The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS B

Wednesday, August 16, 2006 — 8:30 to 11:30 a.m., only

Print Your Name: Steve Sibol

Print Your School's Name: HSCR

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.

The formulas that you may need to answer some questions in this examination are found on page 23. This sheet is perforated so you may remove it from this booklet.

This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice. . .

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

MATHEMATICS B

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [40]

Use this space for computations.

1 The expression $4^{\frac{1}{2}} \cdot 2^3$ is equal to

(1) $4^{\frac{3}{2}}$ $\sqrt{4} \cdot 8$

(2) $8^{\frac{3}{2}}$ $2 \times 8 = 16$

(3) 16

(4) 4

2 What is the solution of the equation $\sqrt{2x-3} - 3 = 6$?

(1) 42

(2) 39

(3) 3

(4) 6

$+3 \quad +3$

$\sqrt{2x-3} = 9$

$2x - 3 = 81$

$\frac{2x}{2} = \frac{84}{2} = 42$

3 What is the minimum point of the graph of the equation $y = 2x^2 + 8x + 9$?

(1) (2,33)

(2) (2,17)

(3) (–2,–15)

(4) (–2,1)

$x = \frac{-b}{2a} = \frac{-8}{2(2)} = -2$

$y = 2x^2 + 8x + 9$

$= 2(-2)^2 + 8(-2) + 9$

$= 1$

4 If x is a positive acute angle and $\cos x = \frac{\sqrt{3}}{4}$, what is the exact value of $\sin x$?

(1) $\frac{\sqrt{3}}{5}$

(2) $\frac{\sqrt{13}}{4}$

(3) $\frac{3}{5}$

(4) $\frac{4}{5}$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x + \left(\frac{\sqrt{3}}{4}\right)^2 = 1$

$\sin^2 x + \frac{3}{16} = 1$

$\sin^2 x = \frac{13}{16}$

$\sin x = \sqrt{\frac{13}{16}}$

$\sin x = \frac{\sqrt{13}}{4}$

5 Which equation does *not* represent a function?

(1) $x = \pi$

(2) $y = 4$

(3) $y = |x|$

(4) $y = x^2 + 5x$

6 The expression $\frac{12}{3+\sqrt{3}}$ is equivalent to

(1) $12-\sqrt{3}$
(2) $6-2\sqrt{3}$
(3) $4-2\sqrt{3}$
(4) $2+\sqrt{3}$

$\frac{12}{3+\sqrt{3}}\left(\frac{3-\sqrt{3}}{3-\sqrt{3}}\right)$

$\frac{12(3-\sqrt{3})}{9-3} = \frac{12(3-\sqrt{3})}{6} = 2(3-\sqrt{3})$

$6-2\sqrt{3}$

Use this space for computations.

7 The function $y = 2^x$ is equivalent to

(1) $x = y \log 2$
(2) $x = \log_2 y$
(3) $y = x \log 2$
(4) $y = \log_2 x$

8 In ΔABC, D is a point on $\overline{AC}$ such that $\overline{BD}$ is a median. Which statement must be true?

(1) $\Delta ABD \cong \Delta CBD$
(2) $\angle ABD \cong \angle CBD$
(3) $\overline{AD} \cong \overline{CD}$
(4) $\overline{BD} \perp \overline{AC}$

9 A designer who is planning to install an elliptical mirror is laying out the design on a coordinate grid. Which equation could represent the elliptical mirror?

Hyperbola (1) $x^2 = 144 + 36y^2$
(2) $x^2 + y^2 = 144$ circle
(3) $x^2 + 4y^2 = 144$
(4) $y = 4y^2 + 144$ linear

10 A solution set of the equation $5 \sin \theta + 3 = 3$ contains all multiples of

(1) 45°
(2) 90°
(3) 135°
(4) 180°

$5\sin\theta + 3 = 3$
$-3 \quad -3$
$\frac{5\sin\theta}{5} = \frac{0}{5}$
$\sin\theta = 0$
$\theta = 0, 180, 360\ldots$

Use this space for computations.

11 What is the total number of points of intersection for the graphs of the equations $y = x^2$ and $y = -x^2$?

(1) 1
(2) 2
(3) 3
(4) 0

12 For which equation is the sum of the roots equal to the product of the roots?

(1) $x^2 + x + 1 = 0$
(2) $x^2 + 3x - 6 = 0$
(3) $x^2 - 8x - 4 = 0$
(4) $x^2 - 4x + 4 = 0$

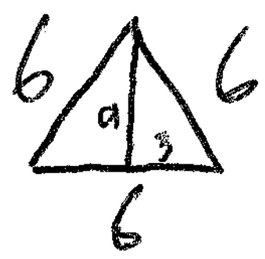
$2+2=2\times2$

$(x-2)(x-2)=0$

$x=2$

13 If the perimeter of an equilateral triangle is 18, the length of the altitude of this triangle is

(1) 6
(2) $6\sqrt{3}$
(3) 3
(4) $3\sqrt{3}$



$a^2+3^2=6^2$

$a^2+9=36$

$a^2=27$

$a=\sqrt{27}$

$\sqrt{9}\sqrt{3}$

$3\sqrt{3}$

14 Jonathan's teacher required him to express the sum $\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}$ using sigma notation. Jonathan proposed four possible answers. Which of these four answers is *not* correct?

(1) $\sum_{k=3}^{7} \frac{k-1}{k}$
(2) $\sum_{k=1}^{5} \frac{k}{k+1}$
(3) $\sum_{k=1}^{5} \frac{k+1}{k+2}$
(4) $\sum_{k=2}^{6} \frac{k}{k+1}$

15 What is the period of the graph of the equation $y = 2\sin\frac{1}{3}x$?

(1) $\frac{2}{3}\pi$
(2) 2π
(3) 6π
(4) $\frac{3\pi}{2}$

$\text{period} = \frac{2\pi}{b} = \frac{2\pi}{1/3} = 6\pi$

16 What is the solution set of the equation $|x^2 - 2x| = 3x - 6$?

(1) $\{2,\pm 3\}$
(2) $\{2\}$
(3) $\{\pm 3\}$
(4) $\{2,3\}$

$x^2-2x=3x-6$
$x^2-5x+6=0$
$(x-3)(x-2)=0$
$3, 2$

Use this space for computations.

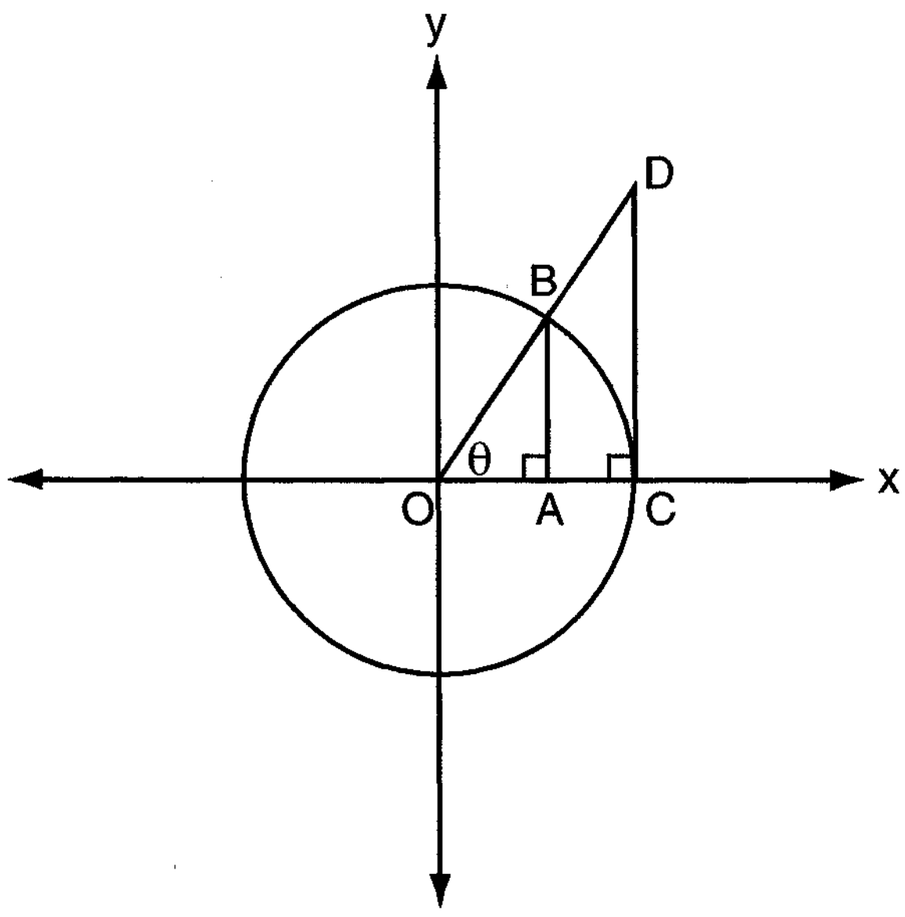
$x^2-2x=-3x+6$
$x^2+x-6=0$
$(x+3)(x-2)=0$
$-3 \quad 2$
extraneous

17 The expression $\frac{\sin 2\theta}{\sin^2 \theta}$ is equivalent to

(1) $\frac{2}{\sin \theta}$
(2) $2 \cos \theta$
(3) $2 \cot \theta$
(4) $2 \tan \theta$

$\frac{2\sin\theta\cos\theta}{\sin^2\theta}$
$\frac{2\cos\theta}{\sin\theta}$

18 The accompanying diagram shows unit circle O, with radius $OB = 1$.



Which line segment has a length equivalent to $\cos \theta$?

(1) $\overline{AB}$
(2) $\overline{CD}$
(3) $\overline{OC}$
(4) $\overline{OA}$

19 The expression $\frac{3y^2 - 12y}{4y^2 - y^3}$ is equivalent to

(1) $\frac{3}{y}$

(2) $-\frac{3}{y}$

(3) $-\frac{9}{4}$

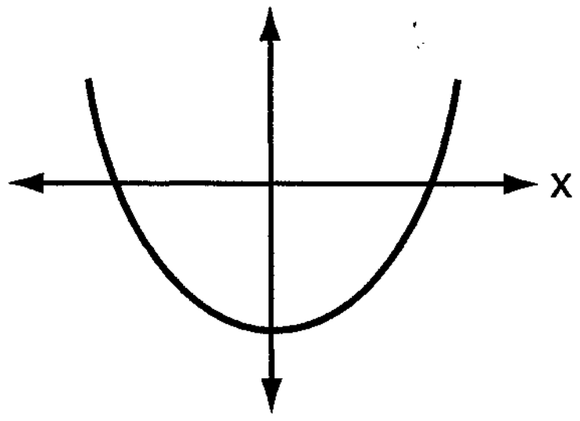
(4) $\frac{3}{4} - \frac{12}{y^2}$

$\frac{-3x(y-4)}{y^2(4-y)}$

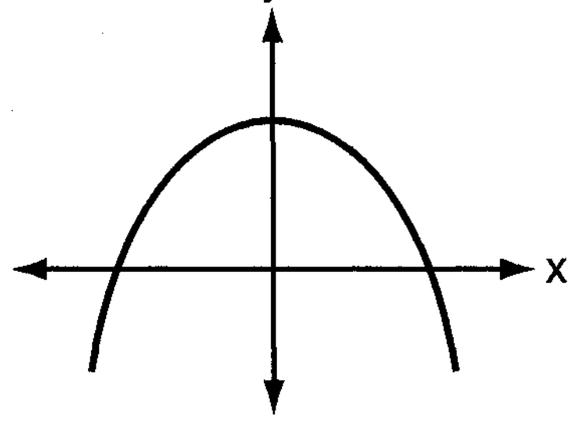
Use this space for computations.

20 Which graph represents a quadratic function with a negative discriminant?

(1)



(3)



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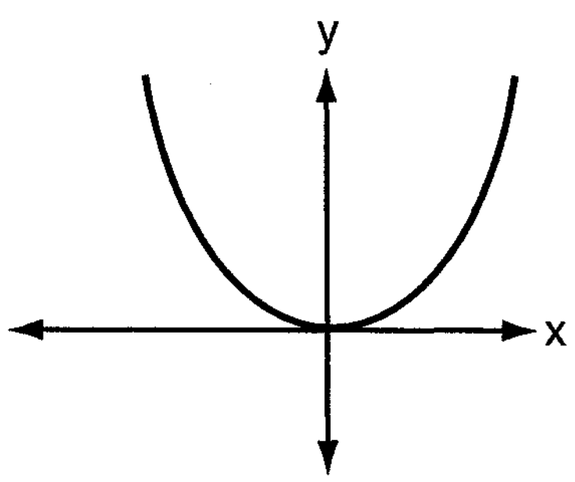

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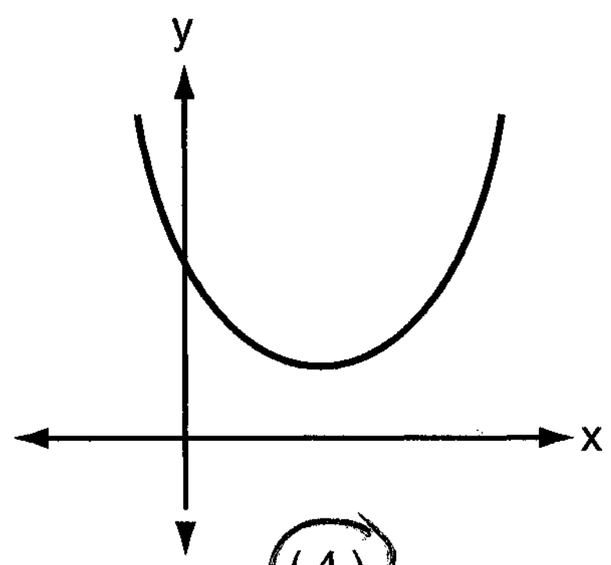


no real roots

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

21 The complex number $c + di$ is equal to $(2 + i)^2$. What is the value of c?

$(2+i)(2+i)$

$4+2i+2i+i^2$

$4+4i-1$

$3+4i$

$c=3$

22 The volume of any spherical balloon can be found by using the formula $V = \frac{4}{3}\pi r^3$.

Write an equation for r in terms of V and π.

$$\frac{V}{\frac{4}{3}\pi} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi}$$

$$\frac{3V}{4\pi} = r^3$$

$$\sqrt[3]{\frac{3V}{4\pi}} = r$$

23 What is the number of degrees in an angle whose radian measure is $\frac{7\pi}{12}$?

$$\frac{7\pi}{12} \cdot \frac{180}{\pi} = 105^\circ$$

24 Solve for x: $\log_b 36 - \log_b 2 = \log_b x$

$\log_b \frac{36}{2} = \log_b x$

$18 = x$

25 Beth's scores on the six Earth science tests she took this semester are 100, 95, 55, 85, 75, and 100. For this population, how many scores are within one standard deviation of the mean?

$\bar{x} = 85$

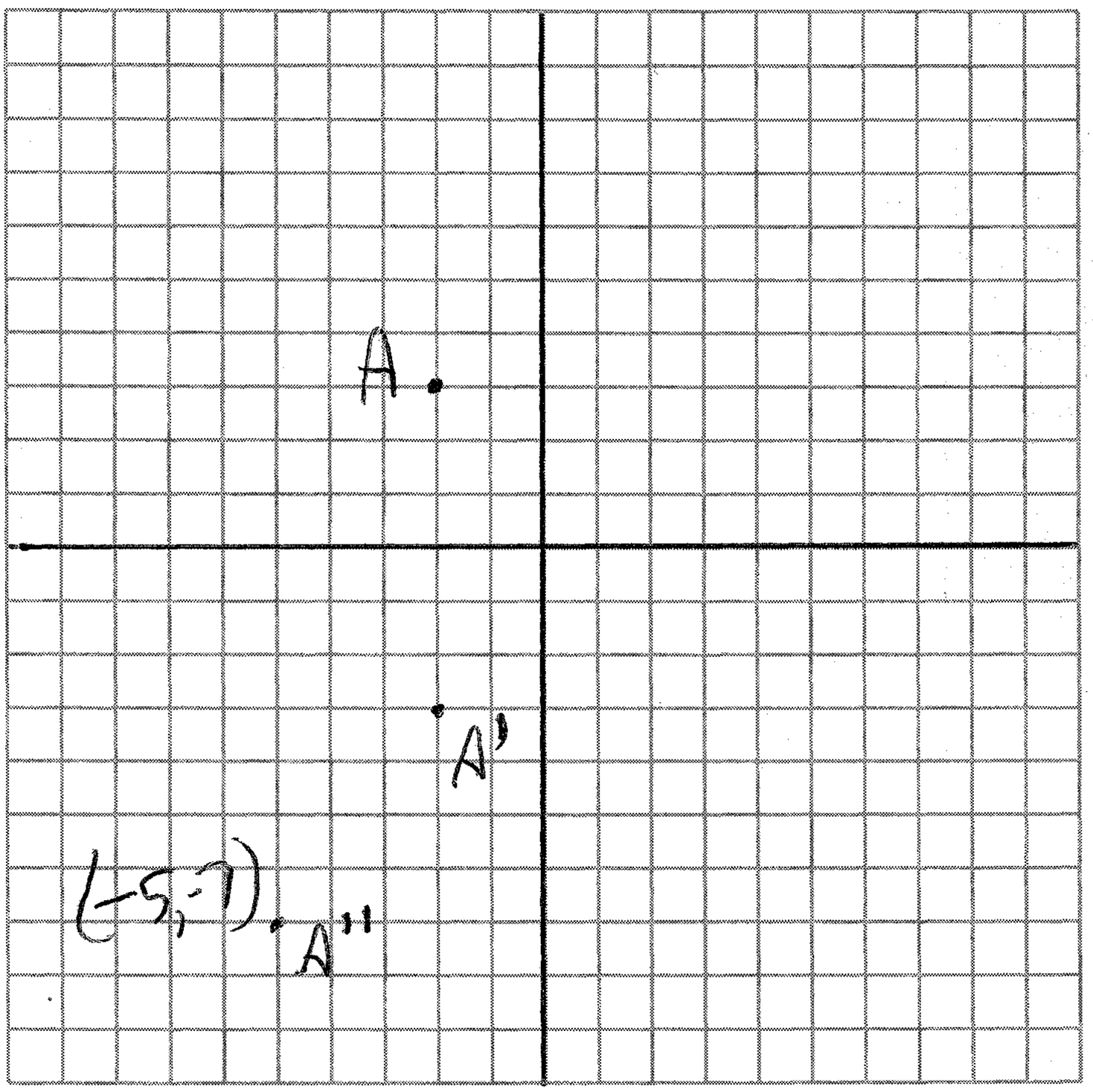
$\sigma_x = 16$

range is $\bar{x} \pm \sigma_x$

69 - 100

5 scores

26 Given point $A(-2,3)$. State the coordinates of the image of A under the composition $T_{-3,-4} \circ r_{x\text{-axis}}$. [The use of the accompanying grid is optional.]

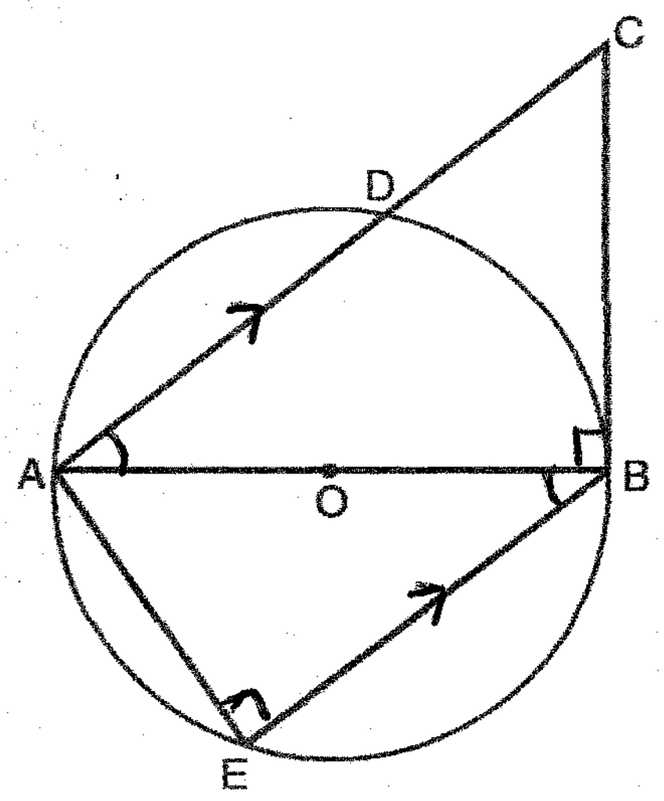


Part III

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [24]

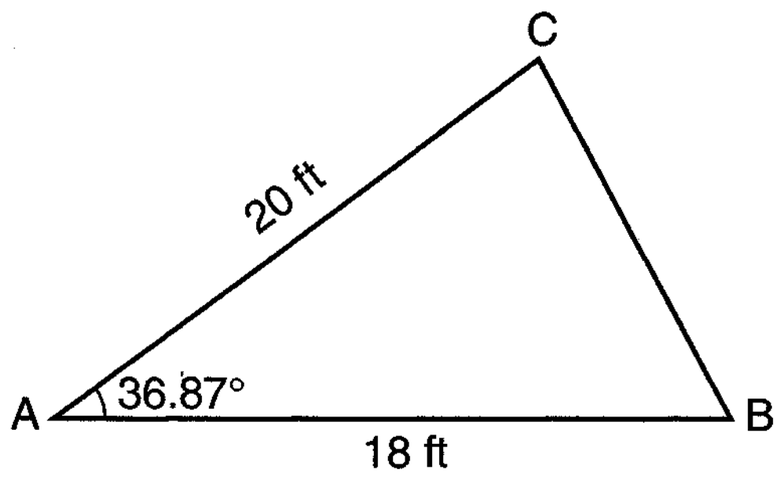
27 In the accompanying diagram of circle O, diameter $\overline{AOB}$ is drawn, tangent $\overline{CB}$ is drawn to the circle at B, E is a point on the circle, and $\overline{BE} \parallel \overline{ADC}$.

Prove: $\Delta ABE \sim \Delta CAB$



STATEMENT	REASON
(1) Circle O, diameter $\overline{AOB}$, tangent $\overline{CB}$, E is point on circle, and $\overline{BE} \parallel \overline{ADC}$	(1) Given
(2) $\angle AEB$ is a right angle	(2) A triangle inscribed in a semi-circle is a right Δ
(3) $\angle CBA$ is a right angle	(3) A tangent drawn to a radius forms a right angle
(4) $\angle AEB \cong \angle CBA$	(4) Right angles are congruent
(5) $\angle CAB \cong \angle ABE$	(5) Alternate interior angles formed by parallel lines and a transversal are congruent
(6) $\Delta ABE \sim \Delta CAB$.	(6) AA

28 The accompanying diagram shows a triangular plot of land that is part of Fran's garden. She needs to change the dimensions of this part of the garden, but she wants the area to stay the same. She increases the length of side AC to 22.5 feet. If angle A remains the same, by how many feet should side AB be *decreased* to make the area of the new triangular plot of land the same as the current one?



Area $= \frac{1}{2}abc\sin A$

$= \frac{1}{2}(20)(18)\sin 36.87$

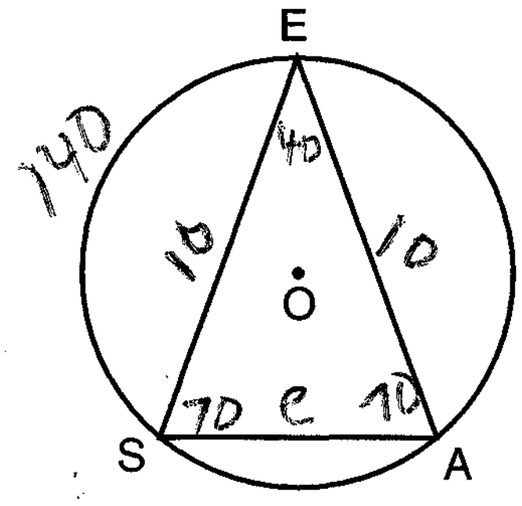
$= 108$

$108 = \frac{1}{2}(22.5)c\sin 36.87$

$C = 16$

side AB must be reduced by 2 feet

29 A machine part consists of a circular wheel with an inscribed triangular plate, as shown in the accompanying diagram. If $\overline{SE} \cong \overline{EA}$, $SE = 10$, and $m\overset{\frown}{SE} = 140$, find the length of $\overline{SA}$ to the *nearest tenth*.



$$\frac{e}{\sin 40} = \frac{10}{\sin 70}$$

$$e = \frac{10 \sin 40}{\sin 70}$$

$$e \approx 6.8$$

30 On mornings when school is in session in January, Sara notices that her school bus is late one-third of the time. What is the probability that during a 5-day school week in January her bus will be late *at least* three times?

$n = 5$

$r = 3, 4, 5$

$p = \frac{1}{3}$

$q = \frac{2}{3}$

$P(3) = {}_5C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^2 = \frac{40}{243}$

$P(4) = {}_5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^1 = \frac{10}{243}$

$P(5) = {}_5C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^0 = \frac{1}{243}$

$\frac{51}{243}$

31 Jean invested \$380 in stocks. Over the next 5 years, the value of her investment grew, as shown in the accompanying table.

Years Since Investment (x)	**Value of Stock, in Dollars** (y)
0	380
1	395
2	411
3	427
4	445
5	462

Write the exponential regression equation for this set of data, rounding all values to *two decimal places*. $y = 379.92(1.04)^x$

Using this equation, find the value of her stock, to the *nearest dollar*, 10 years after her initial purchase.

$y = 379.92(1.04)^{10}$

$\approx \$562$

32 After an oven is turned on, its temperature, T, is represented by the equation $T = 400 - 350(3.2)^{-0.1m}$, where m represents the number of minutes after the oven is turned on and T represents the temperature of the oven, in degrees Fahrenheit.

How many minutes does it take for the oven's temperature to reach 300°F? Round your answer to the *nearest minute*. [The use of the grid on the next page is optional.]

$T = 400 - 350(3.2)^{-0.1m}$

$300 = 400 - 350(3.2)^{-0.1m}$

$-400 \quad -400$

$\frac{-100}{-350} = \frac{-350(3.2)^{-0.1m}}{-350}$

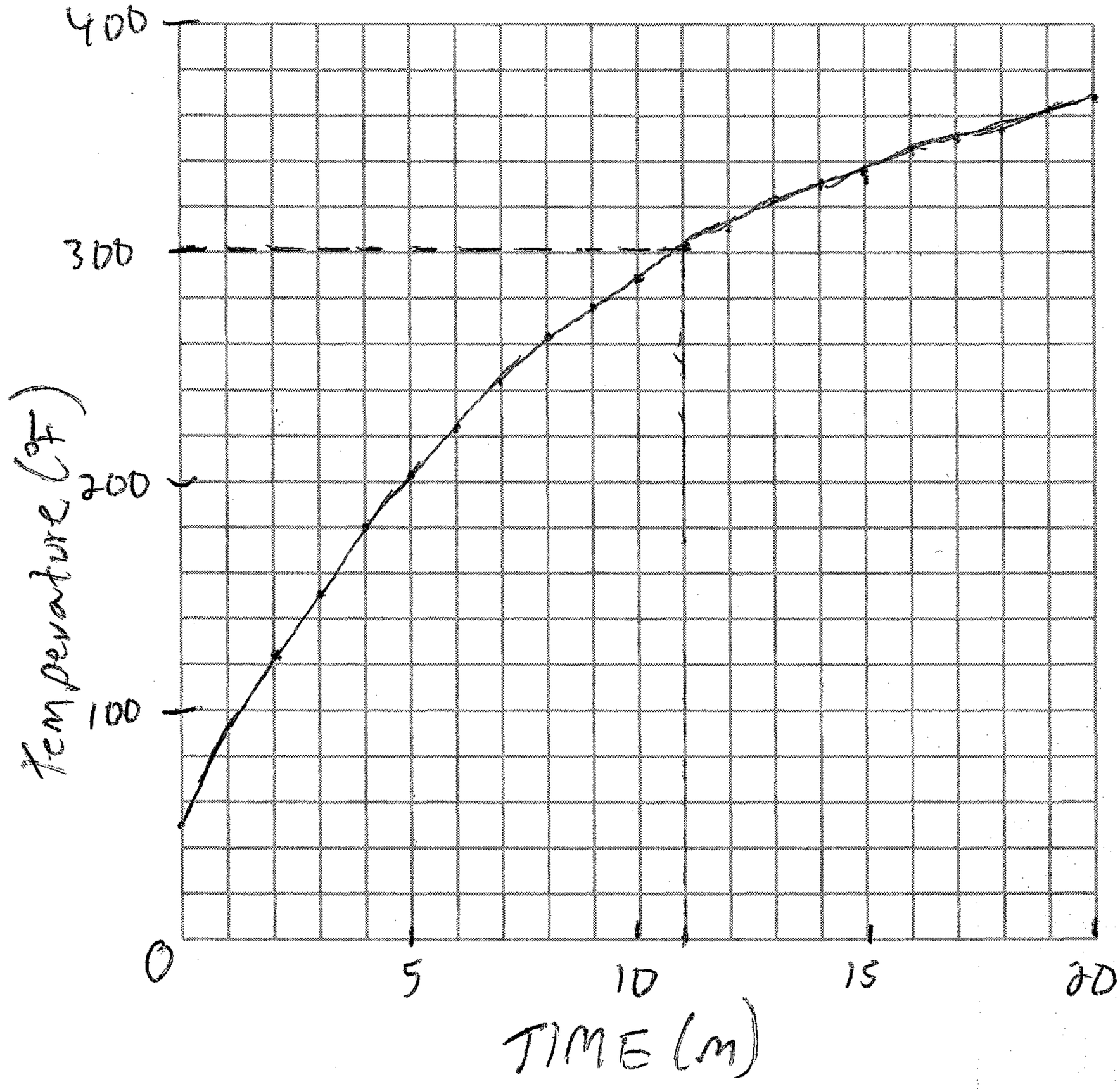
$\frac{2}{7} = (3.2)^{-0.1m}$

$\log\frac{2}{7} = \log(3.2)^{-0.1m}$

$\frac{\log\frac{2}{7}}{-0.1\log 3.2} = \frac{-0.1m\log 3.2}{-0.1\log 3.2}$

$11 \approx m$

Question 32 continued



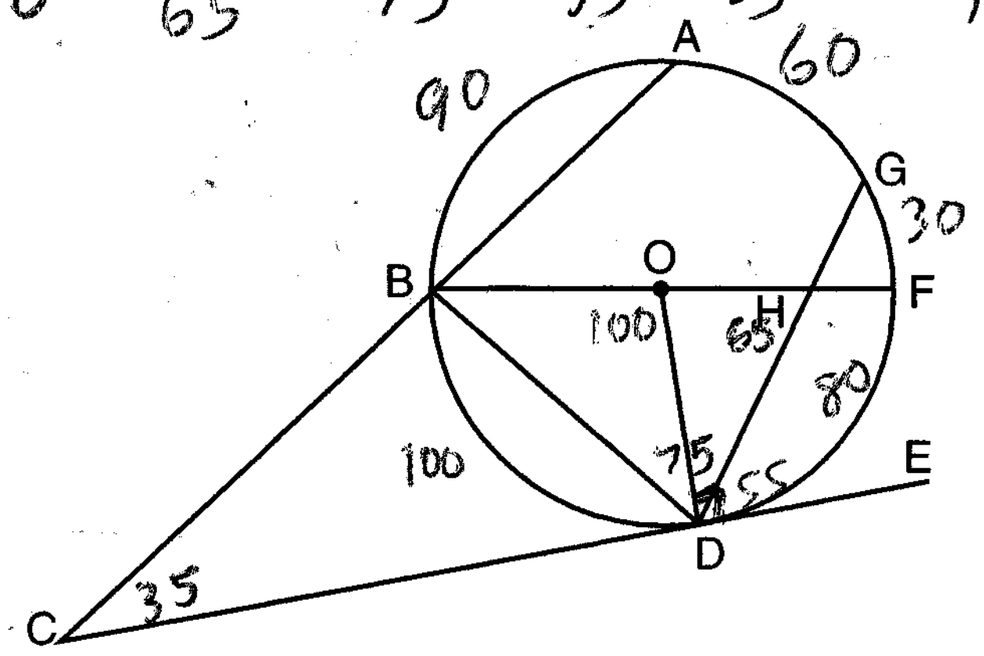
Part IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

33 In the accompanying diagram, circle O has radius $\overline{OD}$, diameter $\overline{BOHF}$, secant $\overline{CBA}$, and chords $\overline{DHG}$ and $\overline{BD}$; $\overline{CE}$ is tangent to circle O at D; $m\overset{\frown}{DF} = 80$; and $m\overset{\frown}{BA} : m\overset{\frown}{AG} : m\overset{\frown}{GF} = 3:2:1$.

Find $m\overset{\frown}{GF}$, $m\angle BHD$, $m\angle BDG$, $m\angle GDE$, $m\angle C$, and $m\angle BOD$.

30 65 75 55 35 100



Since $\overline{BOF}$ is a diameter $m\overset{\frown}{BD} = 100$

$m\overset{\frown}{BA} = \frac{3}{6} \times 180 = 90$

$m\overset{\frown}{AG} = \frac{2}{6} \times 180 = 60$

$m\overset{\frown}{GF} = \frac{1}{6} \times 180 = 30$

$m\angle BHD = \frac{100+30}{2} = 65$

$m\angle BDG = \frac{90+60}{2} = 75$

$m\angle GDE = \frac{80+30}{2} = 55$

$m\angle C = \frac{(60+30+80)-100}{2} = 35$

$m\angle BOD = 100$

34 Barb pulled the plug in her bathtub and it started to drain. The amount of water in the bathtub as it drains is represented by the equation $L = -5t^2 - 8t + 120$, where L represents the number of liters of water in the bathtub and t represents the amount of time, in minutes, since the plug was pulled.

How many liters of water were in the bathtub when Barb pulled the plug? Show your reasoning. Bart pulled the plug at $t=0$

Determine, to the *nearest tenth of a minute*, the amount of time it takes for all the water in the bathtub to drain.

$L = -5t^2 - 8t + 120$

$= -5(0)^2 - 8(0) + 120$

$= 120$

Use Quadratic Formula

$-5t^2 - 8t + 120 = 0$

$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(-5)(120)}}{2(-5)}$$

$$\frac{8 \pm \sqrt{2464}}{10}$$

$$\frac{8 + \sqrt{2464}}{-10}$$

negative solution

$$\frac{8 - \sqrt{2464}}{-10} \approx 4.2$$