The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

# MATHEMATICS B

**Wednesday,** August 13, 2008 — 8:30 to 11:30 a.m., only

**Print Your Name:** Steve Sibol

**Print Your School's Name:** JMAP

**Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.**

**Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.**

**The formulas that you may need to answer some questions in this examination are found on page 23. This sheet is perforated so you may remove it from this booklet.**

**This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.**

**When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.**

**Notice. . .**

**A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.**

**The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.**

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

## Part I

**Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.** [40]

**Use this space for computations.**

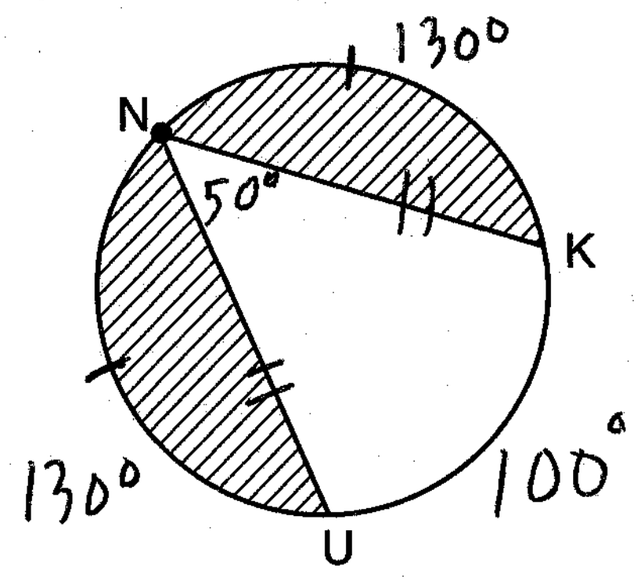
**1** Which transformation of $y = 2^x$ results in the function $y = 2^x - 2$?

(1) $T_{0,-1}$
(2) $T_{0,-2}$
(3) $r_{y\text{-axis}}$
(4) $r_{x\text{-axis}}$

**2** Tanner and Robbie discovered that the means of their grades for the first semester in Mrs. Merrell's mathematics class are identical. They also noticed that the standard deviation of Tanner's scores is 20.7, while the standard deviation of Robbie's scores is 2.7. Which statement must be true?

(1) In general, Robbie's grades are lower than Tanner's grades.
(2) Robbie's grades are more consistent than Tanner's grades. because of a lower S.d.
(3) Robbie had more failing grades during the semester than Tanner had.
(4) The median for Robbie's grades is lower than the median for Tanner's grades.

**3** The NUK Energy Company is designing a new logo, as shown in the accompanying diagram, with $m\overset{\frown}{NK} = 130$ and $m\overset{\frown}{NK} = m\overset{\frown}{NU}$.




Equal chords intercept equal arcs.

$\frac{1}{2} \cdot 100 = 50$

What is the measure of $\angle KNU$?

(1) 50°
(2) 65°
(3) 80°
(4) 100°

**Use this space for computations.**

**4** What is the graph of the function $y = \sqrt{4 - x^2}$?

(1) a circle whose radius is 2 and whose center is at the origin
(2) a circle whose radius is 4 and whose center is at the origin
(3) the upper half of a circle whose radius is 2 and whose center is at the origin
(4) the upper half of a circle whose radius is 4 and whose center is at the origin

$y = \sqrt{4-x^2}$

$y^2 = 4 - x^2$

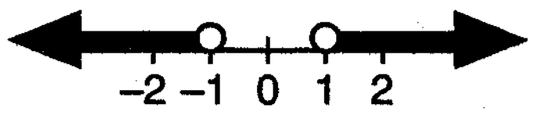
$x^2 + y^2 = 4$

**5** The expression $\frac{6}{y-5} - \frac{y+5}{y^2-25}$ is equivalent to

(1) $\frac{5}{y-5}$
(2) $\frac{5}{y+5}$
(3) $\frac{5y}{y-5}$
(4) $\frac{5y}{y+5}$

$\frac{6}{y-5} - \frac{y+5}{(y+5)(y-5)} = \frac{5}{y-5}$

**6** Which inequality is represented by the accompanying graph?




(1) $|x| > 1$
(2) $|x| \geq 1$
(3) $|x| < 1$
(4) $|x| \leq 1$

$|x| > 1$

$x > 1$ or $x < -1$

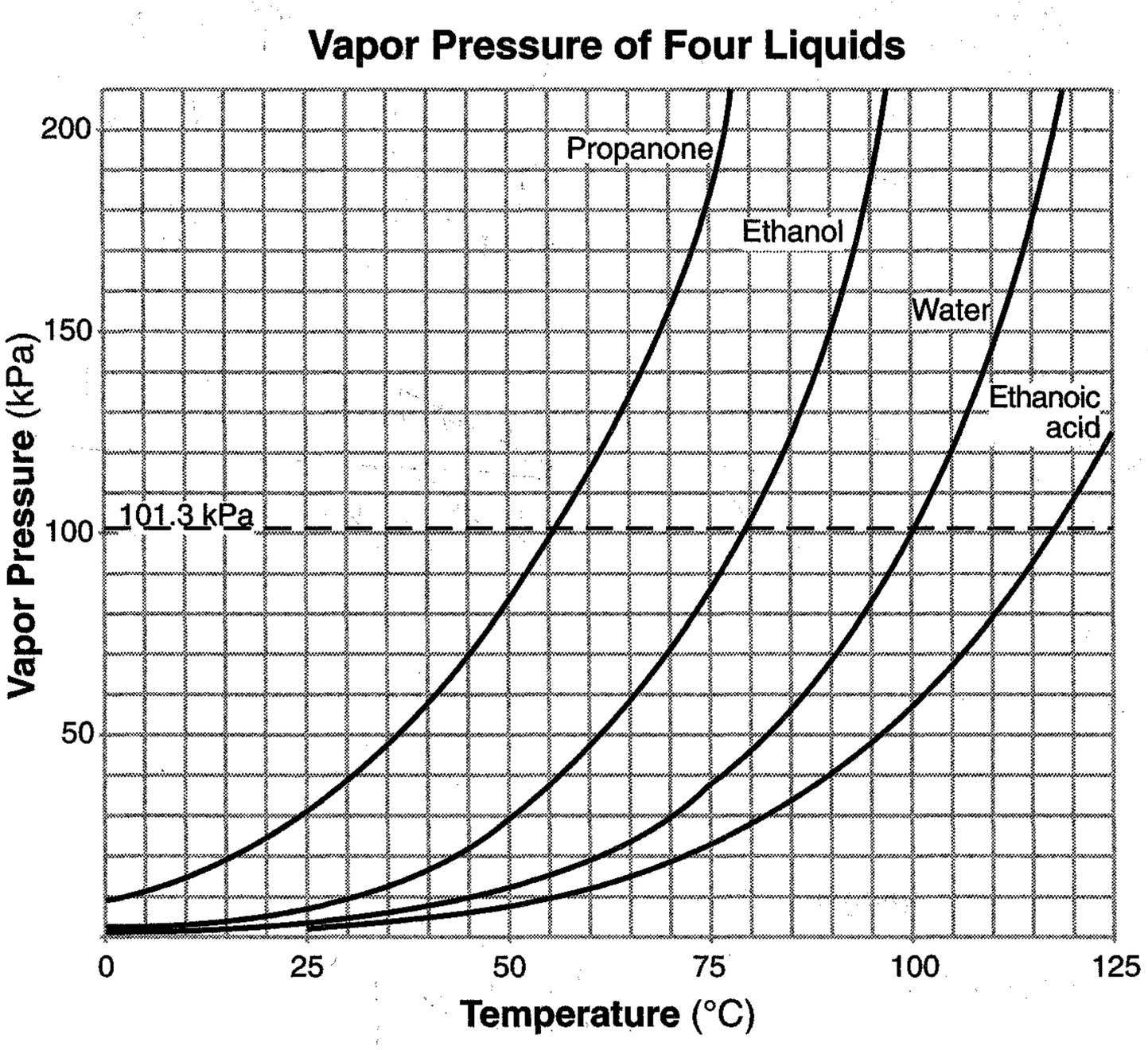
**7** If $n > 0$, the expression $\left(\frac{1}{n}\right)^{-\frac{2}{3}}$ is equal to

(1) $-n^{\frac{2}{3}}$
(2) $-n^{\frac{3}{2}}$
(3) $\sqrt[3]{n^2}$
(4) $\sqrt{n^3}$

$\left(\frac{1}{n}\right)^{-2/3} = n^{2/3} = \sqrt[3]{n^2}$

**Use this space for computations.**

8 The family of curves shown in the accompanying graph illustrates the transformations of a function.




Which type of function could be the original function?

(1) linear
(2) tangent
(3) exponential
(4) sinusoidal

9 The expression $\frac{1}{2}\log m - 3\log n$ is equivalent to

(1) $\log\sqrt{m} + \log n^3$
(2) $\log \frac{1}{2}m - 3\log n$
(3) $\log \frac{m^2}{3\sqrt{n}}$
(4) $\log \frac{\sqrt{m}}{n^3}$

$\frac{1}{2}\log m - 3\log n$

$\log m^{1/2} - \log n^3$

$\log \sqrt{m} - \log n^3$

$\log \frac{\sqrt{m}}{n^3}$

dilation

**10** Under the transformation $(x,y) \rightarrow (2x,2y)$, which property is *not* preserved?

(1) distance
(2) orientation
(3) parallelism
(4) angle measure

**Use this space for computations.**

**11** If $10^k = x$, then $10^{3k}$ is equal to

(1) $x^3$
(2) $3 + x$
(3) $3x$
(4) $1{,}000x$

$10^k = x$

$(10^k)^3 = (x)^3$

$10^{3k} = x^3$

**12** Which equation is *not* a function?

(1) $y = 3x^2 - 4$
(2) $y = \sin x$
(3) $y = \sec x$
(4) $x^2 = 16 - y^2$

U Ψ U
n | n

$x^2 + y^2 = 16$

**13** The expression $1 - \sec x$ is equivalent to

(1) $-\tan x$
(2) $\dfrac{\cos x - 1}{\cos x}$
(3) $\dfrac{\sin x - 1}{\sin x}$
(4) $\dfrac{\tan x}{\sec x - 1}$

$1 - \sec x$

$1 - \frac{1}{\cos x}$

$\frac{\cos x}{\cos x} - \frac{1}{\cos x} = \frac{\cos x - 1}{\cos x}$

**14** The roots of the equation $5x^2 - 2x + 1 = 0$ are

(1) real, rational, and unequal
(2) real, rational, and equal
(3) real, irrational, and unequal
(4) imaginary

$b^2 - 4ac$

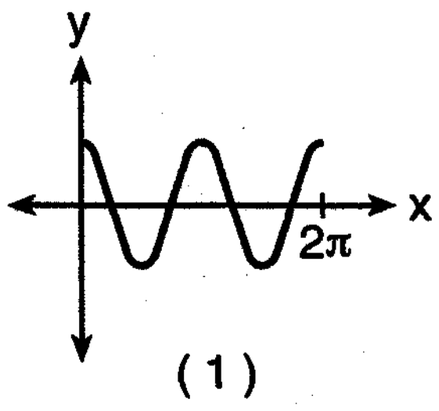
$(-2)^2 - 4(5)(1)$

$4 - 20$

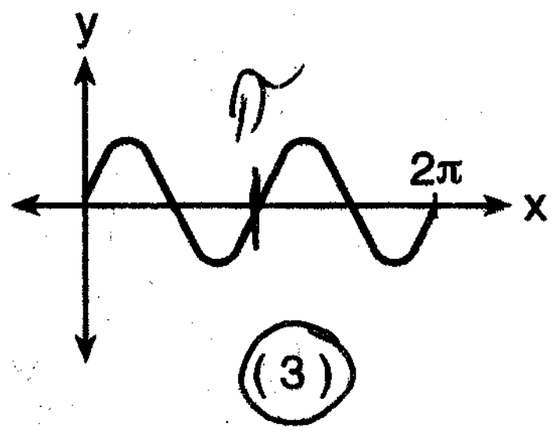
$-16 < 0$

**Use this space for computations.**

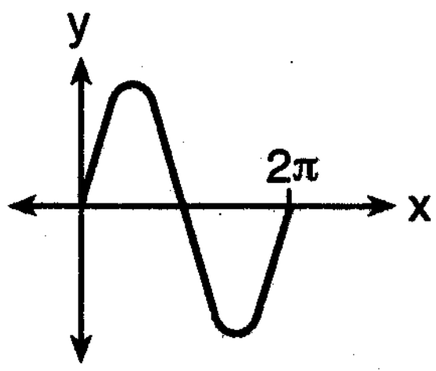
**15** Which graph represents a sound wave that follows a curve whose period is $\pi$ and that is in the form $y = a \sin bx$?




(1)




(3)

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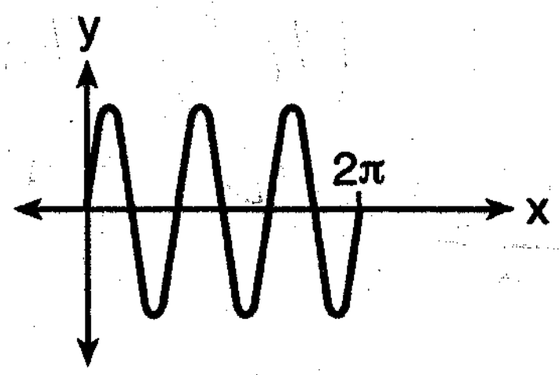



(4)

**16** The expression $\frac{\sqrt{-50}}{\sqrt{2}}$ is equivalent to

(1) $-5i$
(2) $-5$
(3) $5i$
(4) $5$

$$\frac{\sqrt{-50}}{\sqrt{2}} = \frac{\sqrt{2}\sqrt{25}\sqrt{-1}}{\sqrt{2}} = 5i$$

**17** What is the value of $\csc\left(\text{Arc}\sin\frac{3}{4}\right)$?

(1) $\frac{3}{4}$
(2) $\frac{4}{3}$
(3) $\frac{\sqrt{7}}{4}$
(4) $\frac{4}{\sqrt{7}}$

$$\sin\left(\text{Arcsin}\frac{3}{4}\right) = \frac{3}{4}$$
$$\csc\left(\text{Arcsin}\frac{3}{4}\right) = \frac{4}{3}$$

**18** One root of the equation $\frac{3x}{2} + \frac{1}{x} = -\frac{3x}{4}$

(1) $\frac{2}{3}$

(2) $\frac{4i}{9}$

(3) $\frac{2i}{3}$

(4) $\frac{4}{9}$

**Use this space for computations.**

$6x^2 + 4 = -3x^2$

$\frac{9x^2}{9} = \frac{-4}{9}$

$x^2 = \frac{-4}{9}$

$x = \pm\sqrt{\frac{-4}{9}}$

$x = \pm\frac{2i}{3}$

**19** If $2^{(16x^2 - 8x - 3)} = 1$, what does $x$ equal?

(1) $\frac{1}{4}$, only

(2) $\frac{3}{4}$, only

(3) $\frac{1}{4}$ and $-\frac{3}{4}$

(4) $-\frac{1}{4}$ and $\frac{3}{4}$

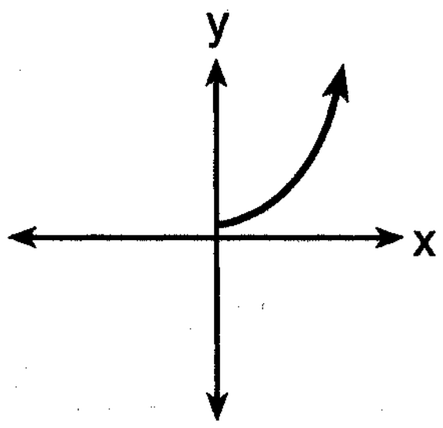
$2^{(16x^2-8x-3)} = 2^0$

$16x^2 - 8x - 3 = 0$

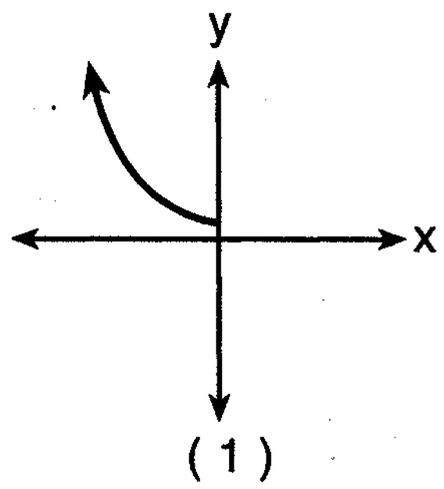
$(4x+1)(4x-3) = 0$

$x = -1/4 \quad x = 3/4$

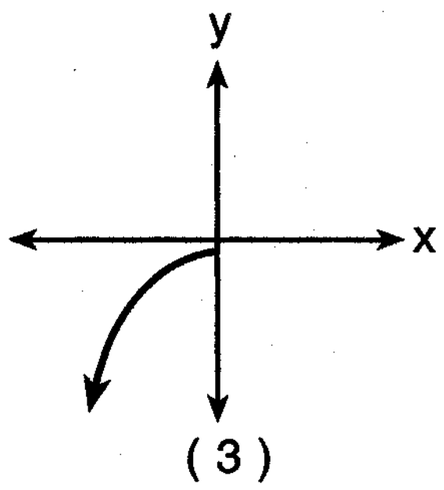
**20** The accompanying graph shows the relationship between kinetic energy, $y$, and velocity, $x$.




The reflection of this graph in the line $y = x$ is




(1)

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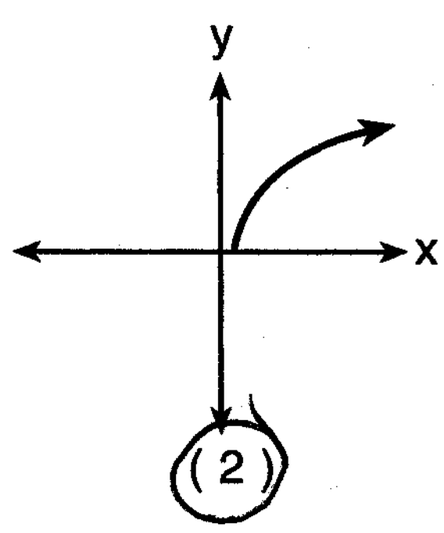


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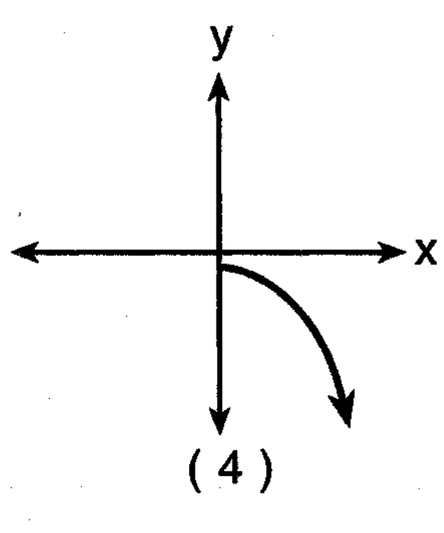



(4)

## Part II

**Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.** [12]

**21** The number of dogs, $D$, housed at a county animal shelter is modeled by the function $D = 4\sqrt{2M} + 50$, where $M$ is the number of months the shelter has been open. How many months will it take for 74 dogs to be housed at the shelter?

$4\sqrt{2m} + 50 = 74$

$\frac{4\sqrt{2m}}{4} = \frac{24}{4}$

$\sqrt{2m} = 6$

$\frac{2m}{2} = \frac{36}{2} = 18$

**22** Solve for all values of $x$: $|3x - 2| = 6$

$3x - 2 = 6$ $\qquad$ $3x - 2 = -6$

$\frac{3x}{3} = \frac{8}{3}$ $\qquad$ $\frac{3x}{3} = \frac{-4}{3}$

**23** Evaluate: $3\sum_{x=2}^{4}\left(x^2-5\right)$

| $x$ | $x^2-5$ |
|---|---|
| 2 | $4-5=-1$ |
| 3 | $9-5=4$ |
| 4 | $16-5=11$ |

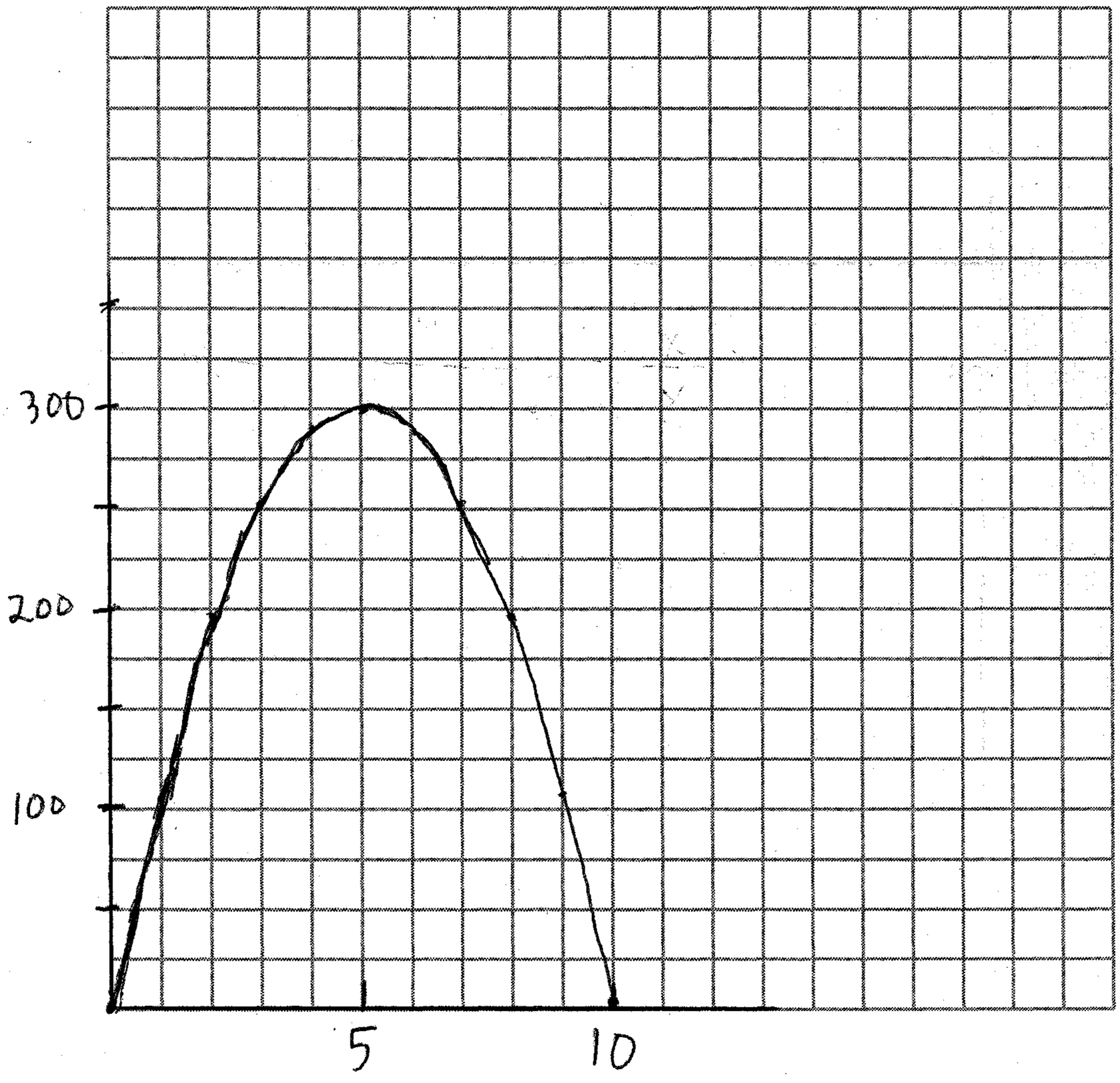
$14 \times 3 = 42$

**24** Express in simplest form: $\dfrac{1-\frac{1}{x}}{x-2+\frac{1}{x}}$

$x\left(\dfrac{x}{x}\right) = \dfrac{x-1}{x^2-2x+1} = \dfrac{\cancel{x-1}}{\cancel{(x-1)}(x-1)} = \dfrac{1}{x-1}$

**25** Each year, the student council at Briarwood High School sponsors a community talent show to raise money. In previous years, the council has discovered that profit from ticket sales, $P(x)$, is a function of the amount charged per ticket, $x$, in dollars, as modeled by the equation $P(x) = 120x - 12x^2$. What amount should the council charge for a ticket to make the greatest profit? [The use of the accompanying grid is optional.]

$$x = \frac{-b}{2a} = \frac{-120}{2(-12)} = 5$$

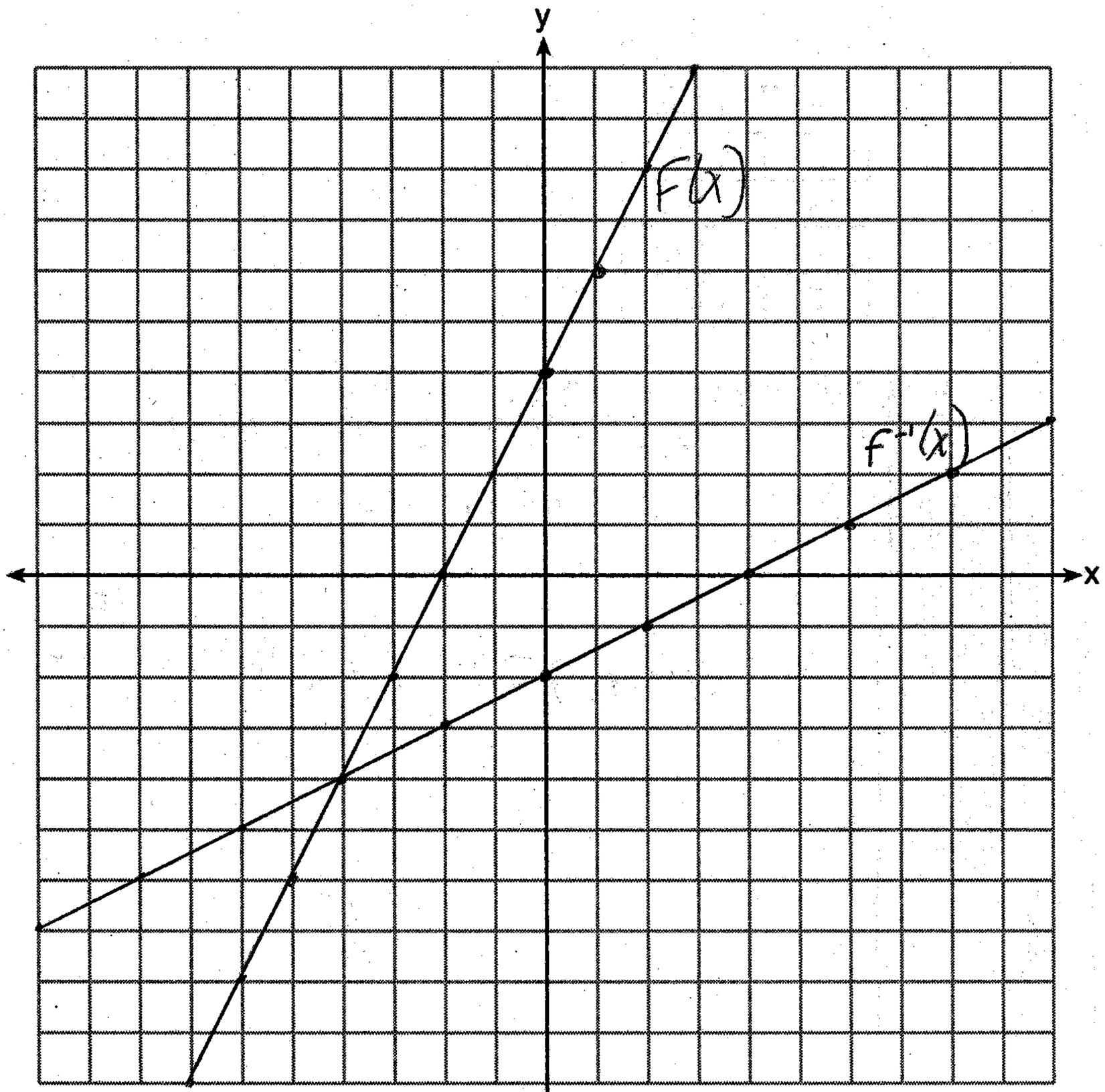


**26** On the accompanying set of axes, graph the function $f(x) = 2x + 4$ and its inverse, $f^{-1}(x)$.

$y = 2x + 4$

$F^{-1}(x)$: $x = 2y + 4$

$y = \frac{1}{2}x - 2$



## Part III

**Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.** [24]

**27** The accompanying table shows the number of bacteria present in a certain culture over a 5-hour period, where $x$ is the time, in hours, and $y$ is the number of bacteria.

| $x$ | $y$ |
|---|---|
| 0 | 1,000 |
| 1 | 1,049 |
| 2 | 1,100 |
| 3 | 1,157 |
| 4 | 1,212 |
| 5 | 1,271 |

Write an exponential regression equation for this set of data, rounding all values to *four decimal places*.

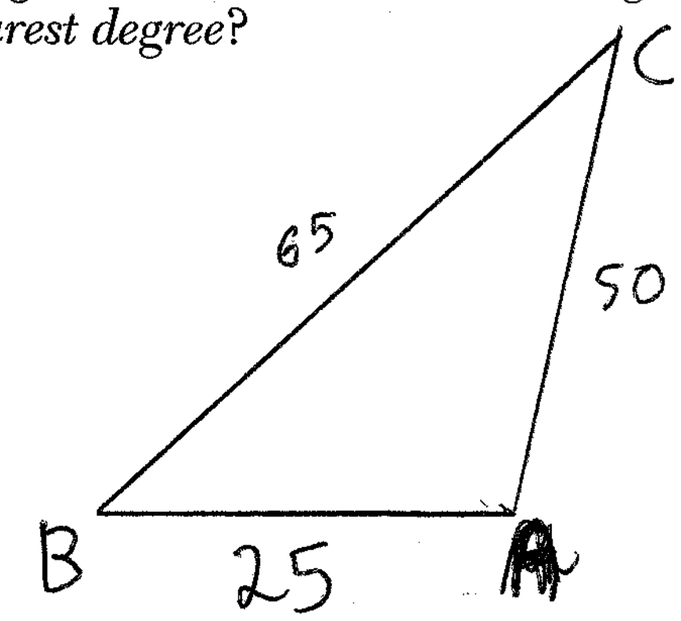
$y = 999.9725(1.0493)^x$

Using this equation, determine the number of whole bacteria present when $x$ equals 6.5 hours.

$999.9725(1.0493)^{6.5} \approx 1367$

**28** During a training exercise in the Mojave Desert, two military vehicles left the base camp at the same time, one traveling at an average speed of 25 miles per hour and the other at an average speed of 50 miles per hour. Each vehicle traveled along a level, straight route.

If the exercise requires the two vehicles to be 65 miles apart after traveling for 1 hour, what must the angle between the two routes be, to the *nearest degree*?




$a^2 = b^2 + c^2 - 2bc\cos A$

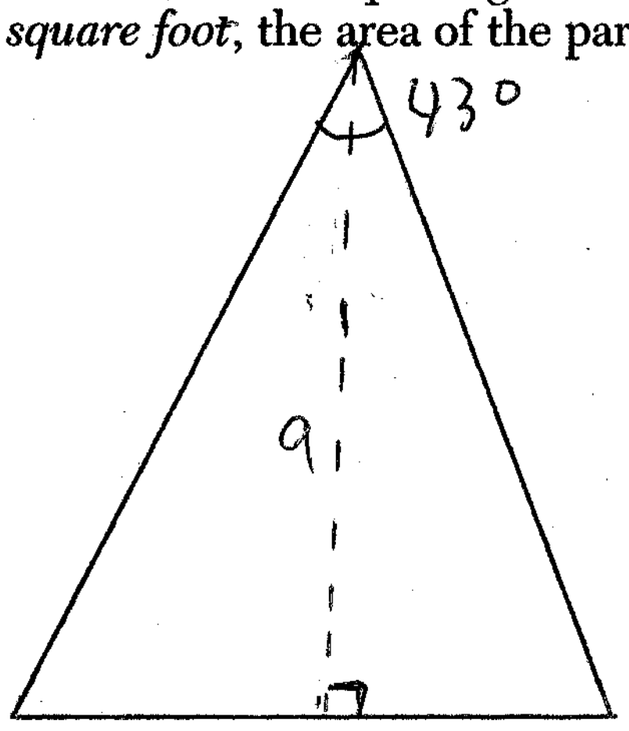
$65^2 = 50^2 + 25^2 - 2(50)(25)\cos A$

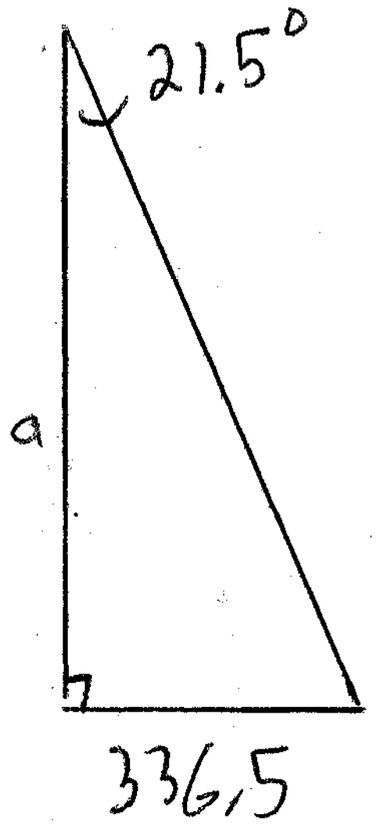
$4225 = 3125 - 2500\cos A$

$\cos A = \left(\frac{-1100}{2500}\right)$

$A \approx 116^\circ$

**29** A parcel of land is in the shape of an isosceles triangle. The base has a length of 673 feet and the two equal legs meet at an angle of 43°. Find, to the *nearest square foot*, the area of the parcel of land.




$\frac{1}{2} \times 43° = 21.5°$

$\frac{1}{2} \times 673 = 336.5$

$\tan 21.5 = \frac{336.5}{a}$

$a \approx 854.2550169$

$A = \frac{1}{2} \text{base} \cdot \text{altitude}$

$\frac{1}{2} \cdot 673 \cdot 854.2550169 \approx 287457$

**30** East West Airlines has a good reputation for being on time. The probability that one of its flights will be on time is .91. If Mrs. Williams flies East West for her next five flights, what is the probability that *at least* three of them will be on time? Round your answer to the *nearest thousandth*.

$n = 5$

$r = 3, 4, 5$

$p = .91$

$q = .09$

$P(3) = {}_5C_3 \cdot .91^3 \cdot .09^2 = .06104$

$P(4) = {}_5C_4 \cdot .91^4 \cdot .09^1 = .30859$

$P(5) = {}_5C_5 \cdot .91^5 \cdot .09^0 = .62403$

$.994$

**31** A landscape architect's designs for a town park call for two parabolic-shaped walkways. When the park is mapped on a Cartesian coordinate plane, the pathways intersect at two points. If the equations of the curves of the walkways are $y = 11x^2 + 23x + 210$ and $y = -19x^2 - 7x + 390$, determine the coordinates of the two points of intersection. [Only an algebraic solution can receive full credit.]

$11x^2 + 23x + 210 = -19x^2 - 7x + 390$

$30x^2 + 30x - 180 = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3 \qquad x = 2$

$y = 11(-3)^2 + 23(-3) + 210 = 240 \qquad (-3, 240)$

$y = 11(2)^2 + 23(2) + 210 = 300 \qquad (2, 300)$

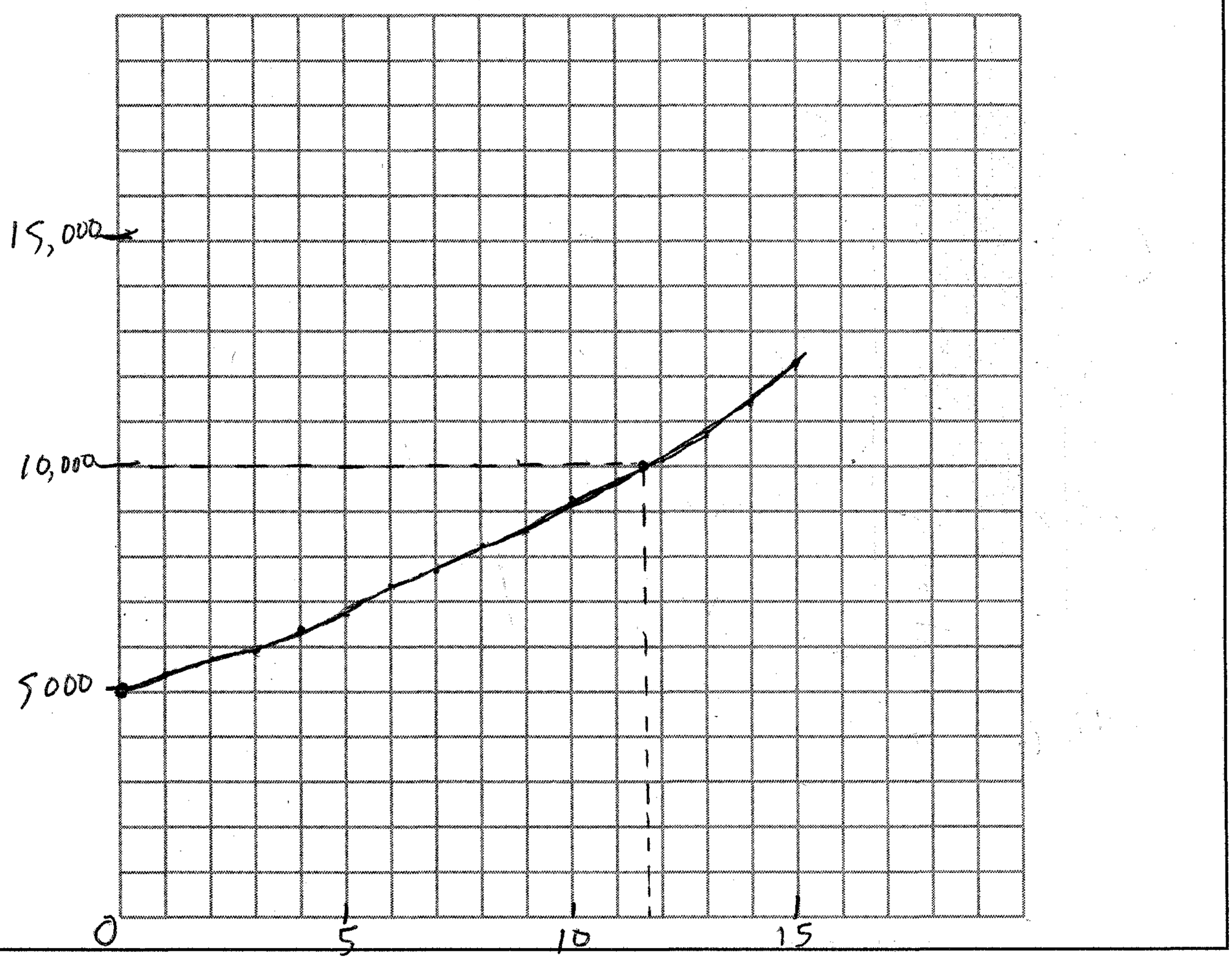
**32** Kristen invests \$5,000 in a bank. The bank pays 6% interest compounded monthly. To the *nearest tenth of a year*, how long must she leave the money in the bank for it to double? (Use the formula $A = P(1 + \frac{r}{n})^{nt}$, where $A$ is the amount accrued, $P$ is the principal, $r$ is the interest rate, $n = 12$, and $t$ is the length of time, in years.) [The use of the accompanying grid is optional.]

$$\frac{10,000}{5,000} = \frac{5000\left(1 + \frac{.06}{12}\right)^{12t}}{5000}$$

$$\log 2 = \log 1.005^{12t}$$

$$\log 2 = 12t \cdot \log 1.005$$

$$t = \frac{\log 2}{12 \log 1.005} \approx 11.6$$



## Part IV

**Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.** [12]

**33** Find all values of $x$ in the interval $0° \leq x < 360°$ that satisfy the equation $3 \cos 2x = \cos x + 2$. Express your answers to the *nearest degree*. [The use of the grid on the next page is optional.]

$3\cos 2x = \cos x + 2$

$3(2\cos^2 x - 1) = \cos x + 2$

$6\cos^2 x - 3 = \cos x + 2$

$6\cos^2 x - \cos x - 5 = 0$

$6x^2 - x - 5 = 0$

$(6x + 5)(x - 1) = 0$

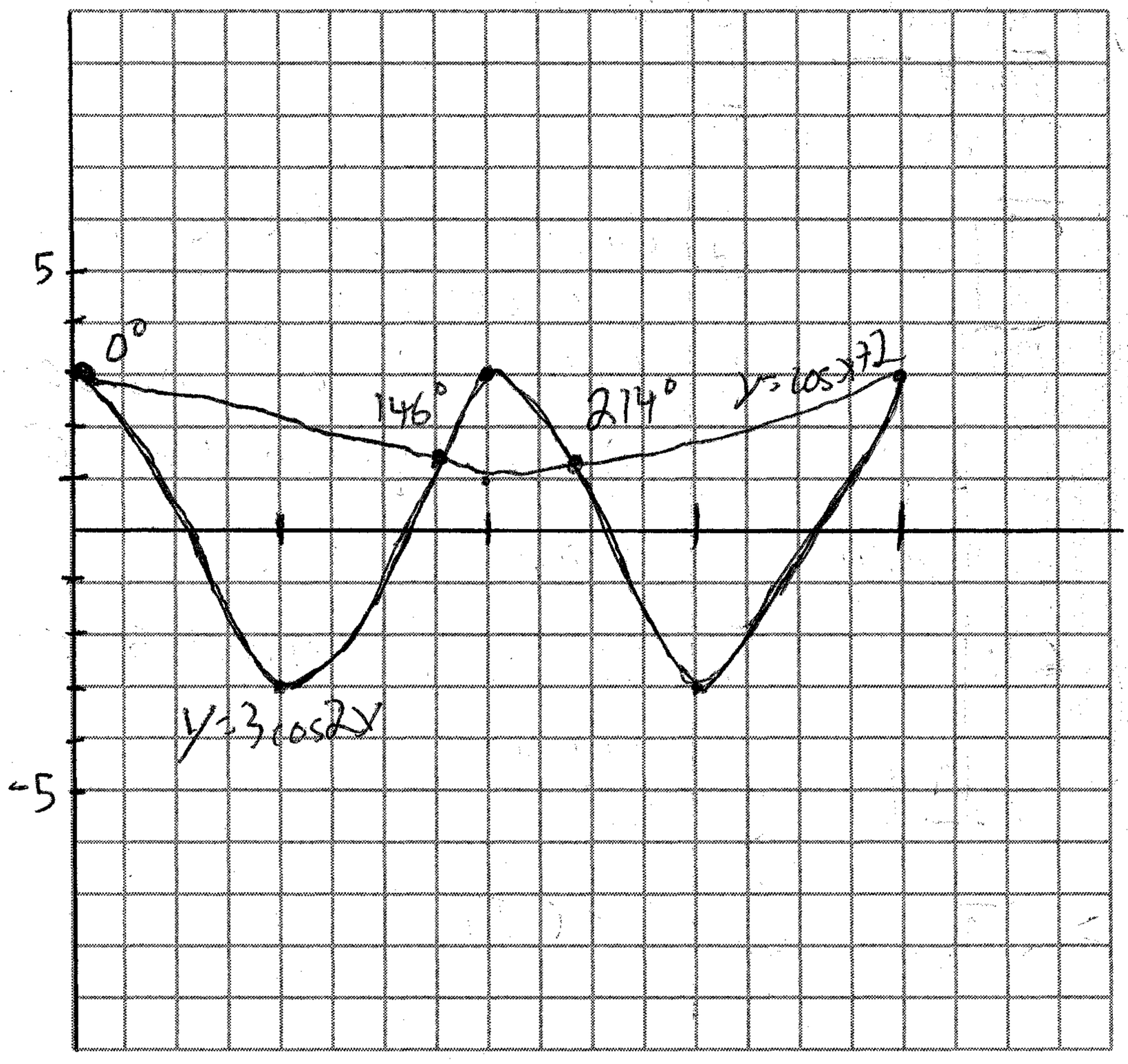
$x = -\frac{5}{6} \qquad x = 1$

$\cos x = \frac{-5}{6} \qquad \cos x = 1$

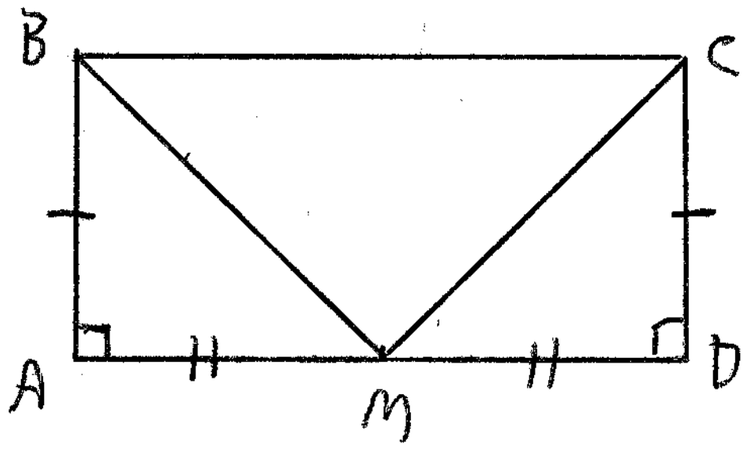
$x = \cos^{-1}\frac{-5}{6} \qquad x = \cos^{-1}$

$x \approx 146°, 214° \qquad x = 0°$

**Question 33 continued**



**34** A tricolored flag is made out of a rectangular piece of cloth whose corners are labeled $A$, $B$, $C$, and $D$. The colored regions are separated by two line segments, $\overline{BM}$ and $\overline{CM}$, that meet at point $M$, the midpoint of side $\overline{AD}$. Prove that the two line segments that separate the regions will always be equal in length, regardless of the size of the flag.




| Statements | Reasons |
|---|---|
| ① Rectangle ABCD, line segments $\overline{BM}$ & $\overline{CM}$ and midpoint M of side $\overline{AD}$ | ① Given |
| ② $\overline{AB} \cong \overline{CD}$ | ② Opposite sides of a rectangle are congruent. |
| ③ $\overline{AM} \cong \overline{DM}$ | ③ Definition of midpoint |
| ④ $\angle A$ and $\angle D$ are right angles | ④ A rectangle has four right angles |
| ⑤ $\angle A \cong \angle D$ | ⑤ All right angles are congruent |
| ⑥ $\triangle ABM \cong \triangle DCM$ | ⑥ SAS |
| ⑦ $\overline{BM} \cong \overline{CM}$ | ⑦ CPCTC |