*The University of the State of New York*

212TH HIGH SCHOOL EXAMINATION

# PLANE GEOMETRY

**Tuesday,** January 19, 1915—9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in plane geometry. The minimum time requirement is five recitations a week for a school year. Name the author of the textbook you have used in your study of plane geometry.

*Answer eight questions, including question 13.*

1 Prove that the exterior angle of a triangle is equal to the sum of the two opposite interior angles. [12]

2 Prove that the diameter perpendicular to a chord bisects the chord and also its subtended arc. [12]

3 Prove that two similar polygons may be divided into the same number of triangles similar each to each and similarly placed. [12]

4 Prove that in any right triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides. [12]

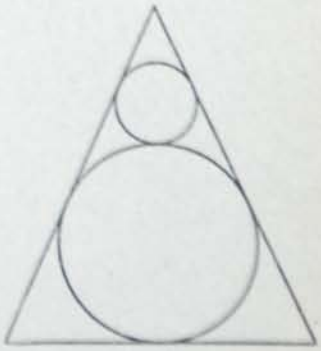
5 For *each* of the following propositions, draw the figure and state the hypothesis and the conclusion in terms of the letters on the figure:

*a* The internal common tangents of two non-intersecting circles meet on their line of centers. [6]

*b* The square of the bisector of an angle of a triangle is equal to the product of the sides of this angle diminished by the product of the segments of the third side made by the bisector. [6]

6 A circle is inscribed in an isosceles trapezoid whose bases are 6 and 18. Find the area of the portion of the trapezoid not included in the circle. [First find the side of the trapezoid.] [12]

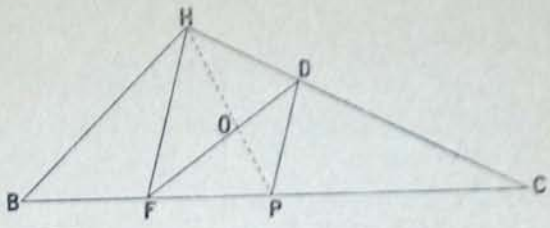
7 Two sides of a triangle are 8 and 12 and the altitude on the third side is 6. A similar triangle has the side homologous to 8 equal to 10. Compute the other two sides and the altitude to the longest side in the second triangle. [12]



8 Prove that if the bisectors of the base angles of an isosceles triangle $ABC$ meet the opposite sides in $E$ and $F$, $EF$ is parallel to the base of the triangle. [12]

9 In an isosceles triangle construct the two circles as shown in the accompanying figure. [All construction lines must be shown.] [12]

10 $F$ is any point on the base $BC$ of the triangle $HBC$. $P$ is the mid-point of $BC$. $PD$ is drawn parallel to $HF$. Prove that the triangle $DFC$ equals one half the triangle $HBC$. [12]




11 *a* If an isosceles triangle is obtuse, which is its longest side? [2]

*b* How many diagonals can be drawn from one vertex of a polygon of $n$ sides? [2]

*c* Under what conditions will the circles having radii $a$ and $b$ respectively, intersect in two distinct points [2] and when will they be externally tangent [2]?

*d* Define similar polygons. [2]

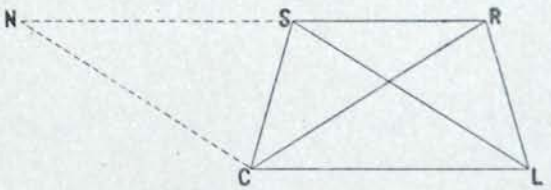
*e* Write *two* theorems that are included in the following statement: If through a point $P$ a line is drawn cutting a circle in $A$ and $B$, then the product $PA \times PB$ is the same for all such lines. [2]

12 Two equal chords are produced till they meet. The angle formed by extending the chords is 18° and the smaller arc intercepted by them is $\frac{1}{15}$ of the circumference. Find each angle of the quadrilateral formed by joining the ends of the chords. [12]

13 Assign a reason to *each* of the eight steps given in the following proof:

THEOREM

*If in a quadrilateral the diagonals are equal and two sides are parallel, the other sides are equal.*




**Given** The quadrilateral CLRS whose diagonals CR and SL are equal and two sides CL and SR parallel.

**To prove** CS = LR

**Proof:** Draw CN ∥ LS meeting RS produced as at N.

1 CLSN is a parallelogram [2]
2 NS = CL and CN = LS [2]
3 CN = CR [2]
4 ∠CNR = ∠CRN [2]
5 ∠RCL = ∠CRN [2]
6 ∠CNR = ∠RCL [2]
7 △CSN = △CLR [2]
8 CS = LR [2]