The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS B

Tuesday, August 17, 2004 — 8:30 to 11:30 a.m., only

Print Your Name: Steven Sibol

Print Your School's Name: HSCR

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will *not* be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.

This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. The formulas that you may need to answer some questions in this examination are found on page 23.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice. . .

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [40]

Use this space for computations.

1 Which condition does *not* prove that two triangles are congruent?

(1) SSS ≅ SSS
(2) SSA ≅ SSA
(3) SAS ≅ SAS
(4) ASA ≅ ASA

The ambiguous case

2 The speed of a laundry truck varies inversely with the time it takes to reach its destination. If the truck takes 3 hours to reach its destination traveling at a constant speed of 50 miles per hour, how long will it take to reach the same location when it travels at a constant speed of 60 miles per hour?

(1) $2\frac{1}{3}$ hours
(2) 2 hours
(3) $2\frac{1}{2}$ hours
(4) $2\frac{2}{3}$ hours

$s_1 t_1 = s_2 t_2$

$\frac{50 \cdot 3}{60} = \frac{60 t_2}{60}$

$\frac{5}{2} = t_2$

3 Which set of ordered pairs is *not* a function?

(1) {(3,1), (2,1), (1,2), (3,2)}
(2) {(4,1), (5,1), (6,1), (7,1)}
(3) {(1,2), (3,4), (4,5), (5,6)}
(4) {(0,0), (1,1), (2,2), (3,3)}

An element of the domain, 3, relates to more than one element in the range, 1 and 2.

4 A circle has the equation $(x + 1)^2 + (y - 3)^2 = 16$. What are the coordinates of its center and the length of its radius?

(1) (–1,3) and 4
(2) (1,–3) and 4
(3) (–1,3) and 16
(4) (1,–3) and 16

$(x-h)^2 + (x-k)^2 = r^2$

$(x-(-1))^2 + y-(3))^2 = 4^2$

5 The mean of a normally distributed set of data is 56, and the standard deviation is 5. In which interval do approximately 95.4% of all cases lie?

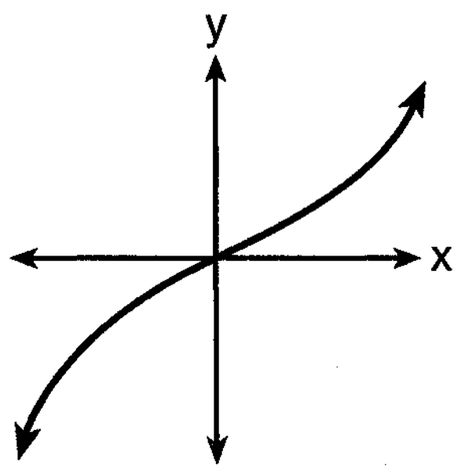
(1) 46–56
(2) 46–66
(3) 51–61
(4) 56–71

95.4% of the cases lie with two standard deviations of the mean. In this case 2 standard deviations is 10

$56 \pm 10 \rightarrow 46\text{-}66$

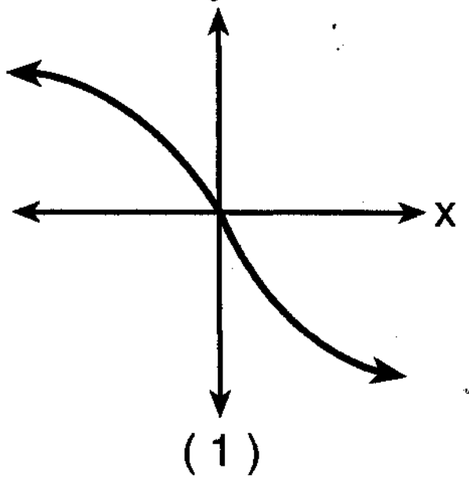
[2]

6 The graph below represents f(x).

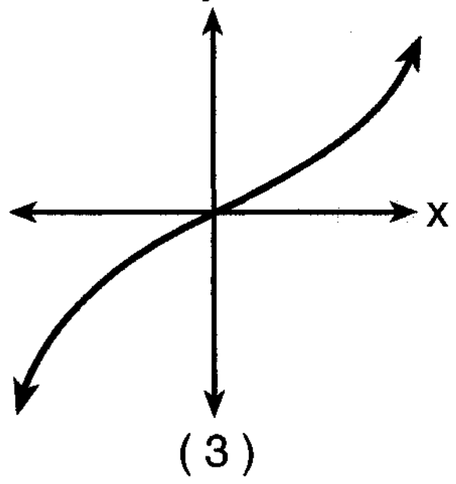


Which graph best represents f($-x$)?

(1)



(3)



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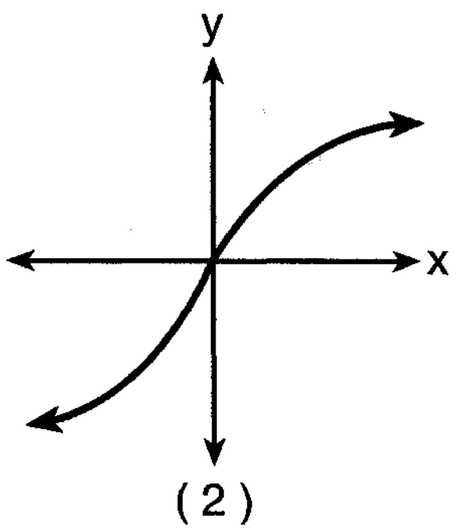
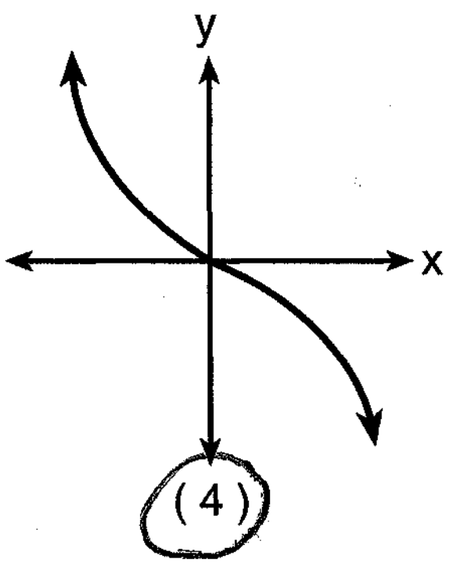


(4)



Use this space for computations.

f(-x) is a reflection in the y-axis

7 When simplified, $i^{27} + i^{34}$ is equal to

(1) i
(2) i^{61}
(3) $-i - 1$
(4) $i - 1$

$\frac{27}{4} = 6$ rem 3 $\quad \frac{34}{4} = 8$ rem 2

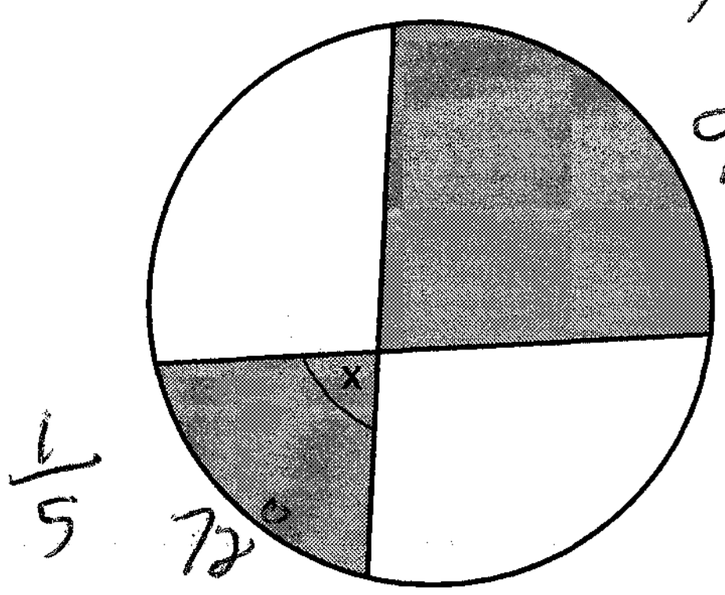
$i^{27} + i^{34}$

$i^3 + i^2$

$-i - 1$

8 The accompanying diagram shows a child's spin toy that is constructed from two chords intersecting in a circle. The curved edge of the larger shaded section is one-quarter of the circumference of the circle, and the curved edge of the smaller shaded section is one-fifth of the circumference of the circle.

Use this space for computations.



1/4
90°
1/5
$x = \frac{90+72}{2} = 81°$

What is the measure of angle x?

(1) 40°
(2) 72°
(3) 81°
(4) 108°

9 If $\sin A = \frac{4}{5}$, $\tan B = \frac{5}{12}$, and angles A and B are in Quadrant I, what is the value of $\sin (A + B)$?

(1) $\frac{63}{65}$
(2) $-\frac{63}{65}$
(3) $\frac{33}{65}$
(4) $-\frac{33}{65}$

$\sin A = \frac{4}{5}$ $\cos A = \frac{3}{5}$ because $\sin^2 A + \cos^2 A = 1$

$\sin(A+B)$

$\sin A \cos B + \cos A \sin B$

$\frac{4}{5}\frac{12}{13} + \frac{3}{5}\frac{5}{13} = \frac{48}{65} + \frac{15}{65} = \frac{63}{65}$

$\tan^2 B + 1 = \sec^2 B$

$\left(\frac{5}{12}\right)^2 + 1 = \sec^2 B$

$\frac{25}{144} + \frac{144}{144} = \sec^2 B$

$\frac{169}{144} = \sec^2 B$

$\frac{13}{12} = \sec B$

$\frac{12}{13} = \cos B$

$\frac{5}{13} = \sin B$

10 If the tangent of an angle is negative and its secant is positive, in which quadrant does the angle terminate?

(1) I
(2) II
(3) III
(4) IV

If secant is positive, then cosine is positive

$\tan\theta = \frac{\sin\theta}{\cos\theta}$. If tangent is negative and cos is positive, sine is negative.

Cos is positive & sine is negative in Quadrant IV.

11 The equation $2x^2 + 8x + n = 0$ has imaginary roots when n is equal to

(1) 10
(2) 8
(3) 6
(4) 4

$a = 2$

$b = 8$

$c = n$

$b^2 - 4ac < 0$

$8^2 - 4(2)(n) < 0$

$64 - 8n < 0$

$\frac{64}{8} < \frac{8n}{8}$ $8 < n$ or $n > 8$.

12 What is the middle term in the expansion of $(x + y)^4$?

(1) x^2y^2
(3) $6x^2y^2$
(2) $2x^2y^2$
(4) $4x^2y^2$

Use this space for computations.

$n = 4$

$r - 1 = 2$

$_nC_{r-1}\, x^{n-(r-1)}\, y^{r-1}$

$_4C_2\, x^{4-2}\, y^2 = 6x^2y^2$

13 What is the image of point (1,1) under $r_{x\text{-axis}} \circ R_{0,90°}$?

(1) (1,1)
(3) (–1,1)
(2) (1,–1)
(4) (–1,–1)

After $R_{0,90°}$ $(x, y) \rightarrow (-y, x)$

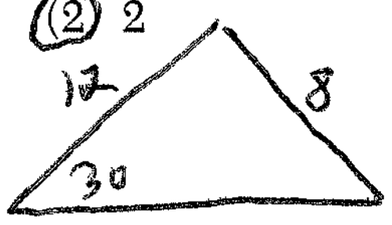
$(1, 1) \rightarrow (-1, 1)$

After $r_{x\text{-axis}}$ $(x, y) \rightarrow (x, -y)$

$(-1, 1) \rightarrow (-1, -1)$

14 How many distinct triangles can be formed if m$\angle A$ = 30, side b = 12, and side a = 8?

(1) 1
(2) 2
(3) 3
(4) 0



$\frac{8}{\sin 30} = \frac{12}{\sin C}$

$C \approx 49°$ or $131°$

$49 + 30 < 180$ ✓

$131 + 30 < 180$ ✓

15 The expression $\frac{(b^{2n+1})^3}{b^n \cdot b^{4n+3}}$ is equivalent to

(1) $\frac{b^n}{2}$
(3) b^{-3n}
(2) b^n
(4) b^{-3n+1}

$\frac{b^{6n+3}}{b^{5n+3}} = b^n$

16 What is the inverse of the function $y = \log_4 x$?

(1) $x^4 = y$
(3) $4^x = y$
(2) $y^4 = x$
(4) $4^y = x$

rewrite as an exponential function

$4^y = x$

inverse: $4^x = y$

17 Which angle is coterminal with an angle of 125°?

(1) –125°
(3) 235°
(2) –235°
(4) 425°

$-235 + 360 = 125$

18 A ball is dropped from a height of 8 feet and allowed to bounce. Each time the ball bounces, it bounces back to half its previous height. The vertical distance the ball travels, d, is given by the formula $d = 8 + 16\sum_{k=1}^{n}\left(\frac{1}{2}\right)^k$, where n is the number of bounces. Based on this formula, what is the total vertical distance that the ball has traveled after four bounces?

(1) 8.9 ft
(2) 15.0 ft
(3) 22.0 ft
(4) 23.0 ft

Use this space for computations.

K	$\left(\frac{1}{2}\right)^K$
1	$\frac{1}{2}$
2	$\frac{1}{4}$
3	$\frac{1}{8}$
4	$\frac{1}{16}$

$8 + 16\left(\frac{15}{16}\right) = 23$

19 The path traveled by a roller coaster is modeled by the equation $y = 27 \sin 13x + 30$. What is the maximum altitude of the roller coaster?

(1) 13
(2) 27
(3) 30
(4) 57

The maximum of a sine wave is 1. The amplitude factor raises the maximum to 27. The translation (+30) raises the maximum to 57.

20 The expression $\frac{11}{\sqrt{3}-5}$ is equivalent to

(1) $\frac{-\sqrt{3}-5}{2}$
(2) $\frac{-\sqrt{3}+5}{2}$
(3) $\frac{\sqrt{3}-5}{2}$
(4) $\frac{\sqrt{3}+5}{2}$

$$\frac{11}{\sqrt{3}-5}\left(\frac{\sqrt{3}+5}{\sqrt{3}+5}\right) = \frac{11(\sqrt{3}+5)}{3-25}$$

$$= \frac{11(\sqrt{3}+5)}{-22}$$

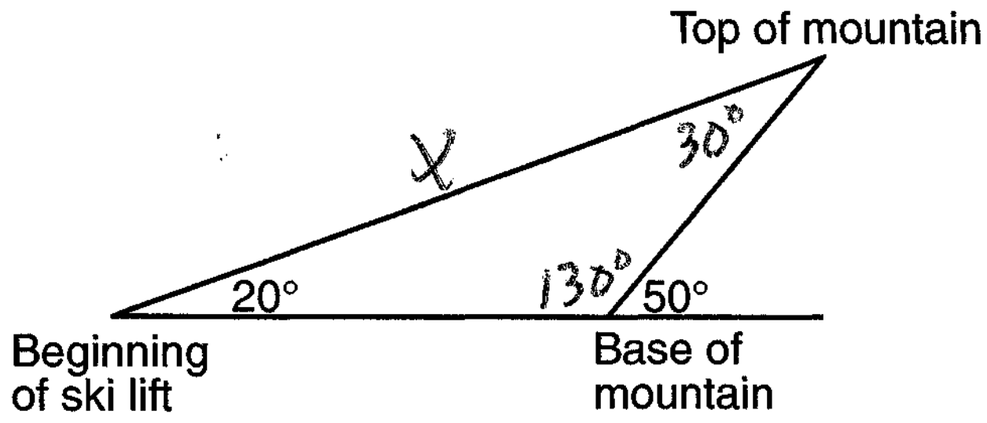
$$= \frac{-(\sqrt{3}+5)}{2}$$

$$= \frac{-\sqrt{3}-5}{2}$$

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

21 A ski lift begins at ground level 0.75 mile from the base of a mountain whose face has a 50° angle of elevation, as shown in the accompanying diagram. The ski lift ascends in a straight line at an angle of 20°. Find the length of the ski lift from the beginning of the ski lift to the top of the mountain, to the *nearest hundredth of a mile*.



$$\frac{x}{\sin 130} = \frac{.75}{\sin 30}$$

$$x = \frac{.75 \sin 130}{\sin 30}$$

$$x \approx 1.15$$

22 Express $\sqrt{-48} + 3.5 + \sqrt{25} + \sqrt{-27}$ in simplest $a + bi$ form.

$\sqrt{16}\sqrt{-1}\sqrt{3} + 3.5 + 5 + \sqrt{9}\sqrt{-1}\sqrt{3}$

$4i\sqrt{3} + 8.5 + 3i\sqrt{3}$

$8.5 + 7i\sqrt{3}$

23 Solve for x: $x^{-3} = \frac{27}{64}$

$x^3 = \frac{64}{27}$

$x = \frac{4}{3}$

24 The profit a coat manufacturer makes each day is modeled by the equation $P(x) = -x^2 + 120x - 2{,}000$, where P is the profit and x is the price for each coat sold. For what values of x does the company make a profit? [The use of the accompanying grid is optional.]

$-x^2+120x-2000>0$

$x^2-120x+2000<0$

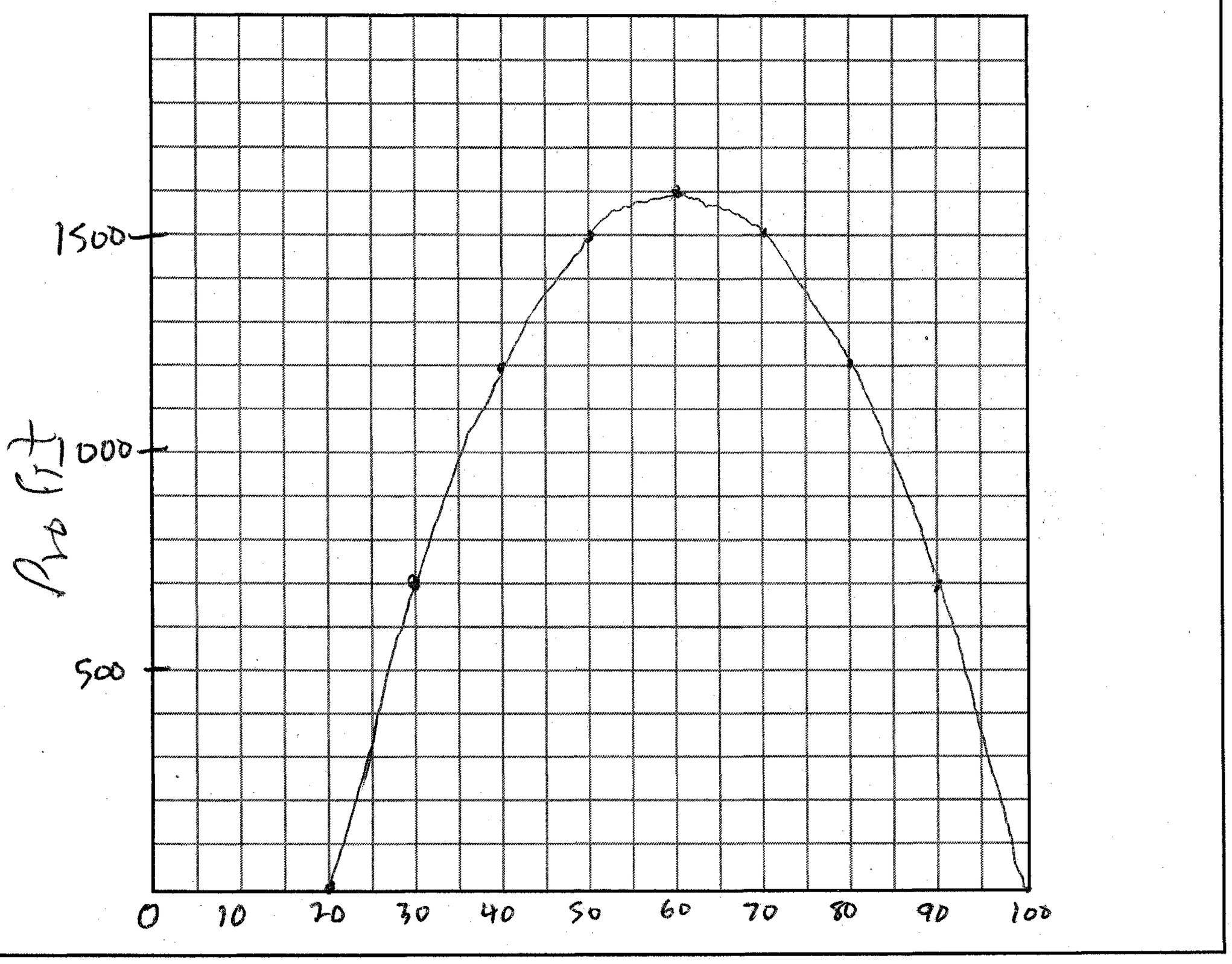
$(x-100)(x-20)<0$

$x-100<0$ and $x-20>0$ or $x-100>100$ and $x-20<0$

$x<100$ and $x>20$ or $x>100$ and $x<20$

not possible

$20<x<100$



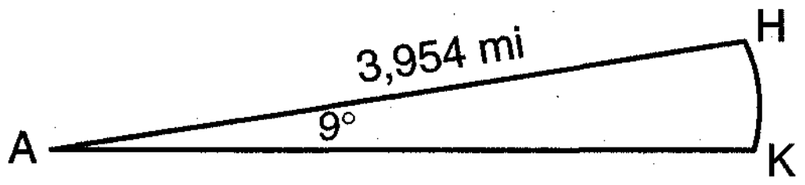
25 Express in simplest form: $\dfrac{\frac{1}{r}-\frac{1}{s}}{\frac{r^2}{s^2}-1}$

$$\frac{\frac{s-r}{rs}}{\frac{r^2-s^2}{s^2}} = \frac{s-r}{rs} \times \frac{s^2}{r^2-s^2}$$

$$-1\frac{s-r}{r} \times \frac{s}{(r-s)(r+s)}$$

$$\frac{-s}{r(r+s)}$$

26 Cities H and K are located on the same line of longitude and the difference in the latitude of these cities is 9°, as shown in the accompanying diagram. If Earth's radius is 3,954 miles, how many miles north of city K is city H along arc HK? Round your answer to the *nearest tenth of a mile*.



(Not drawn to scale)

$$9°\left(\frac{\pi}{180°}\right) = \frac{\pi}{20} \text{ radians}$$

$$\theta = \frac{s}{r}$$

$$\frac{\pi}{20} = \frac{s}{3954}$$

$$\frac{20}{20}s = \frac{3954\pi}{20}$$

$$s \approx 621.1$$

Part III

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [24]

27 A depth finder shows that the water in a certain place is 620 feet deep. The difference between d, the actual depth of the water, and the reading is $|d - 620|$ and must be less than or equal to $0.05d$. Find the minimum and maximum values of d, to the *nearest tenth of a foot*.

$|d-620| \leq .05d$

$d-620 \leq .05d$

$\frac{.95d}{.95} \leq \frac{620}{.95}$

$d \leq 652.6$

$d-620 \geq -.05d$

$\frac{1.05d}{1.05} \geq \frac{620}{1.05}$

$d \geq 590.5$

28 An amount of P dollars is deposited in an account paying an annual interest rate r (as a decimal) compounded n times per year. After t years, the amount of money in the account, in dollars, is given by the equation $A = P\left(1 + \frac{r}{n}\right)^{nt}$.

Rachel deposited \$1,000 at 2.8% annual interest, compounded monthly. In how many years, to the *nearest tenth of a year*, will she have \$2,500 in the account? [The use of the grid on the next page is optional.]

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

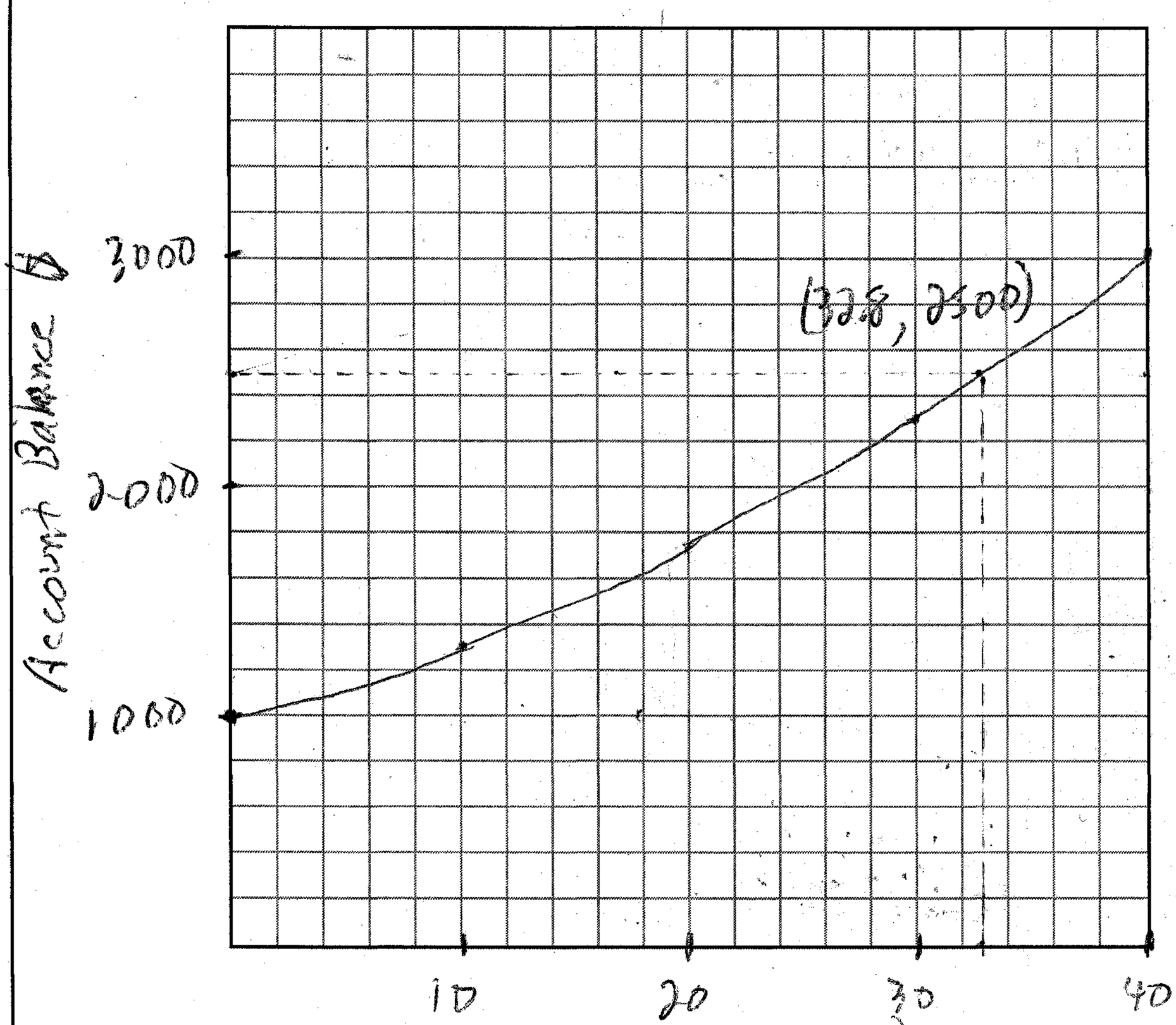
$\frac{2500}{1000} = \frac{1000\left(1 + \frac{.028}{12}\right)^{12t}}{1000}$

$\log 2.5 = \log\left(1 + \frac{.007}{3}\right)^{12t}$

$\frac{\log 2.5}{12\log\left(1 + \frac{.007}{3}\right)} = \frac{12t \log\left(1 + \frac{.007}{3}\right)}{12\log\left(1 + \frac{.007}{3}\right)}$

$32.8 \approx t$

Question 28 continued



29 A box containing 1,000 coins is shaken, and the coins are emptied onto a table. Only the coins that land heads up are returned to the box, and then the process is repeated. The accompanying table shows the number of trials and the number of coins returned to the box after each trial.

Trial	0	1	3	4	6
Coins Returned	1,000	610	220	132	45

Write an exponential regression equation, rounding the calculated values to the *nearest ten-thousandth*. $y = 1018.2839(0.5969)^x$

Use the equation to predict how many coins would be returned to the box after the eighth trial.

$y = 1018.2839(0.5969)^8$

≈ 16

30 Tim Parker, a star baseball player, hits one home run for every ten times he is at bat. If Parker goes to bat five times during tonight's game, what is the probability that he will hit *at least* four home runs?

$n = 5$

$r = 4, 5$

$p = \frac{1}{10}$

$q = \frac{9}{10}$

$P(4) = {}_5C_4 \left(\frac{1}{10}\right)^4 \left(\frac{9}{10}\right)^1 = \frac{45}{100,000}$

$P(5) = {}_5C_5 \left(\frac{1}{10}\right)^5 \left(\frac{9}{10}\right)^0 = \frac{1}{100,000}$

$\frac{46}{100,000}$

31 A rectangular piece of cardboard is to be formed into an uncovered box. The piece of cardboard is 2 centimeters longer than it is wide. A square that measures 3 centimeters on a side is cut from each corner. When the sides are turned up to form the box, its volume is 765 cubic centimeters. Find the dimensions, in centimeters, of the original piece of cardboard.

$V = L\,W\,H$

$765 = (w+2)(w)(3)$

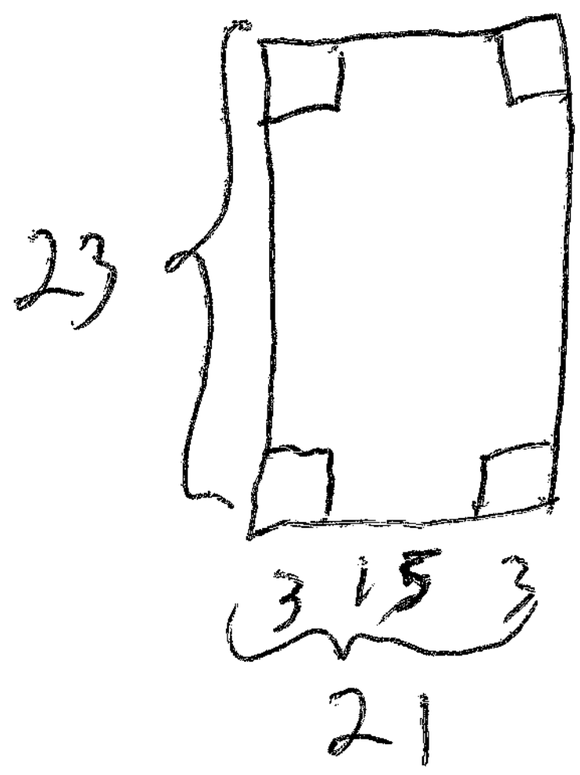
$765 = 3w^2 + 6w$

$3w^2 + 6w - 765 = 0$

$w^2 + 2w - 255 = 0$

$(w+17)(w-15) = 0$

reject $w = -17$ $w = 15$



21×23

32 Solve algebraically for all values of θ in the interval $0° \leq \theta < 360°$ that satisfy the equation $\frac{\sin^2 \theta}{1 + \cos \theta} = 1$.

cross multiply $\sin^2\theta = 1 + \cos\theta$

$1 - \cos^2\theta = 1 + \cos\theta$

$\cos^2\theta + \cos\theta = 0$

$\cos\theta(\cos\theta + 1) = 0$

$\cos\theta = 0$

$\theta = 90°, 270°$

$\cos\theta + 1 = 0$

$\cos\theta = -1$

Extraneous solution

If $\cos\theta = -1$, the denominator of the equation equals 0.

Part IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

33 The tide at a boat dock can be modeled by the equation $y = -2\cos\left(\frac{\pi}{6}t\right) + 8$, where t is the number of hours past noon and y is the height of the tide, in feet. For how many hours between $t = 0$ and $t = 12$ is the tide at least 7 feet? [The use of the grid on the next page is optional.]

$-2\cos\left(\frac{\pi}{6}t\right) + 8 \geq 7$

$-8 \quad -8$

$\frac{-2\cos\left(\frac{\pi}{6}t\right)}{-2} \geq \frac{-1}{-2}$

$\cos\frac{\pi}{6}t \leq \frac{1}{2}$

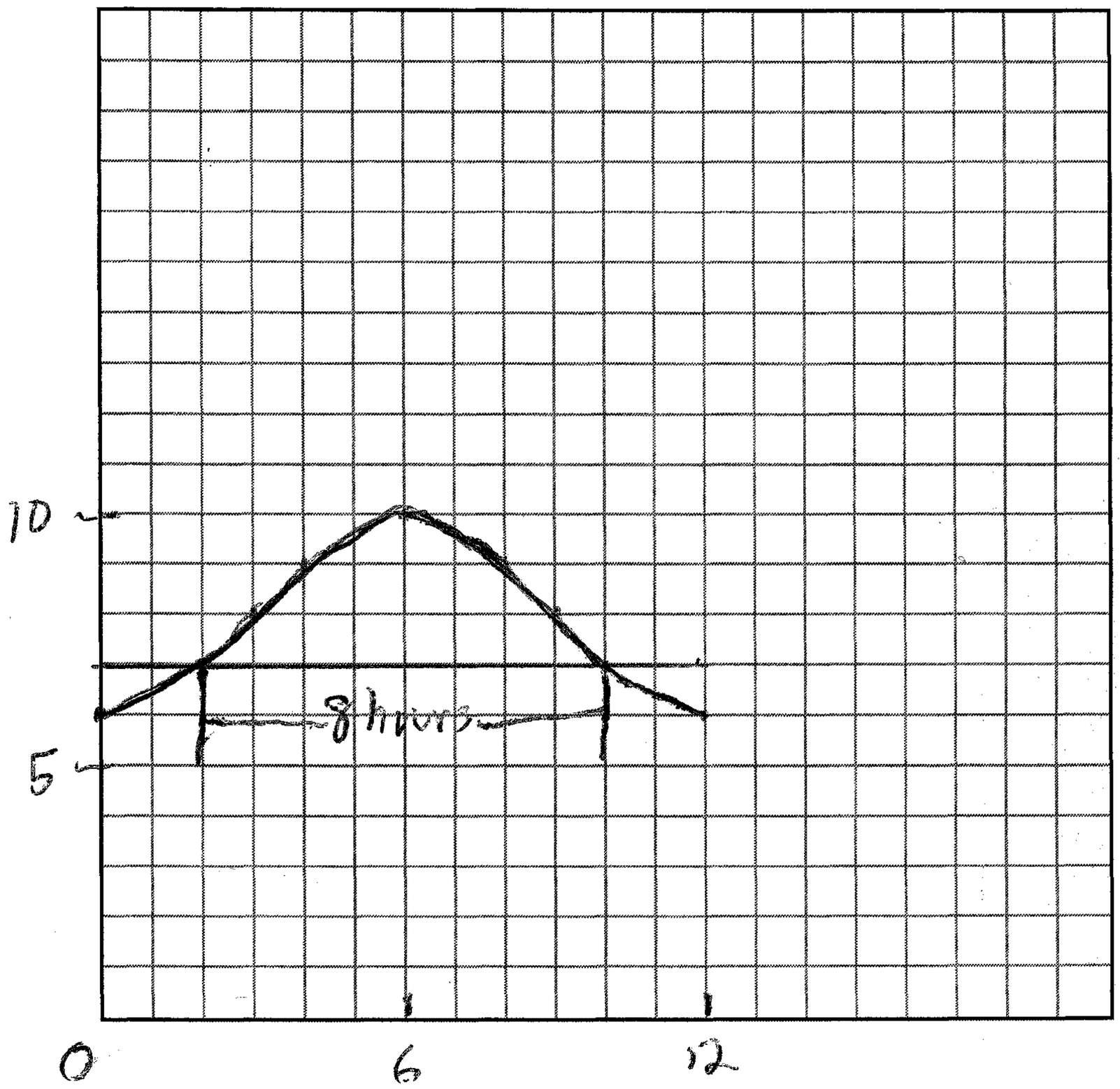
$\frac{\pi}{6}t \leq \cos^{-1}\left(\frac{1}{2}\right)$

$\frac{6}{\pi}\left(\frac{\pi}{3} \leq \frac{\pi}{6}t \leq \frac{5\pi}{3}\right)$

$2 \leq t \leq 10$

8 hours

Question 33 continued



34 The coordinates of quadrilateral *JKLM* are *J*(1,–2), *K*(13,4), *L*(6,8), and *M*(–2,4). Prove that quadrilateral *JKLM* is a trapezoid but *not* an isosceles trapezoid. [The use of the grid on the next page is optional.]

	STATEMENT	REASON
(1)	Quadrilateral JKLM with J(1,-2), K(13,4), L(6,8), and M(-2,4)	(1) Given
(2)	$m_{\overline{JK}} = \frac{4-(-2)}{13-1} = \frac{1}{2}$ $m_{\overline{LM}} = \frac{8-4}{6-(-2)} = \frac{1}{2}$ $m_{\overline{JM}} = \frac{-2-4}{1-(-2)} = -2$ $m_{\overline{KL}} = \frac{4-8}{13-6} = -\frac{4}{7}$	(2) Definition of slope
(3)	$\overline{JK} \parallel \overline{LM}$, $\overline{JM} \nparallel \overline{KL}$	(3) Parallel lines have equal slope.
(4)	JKLM is a trapezoid	(4) A trapezoid is a quadrilateral with one and only one pair of parallel sides.
(5)	$d_{\overline{JM}} = \sqrt{(1-(-2))^2+(-2-4)^2} = \sqrt{45}$ $d_{\overline{KL}} = \sqrt{(13-6)^2+(4-8)^2} = \sqrt{65}$	(5) $\overline{JM} \neq \overline{KL}$
(6)	JKLM is not an isosceles trapezoid	(6) The non-parallel sides are also not congruent.

Question 34 continued