions by writing the letter a, b or c in the space at the right.

10 A circle can always be circumscribed about a quadrilateral if the opposite angles of the quadrilateral are (a) complementary, (b) supplementary or (c) equal. *Ans*.....

11 Two circles are drawn so that they have four common tangents. The line segment joining the centers is (a) equal to the sum of the radii, (b) greater than the sum of the radii or (c) less than the sum of the radii. *Ans*.....

[OVER]

12 Corresponding sides in two similar polygons are in the ratio 1:4. The area of the larger polygon is (*a*) twice, (*b*) four times or (*c*) sixteen times, the area of the smaller polygon. *Ans*.....

13 A regular polygon and a triangle have equal areas. If the perimeter of the polygon is 10 and the base of the triangle is 8, the apothem of the polygon is (*a*) equal to, (*b*) greater than or (*c*) less than, the height of the triangle. *Ans*.....

Directions (questions 14–18) — Indicate whether each of the following statements is *always* true, *sometimes* true or *never* true by writing the word *always, sometimes* or *never* on the dotted line at the right.

14 If two rectangles have equal bases, their areas are to each other as their altitudes. *Ans*.....

15 If the three sides of a triangle are unequal, the altitude upon any side is equal to the median upon that side. *Ans*.....

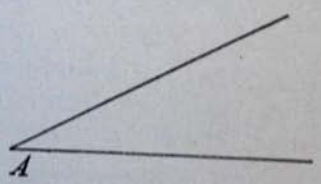
16 Similar triangles inscribed in the same circle or in equal circles have their corresponding sides equal. *Ans*.....

17 If a circle is circumscribed about a triangle, the center of the circle lies inside the triangle. *Ans*.....

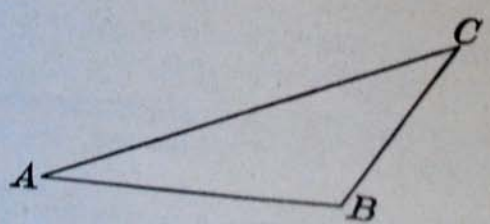
18 If an altitude of an equilateral triangle is represented by $3x$, then x represents the radius of the inscribed circle. *Ans*.....

Directions (questions 19–20) — Leave all construction lines on the paper.

19 Construct the locus of the centers of circles each of which is tangent to both sides of the given angle A.



20 Construct the altitude of triangle ABC upon side AB.



See instructions for groups II and III on page 1.

Group II

Answer three questions from this group.

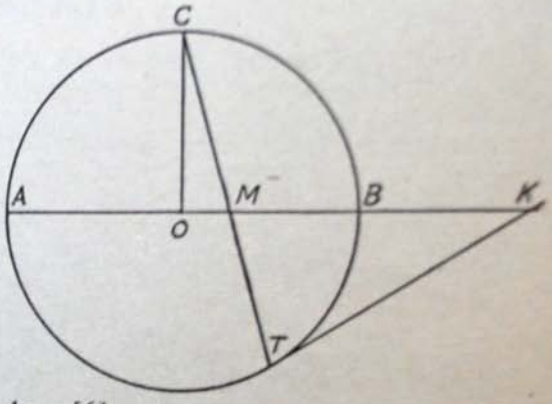
21 Prove that if the opposite sides of a quadrilateral are equal, the figure is a parallelogram. [10]

22 Prove that if two triangles have an angle of one equal to an angle of the other and the sides including these angles proportional, the triangles are similar. [10]

23 Given the isosceles triangle ABC with AB and AC the equal sides, D a point on AB, and AF the altitude upon BC. A line through D perpendicular to AB meets BC, extended if necessary, at E.

Prove: $\frac{FC}{BD} = \frac{AC}{BE}$ [10]

24 Given a circle whose center is O with AB a diameter and OC a radius perpendicular to AB; a line is drawn from C through M, any point on AB, cutting the circle at T. A tangent to the circle at T meets the diameter AB extended at K. Prove that KM and KT are equal. [10]



25 *a* Transform a given quadrilateral into a triangle. [6]

b Transform the triangle obtained in answer to *a* into an isosceles triangle. [4]

[Show all construction lines in both *a* and *b*.]

Group III

Answer two questions from this group.

26 The perimeter of the parallelogram $ABCD$ is 10 feet and the angle A is 45°. The altitude on base AB is represented by x.

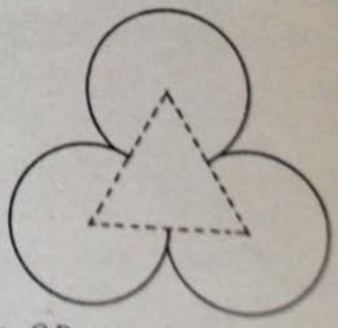
a Express the side AD in terms of x. [2½]

b Express AB in terms of x. [2½]

c Express the area K of the parallelogram in terms of x. [2½]

d Find, correct to the *nearest square foot*, the value of K if the value of x is 2 feet. [2½]

27 A church window has the form of a trefoil as shown in the figure. The triangle used in the construction is equilateral, the vertices of the triangle are the centers of the circular arcs and the radius of each arc is one half the side of the triangle. The side of the triangle is 8 inches. Find (*a*) the perimeter of the figure [3], (*b*) the area of the figure [7]. [Answers may be left in terms of π and radicals.]



28 AB is a diameter of a circle whose center is O. On OB extended a point P is taken 10 inches from O. Through P a secant is drawn intersecting the circle at C and D so that the arc $BC = 10°$ and the arc $AD = 60°$. Find, correct to the *nearest tenth of an inch*, the distance of the secant from the center of the circle. [Use numerical trigonometry.] [10]