The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

# MATHEMATICS B

**Tuesday,** August 16, 2005 — 8:30 to 11:30 a.m., only

**Print Your Name:** Steve Sibol

**Print Your School's Name:** HSCR

**Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.**

**Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will *not* be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.**

**This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. The formulas that you may need to answer some questions in this examination are found on page 23.**

**When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.**

| |
|---|
| **Notice. . .** |
| **A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.** |

**The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.**

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

MATHEMATICS B

## Part I

**Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.** [40]

**Use this space for computations.**

**1** What is the turning point, or vertex, of the parabola whose equation is $y = 3x^2 + 6x - 1$?

(1) (1,8) (3) (–3,8)
(2) (–1,–4) (4) (3,44)

$x = \frac{-b}{2a} = \frac{-6}{2(3)} = -1$

$3(-1)^2 + 6(-1) - 1$

$= -4$

**2** The growth of bacteria in a dish is modeled by the function $f(t) = 2^{\frac{t}{3}}$. For which value of $t$ is $f(t) = 32$?

(1) 8 (3) 15
(2) 2 (4) 16

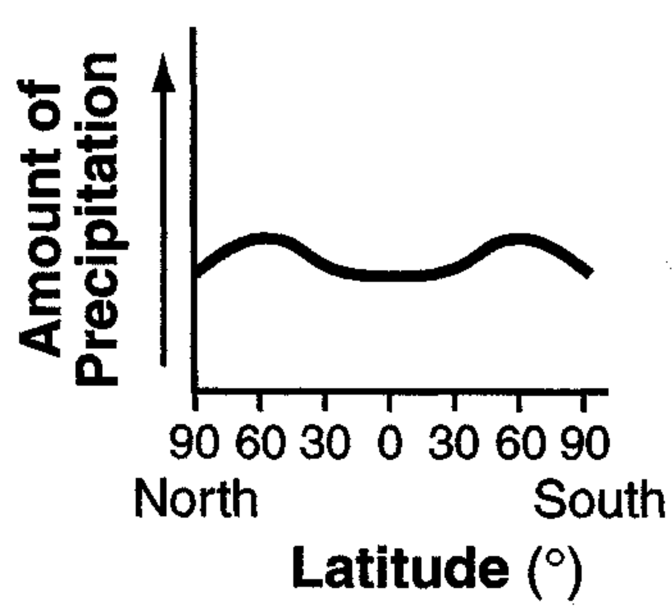
$32 = 2^{\frac{t}{3}}$

$2^5 = \frac{t}{3}$

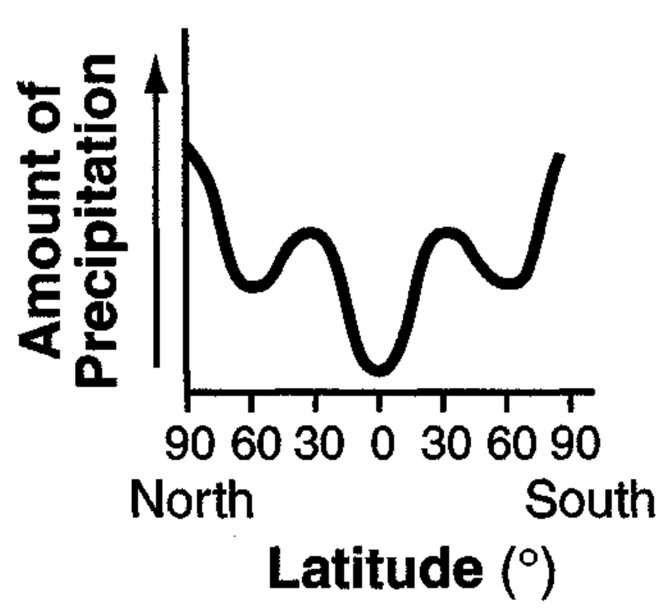
$5 = \frac{t}{3}$

$t = 15$

**3** The graphs below show the average annual precipitation received at different latitudes on Earth. Which graph is a translated cosine curve?



(1)



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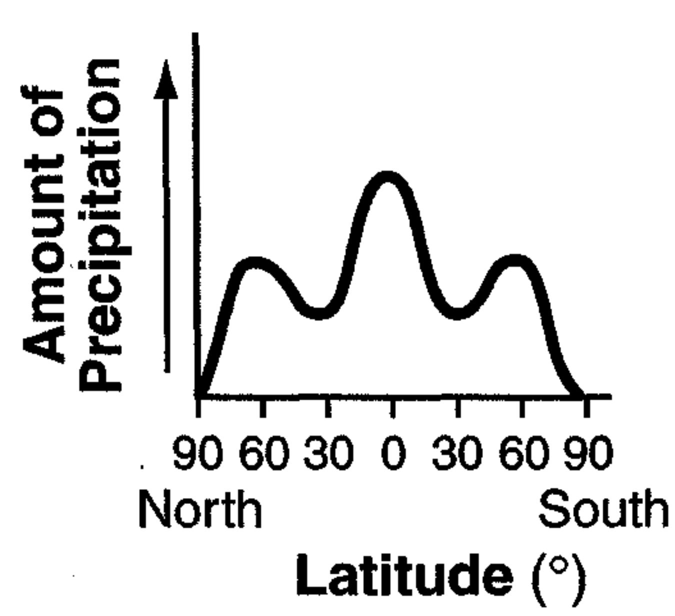
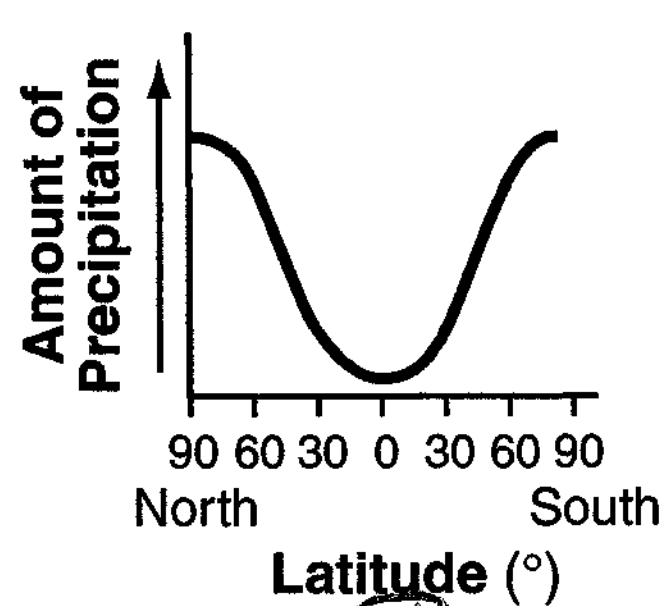


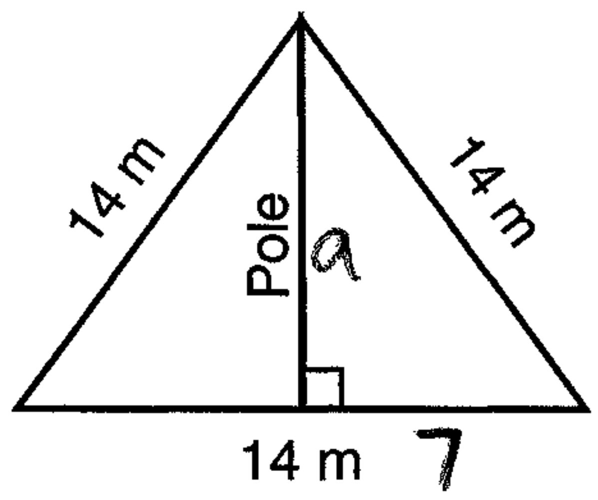
(2)



(4)

4 The accompanying diagram shows two cables of equal length supporting a pole. Both cables are 14 meters long, and they are anchored to points in the ground that are 14 meters apart.

**Use this space for computations.**




$a^2+7^2=14^2$

$a^2+49=196$

$a^2=147$

$a=\sqrt{147}$

$a=\sqrt{49}\sqrt{3}$

$a=7\sqrt{3}$

What is the exact height of the pole, in meters?

(1) 7
(2) $7\sqrt{2}$
(3) $7\sqrt{3}$
(4) 14

5 What is the sum of $(y-5)+\frac{3}{y+2}$?

(1) $y-5$
(2) $\frac{y^2-7}{y+2}$
(3) $\frac{y-2}{y+2}$
(4) $\frac{y^2-3y-7}{y+2}$

$\frac{(y-5)(y+2)+3}{y+2}=\frac{y^2+2y-5y-10+3}{y+2}$

$\frac{y^2-3y-7}{y+2}$

6 The expression $\frac{1}{5-\sqrt{13}}$ is equivalent to

(1) $\frac{5+\sqrt{13}}{12}$
(2) $\frac{5+\sqrt{13}}{-12}$
(3) $\frac{5+\sqrt{13}}{8}$
(4) $\frac{5+\sqrt{13}}{-8}$

$\frac{1}{5-\sqrt{13}}\cdot\frac{5+\sqrt{13}}{5+\sqrt{13}}=\frac{5+\sqrt{13}}{25-13}=\frac{5+\sqrt{13}}{12}$

7 When expressed as a monomial in terms of $i$, $2\sqrt{-32}-5\sqrt{-8}$ is equivalent to

(1) $2\sqrt{2i}$
(2) $2i\sqrt{2}$
(3) $-2i\sqrt{2}$
(4) $18i\sqrt{2}$

$2\sqrt{16}\sqrt{-1}\sqrt{2}-5\sqrt{4}\sqrt{-1}\sqrt{2}$

$8i\sqrt{2}-10i\sqrt{2}$

$-2i\sqrt{2}$

$(x, y) \rightarrow (x+2, y-6)$

8 The image of the origin under a certain translation is the point (2,–6). The image of point (–3,–2) under the same translation is the point

**Use this space for computations.**

(1) (–6,12)
(2) (–5,4)
(3) $\left(-\frac{3}{2}, \frac{1}{3}\right)$
(4) (–1,–8)

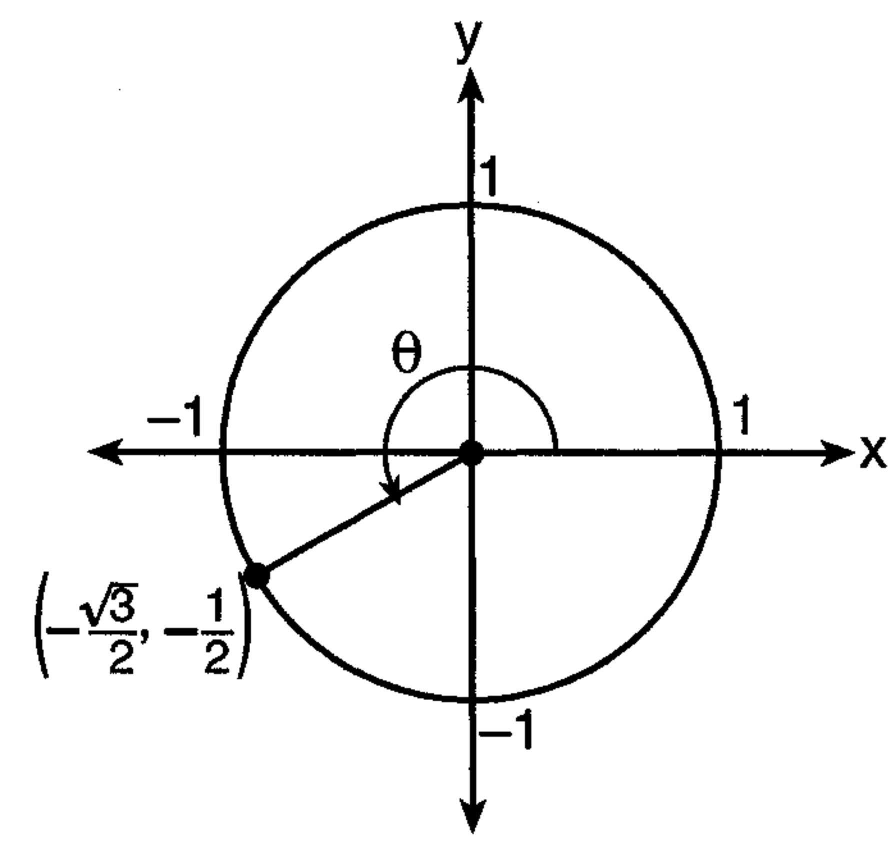
9 The solution of $|2x - 3| < 5$ is

(1) $x < -1$ or $x > 4$
(2) $-1 < x < 4$
(3) $x > -1$
(4) $x < 4$

$2x - 3 < 5$
$\underline{+3 \quad +3}$
$\frac{2x}{2} < \frac{8}{2}$
$x < 4$

$2x - 3 > -5$
$\underline{+3 \quad +3}$
$\frac{2x}{2} > \frac{-2}{2}$
$x > -1$

10 In the accompanying diagram of a unit circle, the ordered pair $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ represents the point where the terminal side of θ intersects the unit circle.




$\cos\theta = \frac{-\sqrt{3}}{2}$
$\theta = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$
$150^\circ$ and $210^\circ$

$\sin\theta = -\frac{1}{2}$
$\theta = \sin^{-1}\left(-\frac{1}{2}\right)$
$210^\circ$ and $330^\circ$

What is m∠θ?

(1) 210
(2) 225
(3) 233
(4) 240

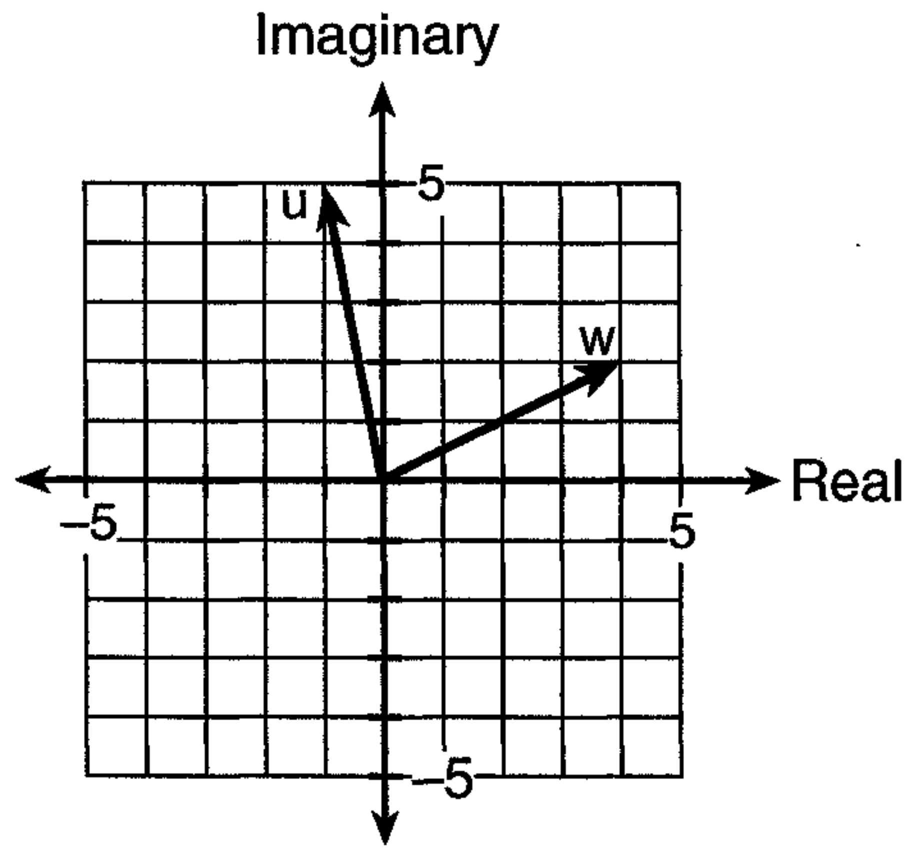
11 Two straight roads intersect at an angle whose measure is 125°. Which expression is equivalent to the cosine of this angle?

(1) cos 35°
(2) –cos 35°
(3) cos 55°
(4) –cos 55°

$\cos\theta = -\cos(180^\circ - \theta)$
$\cos 125^\circ = -\cos(180^\circ - 125^\circ)$
$= -\cos 55^\circ$

**12** Two complex numbers are graphed below.

**Use this space for computations.**




$w = 4 + 2i$

$u = -1 + 5i$

$3 + 7i$

What is the sum of $w$ and $u$, expressed in standard complex number form?

(1) $7 + 3i$
(2) $3 + 7i$
(3) $5 + 7i$
(4) $-5 + 3i$

**13** When simplified, the complex fraction $\dfrac{1 + \frac{1}{x}}{\frac{1}{x} - x}$, $x \neq 0$, is equivalent to

(1) 1
(2) $-1$
(3) $\dfrac{1}{1 - x}$
(4) $\dfrac{1}{x - 1}$

$$\frac{\frac{x+1}{x}}{\frac{1-x^2}{x}} = \frac{x+1}{x} \div \frac{(1-x)(1+x)}{x}$$

$$\frac{\cancel{x+1}}{\cancel{x}} \times \frac{\cancel{x}}{(1-x)\cancel{(1+x)}}$$

**14** A certain radio wave travels in a path represented by the equation $y = 5 \sin 2x$. What is the period of this wave?

(1) 5
(2) 2
(3) $\pi$
(4) $2\pi$

period $= \dfrac{2\pi}{b} = \dfrac{\cancel{2}\pi}{\cancel{2}} = \pi$

**15** The mean score on a normally distributed exam is 42 with a standard deviation of 12.1. Which score would be expected to occur *less than* 5% of the time?

(1) 25
(2) 32
(3) 60
(4) 67

The score expected to occur least is farthest from the mean.

$a=4 \quad b=m \quad c=9$

**16** For which positive value of $m$ will the equation $4x^2 + mx + 9 = 0$ have roots that are real, equal, and rational?

**Use this space for computations.**

(1) 12 (3) 3
(2) 9 (4) 4

$b^2 - 4ac = 0$

$m^2 - 4(4)(9) = 0$

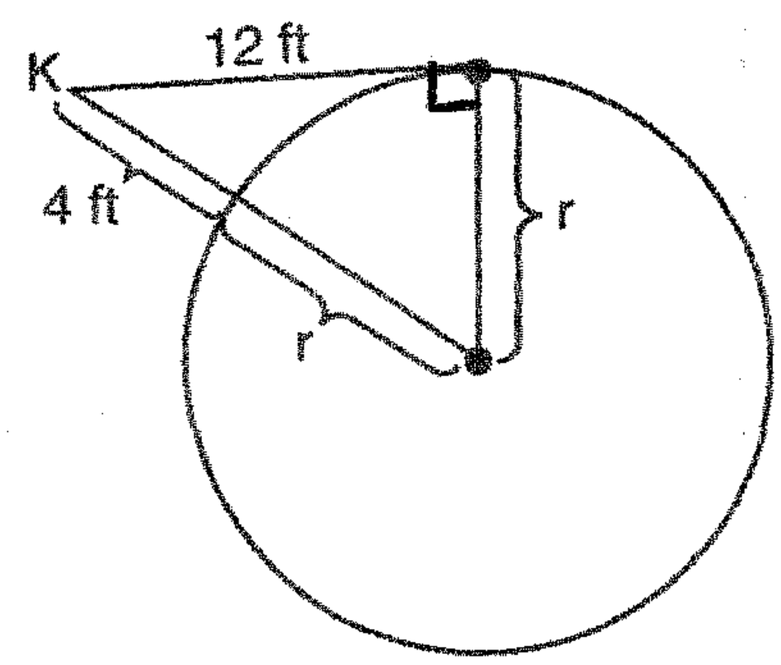
$m^2 = 144$

$m = 12$

**17** An object orbiting a planet travels in a path represented by the equation $3(y + 1)^2 + 5(x + 4)^2 = 15$. In which type of pattern does the object travel?

(1) hyperbola (3) circle
(2) ellipse (4) parabola

**18** Kimi wants to determine the radius of a circular pool without getting wet. She is located at point $K$, which is 4 feet from the pool and 12 feet from the point of tangency, as shown in the accompanying diagram.




$a^2 + b^2 = c^2$

$r^2 + 12^2 = (r+4)^2$

$\cancel{r^2} + 144 = \cancel{r^2} + 8r + 16$

$\frac{\cancel{8}r}{\cancel{8}} = \frac{128}{8}$

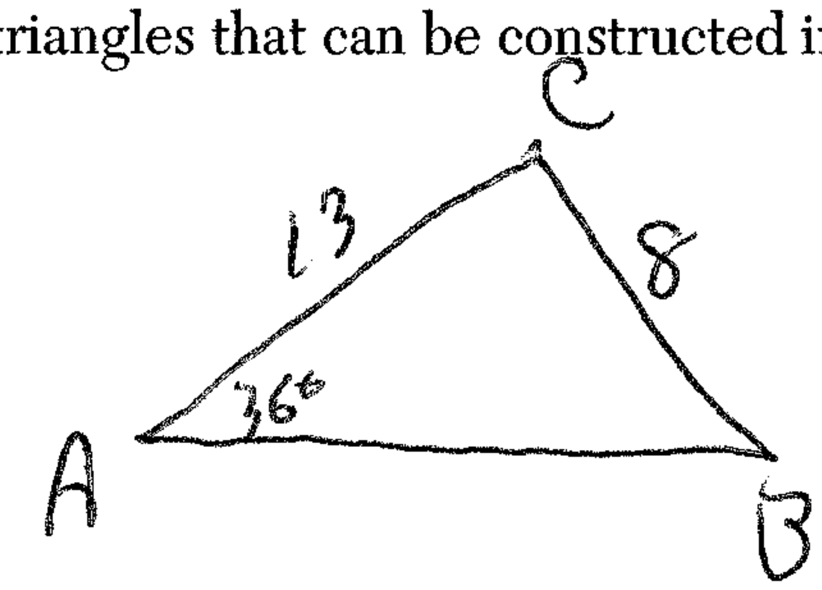
$r = 16$

What is the radius of the pool?

(1) 16 ft (3) 32 ft
(2) 20 ft (4) $4\sqrt{10}$ ft

19 What is the total number of distinct triangles that can be constructed if $AC = 13$, $BC = 8$, and $m\angle A = 36$?

(1) 1
(2) 2
(3) 3
(4) 0




**Use this space for computations.**

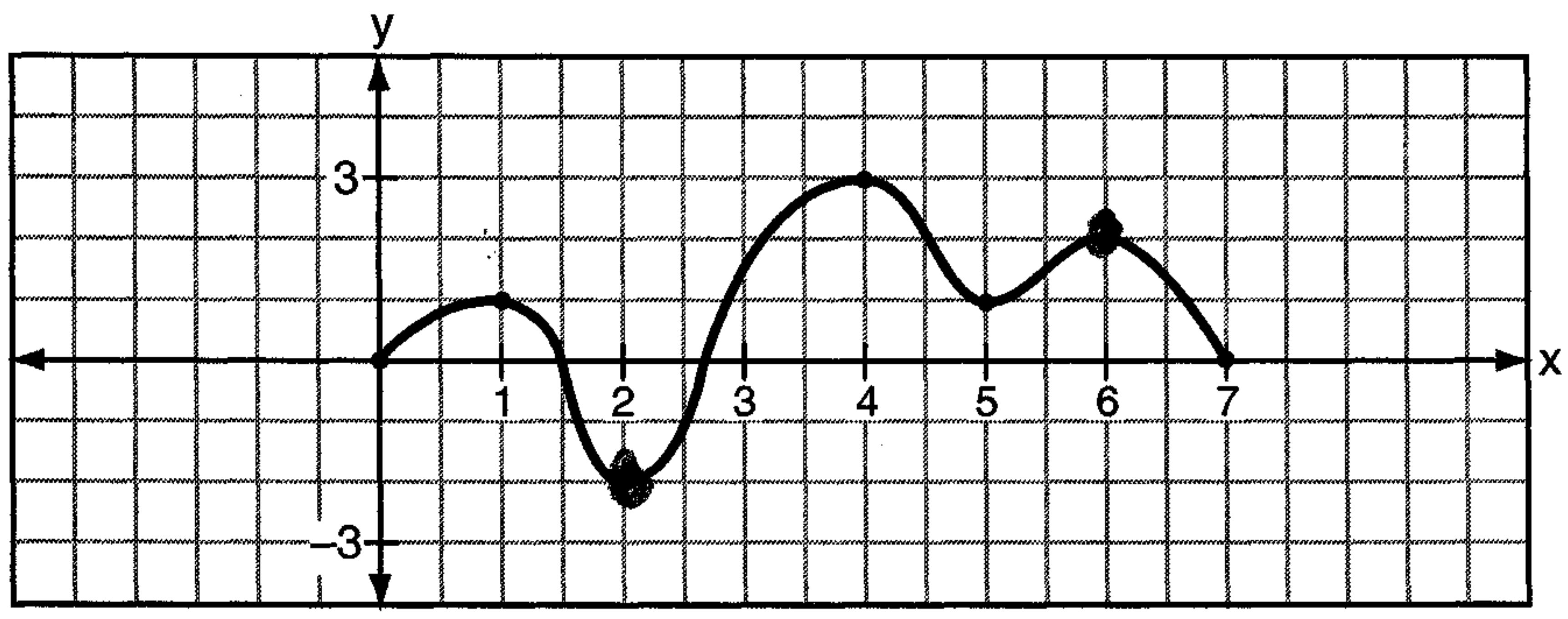
$$\frac{8}{\sin 36} = \frac{13}{\sin B}$$

$B \approx 73^\circ$ or $107^\circ$

$73 + 36 < 180$ ✓

$107 + 36 < 180$ ✓

20 The accompanying graph is a sketch of the function $y = f(x)$ over the interval $0 \le x \le 7$.




What is the value of $(f \circ f)(6)$?

(1) 1
(2) 2
(3) 0
(4) −2

$f(6) = 2$

$f(2) = -2$

## Part II

**Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.** [12]

**21** Evaluate: $\sum_{n=1}^{5}(n^2 + n)$

| $n$ | $n^2+n$ |
|---|---|
| 1 | 2 |
| 2 | 6 |
| 3 | 12 |
| 4 | 20 |
| 5 | 30 |
| | 70 |

**22** The Coolidge family's favorite television channels are 3, 6, 7, 10, 11, and 13. If the Coolidge family selects a favorite channel at random to view each night, what is the probability that they choose *exactly* three even-numbered channels in five nights? Express your answer as a fraction or as a decimal rounded to *four decimal places*.

$n = 5$

$r = 3$

$p = \frac{2}{6} = \frac{1}{3}$

$q = \frac{2}{3}$

$_5C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^2 = \frac{40}{243}$

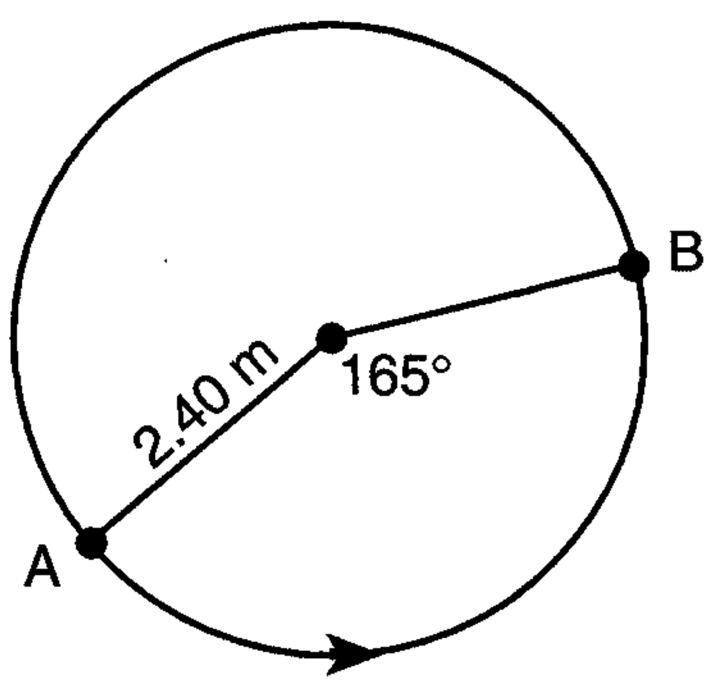
**23** Boyle's Law states that the pressure of compressed gas is inversely proportional to its volume. The pressure of a certain sample of a gas is 16 kilopascals when its volume is 1,800 liters. What is the pressure, in kilopascals, when its volume is 900 liters?

$P_1V_1 = P_2V_2$

$\frac{16 \times 1800}{900} = \frac{P_2 \times 900}{900}$

$32 = P_2$

**24** The accompanying diagram shows the path of a cart traveling on a circular track of radius 2.40 meters. The cart starts at point $A$ and stops at point $B$, moving in a counterclockwise direction. What is the length of minor arc $AB$, over which the cart traveled, to the *nearest tenth of a meter*?




$165° \cdot \frac{\pi}{180} = \frac{11\pi}{12}$ radians

$\theta = \frac{S}{R}$

$\frac{11\pi}{12} = \frac{S}{2.4}$

$\frac{12S}{12} = \frac{26.4\pi}{12}$

$S \approx 6.9$

**25** Given the function $y = \mathrm{f}(x)$, such that the entire graph of the function lies above the $x$-axis. Explain why the equation $\mathrm{f}(x) = 0$ has no real solutions.

Since the graph lies entirely above the x-axis, there is no point on the graph where $y=0$.

**26** Express in simplest terms: $\frac{2-2\ \sin^2 x}{\cos\ x}$

$$\frac{2(1-\sin^2 x)}{\cos x}$$

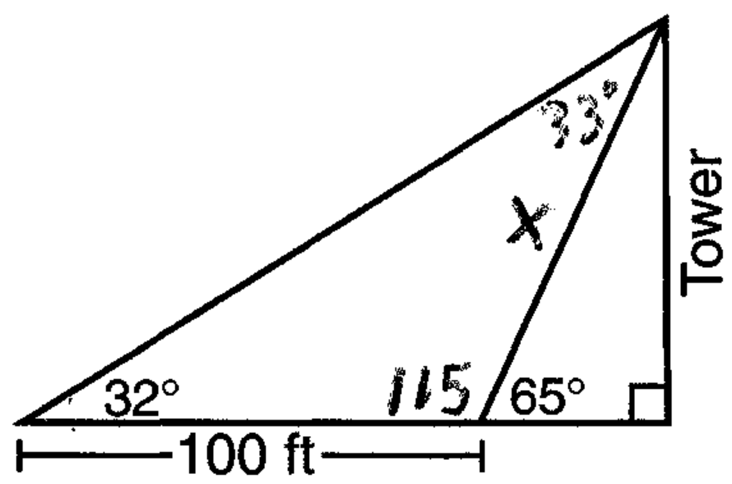
$$\frac{2\cos^2 x}{\cos x}$$

$$2\cos x$$

## Part III

**Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.** [24]

**27** The accompanying diagram shows the plans for a cell-phone tower that is to be built near a busy highway. Find the height of the tower, to the *nearest foot*.




$$\frac{x}{\sin 32} = \frac{100}{\sin 33}$$

$$x = \frac{100 \sin 32}{\sin 33}$$

$$x \approx 97.3$$

$$\sin 65 \approx \frac{T}{97.3}$$

$$T \approx 88$$

**28** The lateral surface area of a right circular cone, $s$, is represented by the equation $s = \pi r \sqrt{r^2 + h^2}$, where $r$ is the radius of the circular base and $h$ is the height of the cone. If the lateral surface area of a large funnel is 236.64 square centimeters and its radius is 4.75 centimeters, find its height, to the *nearest hundredth of a centimeter*.

$$s = \pi r \sqrt{r^2 + h^2}$$

$$\frac{236.64}{4.75\pi} = \frac{\cancel{4.75\pi}\sqrt{4.75^2 + h^2}}{\cancel{4.75\pi}}$$

$$\frac{236.64}{4.75\pi} = 4.75^2 + h^2$$

$$\sqrt{\frac{236.64}{4.75\pi} - 4.75^2} = h$$

$$15.13 \approx h$$

**29** Solve for all values of $x$: $\frac{9}{x} + \frac{9}{x-2} = 12$

$$\frac{9(x-2)+9x}{x(x-2)} = 12$$

$$\frac{9x-18+9x}{x^2-2x} = 12$$

$$18x-18 = 12x^2-24x$$

$$0 = 12x^2-42x+18$$

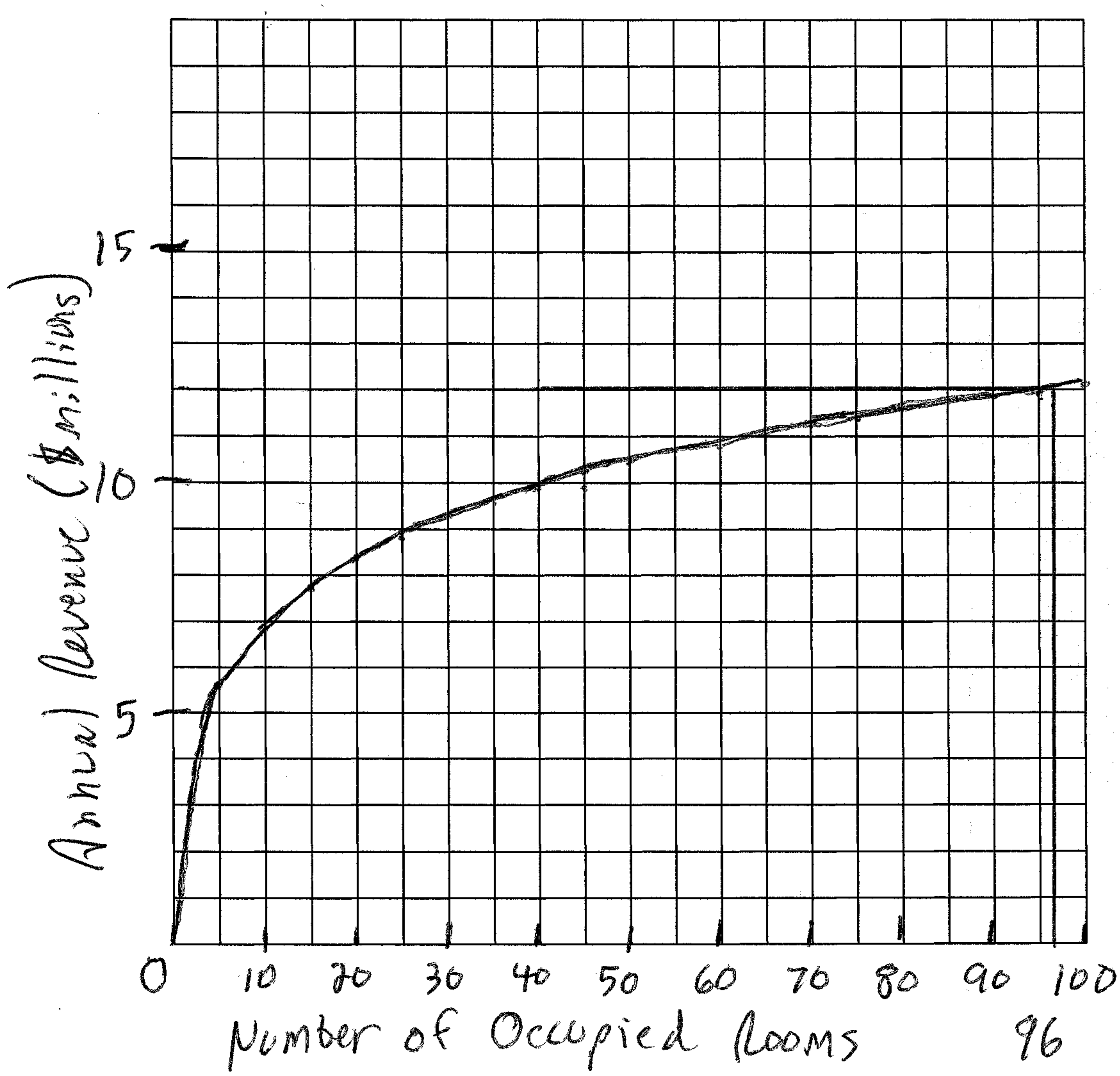
$$0 = 2x^2-7x+3$$

$$0 = (2x-1)(x-3)$$

$$x = \frac{1}{2} \qquad x = 3$$

**30** A hotel finds that its total annual revenue and the number of rooms occupied daily by guests can best be modeled by the function $R = 3 \log (n^2 + 10n)$, $n > 0$, where $R$ is the total annual revenue, in millions of dollars, and $n$ is the number of rooms occupied daily by guests. The hotel needs an annual revenue of \$12 million to be profitable. Graph the function on the accompanying grid over the interval $0 < n \leq 100$.

Calculate the minimum number of rooms that must be occupied daily for the hotel to be profitable. [Additional space is provided on the next page for your calculations.]



**Question 30 continued**

$$\frac{\cancel{3}\log(n^2+10n)}{\cancel{3}} = \frac{12}{3}$$

$$\log(n^2+10n) = 4$$

$$n^2+10n = 10^4$$

$$n^2+10n-10000 = 0$$

$$x = \frac{-10 \pm \sqrt{10^2-4(-10000)}}{2}$$

$$\frac{-10+\sqrt{40,100}}{2} \approx 95.1 \qquad \frac{-10-\sqrt{40,100}}{2}$$

negative solution

96 rooms

**31** The profit, $P$, for manufacturing a wireless device is given by the equation $P = -10x^2 + 750x - 9{,}000$, where $x$ is the selling price, in dollars, for each wireless device. What range of selling prices allows the manufacturer to make a profit on this wireless device? [The use of the grid on the next page is optional.]

$$-10x^2 + 750x - 9000 > 0$$

$$x^2 - 75x + 900 < 0$$

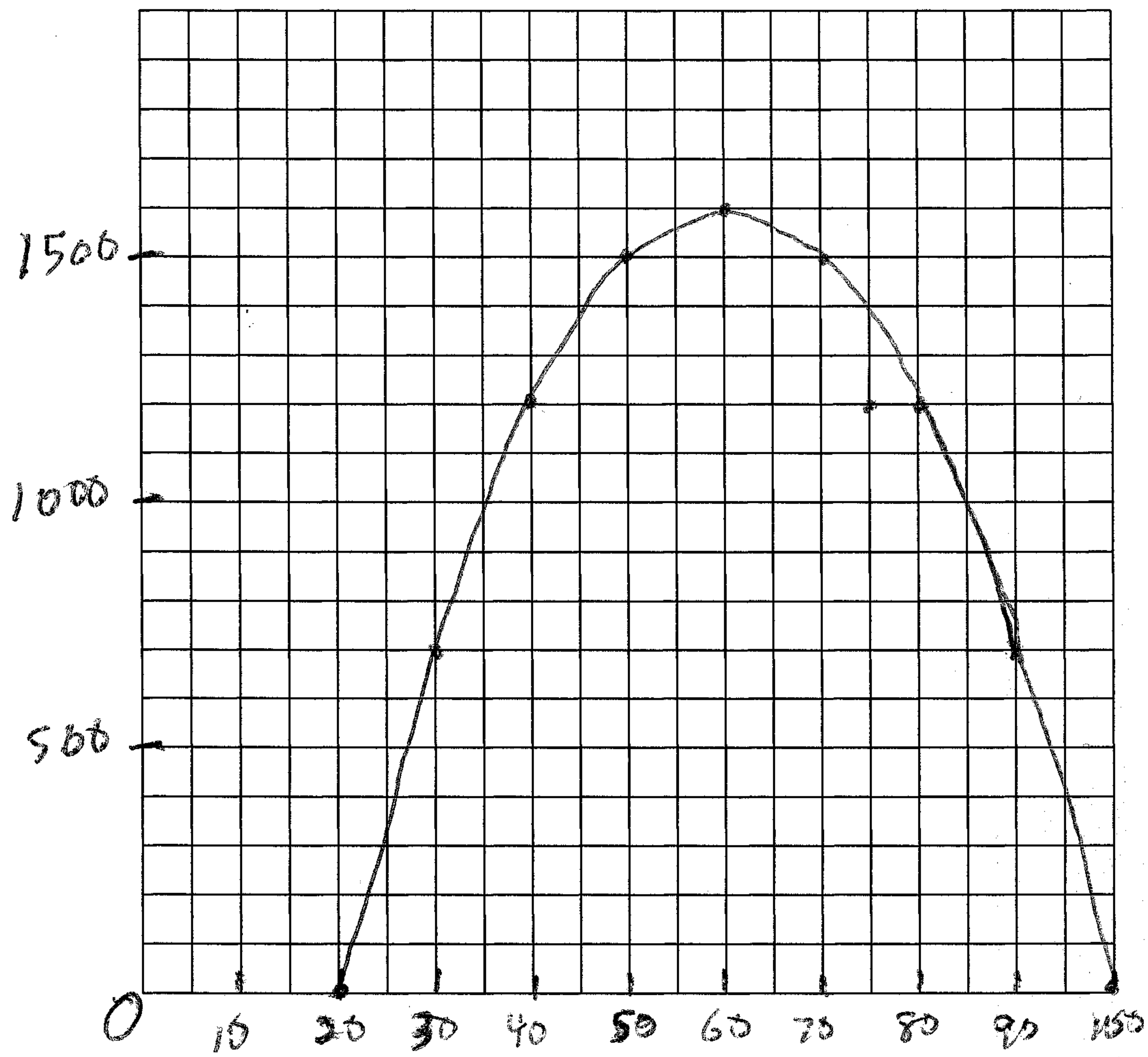
$$(x-60)(x-15) < 0$$

$$x - 60 < 0 \qquad x - 15 > 0$$

$$x < 60 \text{ and } x > 15$$

$$15 < x < 60$$

**Question 31 continued**



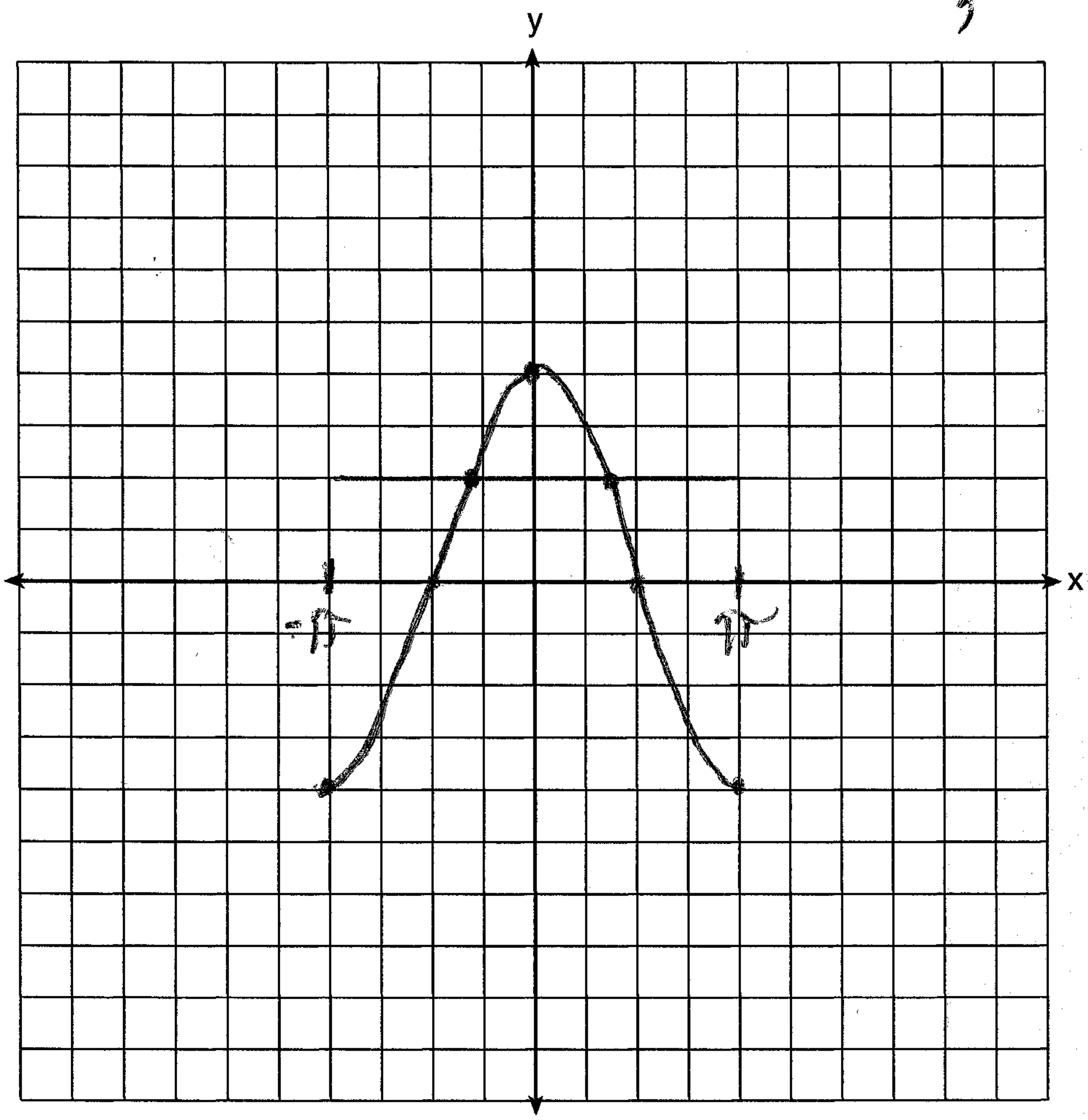

$20 < x < 100$

**32** On the accompanying set of axes, graph the equations $y = 4 \cos x$ and $y = 2$ in the domain $-\pi \leq x \leq \pi$.

Express, in terms of $\pi$, the interval for which $4 \cos x \geq 2$.

$\cos x \geq \frac{2}{4}$

$\frac{-\pi}{3} \leq x \leq \frac{\pi}{3}$



## Part IV

**Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.** [12]

**33** The accompanying table illustrates the number of movie theaters showing a popular film and the film's weekly gross earnings, in millions of dollars.

| **Number of Theaters ($x$)** | 443 | 455 | 493 | 530 | 569 | 657 | 723 | 1,064 |
|---|---|---|---|---|---|---|---|---|
| **Gross Earnings ($y$)** (millions of dollars) | 2.57 | 2.65 | 3.73 | 4.05 | 4.76 | 4.76 | 5.15 | 9.35 |

Write the linear regression equation for this set of data, rounding values to *five decimal places*. $y = .01021x - 1.66787$

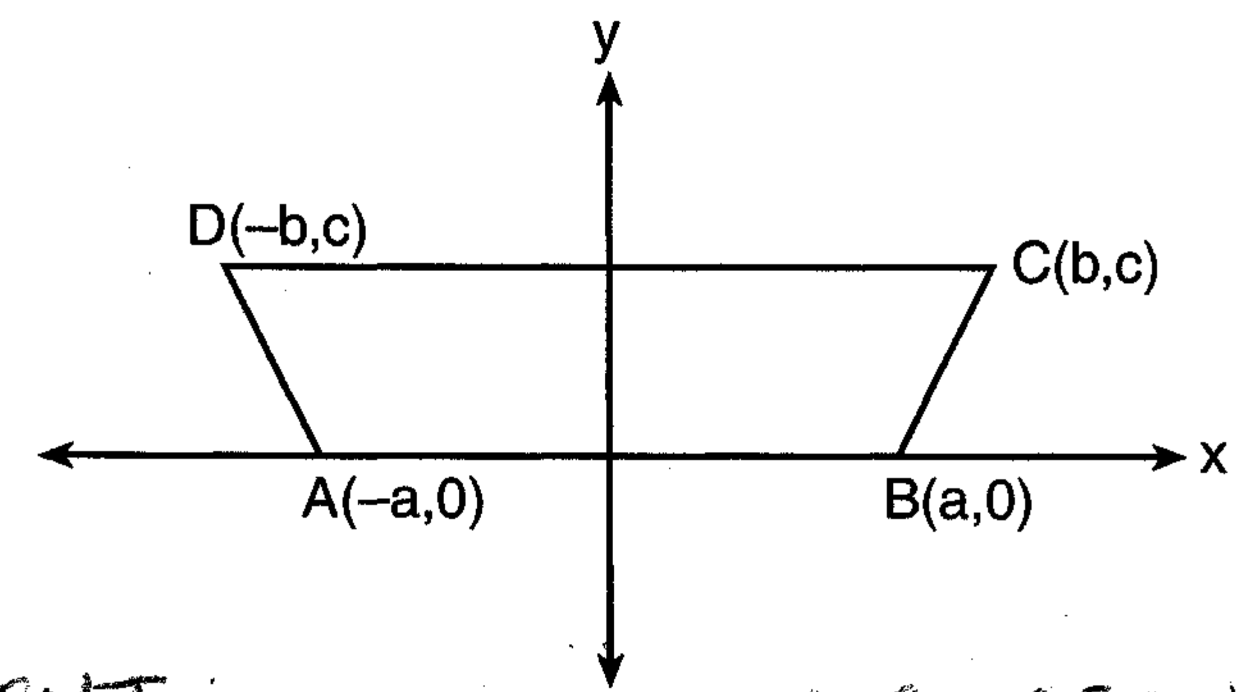
Using this linear regression equation, find the approximate gross earnings, in millions of dollars, generated by 610 theaters. Round your answer to *two decimal places*. $y = .01021(610) - 1.66787 \approx 4.56$

Find the minimum number of theaters that would generate at least 7.65 million dollars in gross earnings in one week.

$7.65 = .01021x - 1.66787$

$x \approx 913$

**34** In the accompanying diagram of $ABCD$, where $a \neq b$, prove $ABCD$ is an isosceles trapezoid.




| STATEMENT | REASON |
|---|---|
| (1) Quadrilateral ABCD with vertices $A(-a,0)$, $B(a,0)$, $C(b,c)$, $D(-b,c)$, $a \neq b$ | (1) Given |
| (2) $m_{\overline{AB}} = \frac{0-0}{-a-a} = \frac{0}{-2a} = 0$<br>$m_{\overline{CD}} = \frac{c-c}{-b-b} = \frac{0}{-2b} = 0$<br>$m_{\overline{AD}} = \frac{c-0}{-b-(-a)} = \frac{c}{-b+a}$<br>$m_{BC} = \frac{c-0}{b-a} = \frac{c}{b-a}$ | (2) Definition of slope |
| (3) $\overline{AB} \parallel \overline{CD}$ | (3) Parallel lines have the same slope. |
| (4) If $\overline{AD} \parallel \overline{BC}$, then $\frac{c}{-b+a} = \frac{c}{b-a}$<br>$b-a = -b+a$<br>$2a = 2b$<br>$a = b$<br>but $a \neq b$, so $\overline{AD}$ and $\overline{BC}$ are not parallel | (4) Assumption |
| (5) ABCD is a trapezoid | (5) A trapezoid has one and only one pair of parallel sides. |



continued →

6 $d_{\overline{BC}} = \sqrt{(b-a)^2 + (c-0)^2}$

$= \sqrt{b^2 - 2ab + a^2 + c^2}$

$= \sqrt{a^2 + b^2 - 2ab + c^2}$

$d_{\overline{AD}} = \sqrt{(a-b)^2 + c^2}$

$= \sqrt{a^2 - 2ab + b^2 + c^2}$

$= \sqrt{a^2 + b^2 - 2ab + c^2}$

6 Distance Formula

7 $\overline{AD} \cong \overline{BC}$

7 Sides have equal length.

8 $\overline{ABCD}$ is an isosceles trapezoid

8 The non-parallel sides are congruent.