The University of the State of New York

265TH HIGH SCHOOL EXAMINATION

PLANE GEOMETRY

Tuesday, January 21, 1936 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for a school year.

See instructions for groups II and III on page 1.

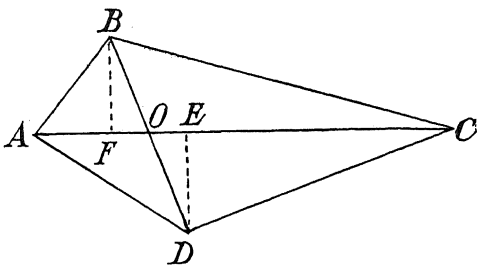
Group II

Answer three questions from this group.

21 Prove that an angle formed by a secant and a tangent, intersecting outside a circle, is measured by one half the difference of the intercepted arcs. [10]

22 Prove that the area of a trapezoid is equal to one half the product of its altitude and the sum of its bases. [10]

23 If diagonal AC of quadrilateral $ABCD$ bisects diagonal BD, then triangle ABC is equal in area to triangle ADC.



Given: Quadrilateral $ABCD$ with diagonal AC bisecting diagonal BD

To prove: $\triangle ABC = \triangle ADC$

Construction: Draw DE and BF each $\perp AC$

Below are given the statements for a proof of the above theorem but they are not arranged in logical order. *Do not copy the above material.* Rearrange the statements in logical order on your answer paper and assign a reason for *each* statement. [10]

$\angle BFO = \angle DEO$	$\angle BFO$ and $\angle DEO$ are right angles
$\triangle FOB \cong \triangle EOD$	$AC = AC$
$\triangle ABC = \triangle ADC$	$OB = OD$
$\angle FOB = \angle EOD$	$BF = DE$

24 On sides BA and BC of any triangle ABC, equilateral triangles BAE and BCD are constructed, both lying outside the triangle. If AD and CE are drawn, prove that these two lines are equal. [10]

25 AB is the hypotenuse of a right triangle ABC; a perpendicular to AB at A meets BC extended at E; a perpendicular to AB at B meets AC extended at D.
Prove that $AE : AB = BC : CD$ [10]

Group III

Answer two questions from this group.

Leave all work on the paper; merely writing the answers is not sufficient. Irrational results may be left in the form of π and radicals unless otherwise stated.

26 A circle and an equilateral triangle each have a perimeter of 132 feet. Find, correct to the *nearest square foot*, the difference between their areas. [Use $\pi = \frac{22}{7}$ and $\sqrt{3} = 1.73$] [10]

27 The diagonals of a rhombus are 18 and 30. Find one of the angles of the rhombus correct to the *nearest degree*. [Use numerical trigonometry.] [10]

28 Given the rectangle $ABCD$ with $AB = 2$ inches and $AD = 3$ inches

a Explain how you would find the locus of points equidistant from AB and AD. [4]

b Explain how you would find the locus of points $1\frac{1}{2}$ inches from C. [4]

c How many points are there within the rectangle that satisfy both conditions given in *a* and *b*? [2]

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 The apothem of a regular polygon is 3 inches and its perimeter is 32 inches; find the area of the polygon in square inches. *Ans*.....

2 The hypotenuse of a right triangle is 18 inches and one of its angles is 30°; find the length of the side opposite the 30° angle. *Ans*.....

3 If the vertex angle of an isosceles triangle is 70°, find the number of degrees in each base angle. *Ans*.....

4 An angle formed by a tangent to a circle and a chord drawn from the point of contact is 70°; find the number of degrees in the minor intercepted arc. *Ans*.....

5 In a given circle, chord MN is perpendicular to chord RS. How many degrees are there in the sum of arcs RN and MS? *Ans*.....

6 In triangle ABC, angle $C = 90°$, $AB = 15$ feet and $BC = 9$ feet; find angle A correct to the *nearest degree*. *Ans*.....

7 In triangle ABC, the altitudes AD and BE are drawn and intersect at F. What triangle is similar to triangle AFE? *Ans*.....

8 A sector of a circle has an angle of 30°. If the area of the circle is 24, what is the area of the sector? *Ans*.....

9 In triangle ABC, angle $B = 95°$; which is the longest side of the triangle? *Ans*.....

10 The locus of points at a given distance from a given straight line is (a) a circle, (b) one straight line, (c) two parallel lines or (d) *two* intersecting lines. Which is correct, (a), (b), (c) or (d)? *Ans*.....

11 The altitude on the hypotenuse of any right triangle divides it into two triangles which are (a) equal, (b) similar or (c) congruent. Which is correct, (a), (b) or (c)? *Ans*.....

12 As the number of sides of a regular polygon increases, each exterior angle (a) increases, (b) decreases or (c) remains the same. Which is correct, (a), (b) or (c)? *Ans*.....

13 The area of a square is 81 square inches; find the length of its diagonal. [Answer may be left in radical form.] *Ans*.....

14 From a point on a circle whose diameter is 10 inches long two chords are drawn to the ends of a diameter. If one chord is 8 inches long, how long is the other chord? *Ans*.....

15 In triangle ABC, $BC = 10$ inches, $AC = 15$ inches. A line parallel to AC and terminating in the other two sides is 12 inches long. How long is the longer segment of BC? *Ans*.....

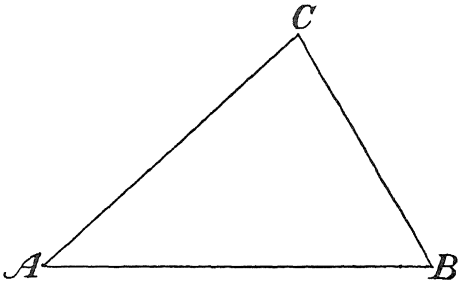
16 If the mid-points of the sides of any quadrilateral are joined in order, the figure thus formed must be (a) a square, (b) a rhombus, (c) a rectangle or (d) a parallelogram. Which is correct, (a), (b), (c) or (d)? *Ans*.....

17 If the corresponding altitudes of two similar triangles are 2 inches and 3 inches, find the ratio of the area of the smaller triangle to that of the larger triangle. *Ans*.....

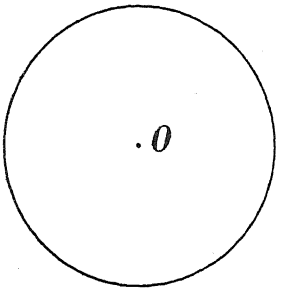
[OVER]

Directions (questions 18–20) — Leave all construction lines on the paper.

18 In triangle ABC, construct the altitude on side AB.



19 Inscribe a square in circle O.



20 AB is the perimeter of an equilateral triangle. Find by construction the length of one side.