The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS B

Wednesday, August 13, 2003 — 8:30 to 11:30 a.m., only

Print Your Name: *Steve Sibol*

Print Your School's Name: *HSCR*

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will *not* be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.

This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. The formulas that you may need to answer some questions in this examination are found on page 19.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice. . .

A graphing calculator, a straightedge (ruler), and a compass must be available for your use while taking this examination.
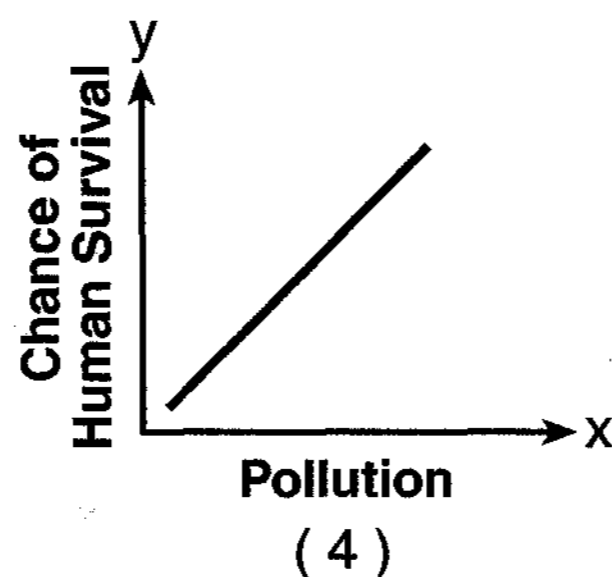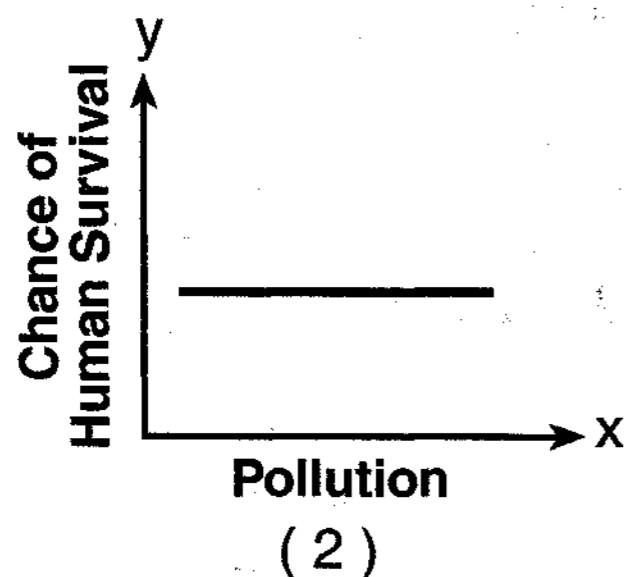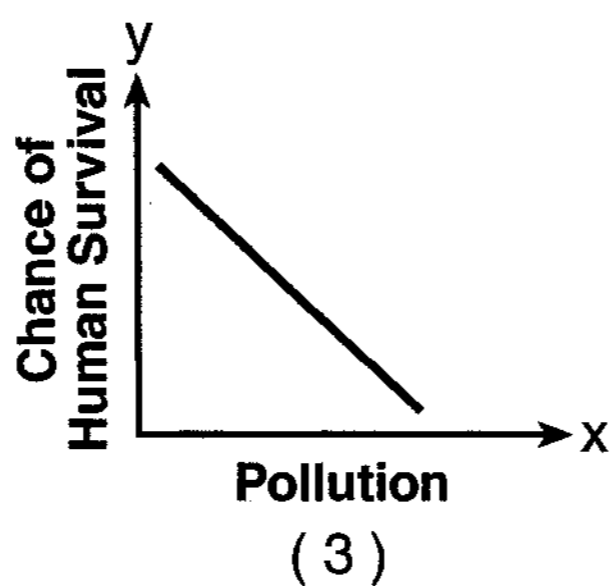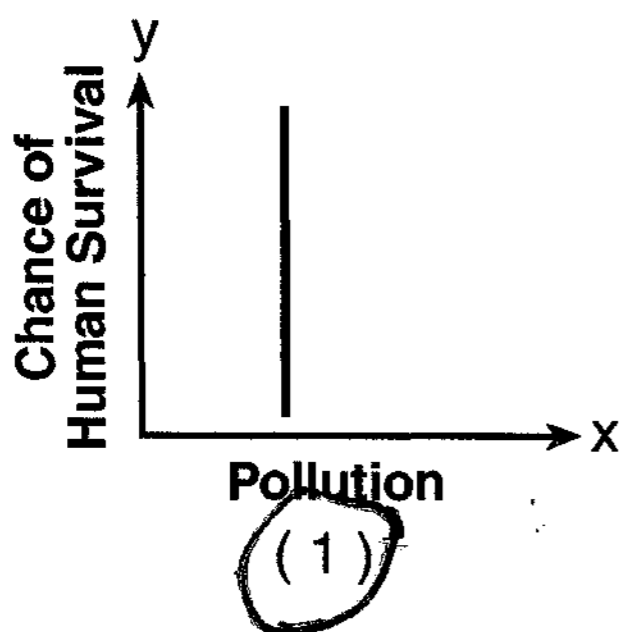
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [40]

1 Which graph does *not* represent a function of x?

(handwritten) (1) Fails the vertical line test

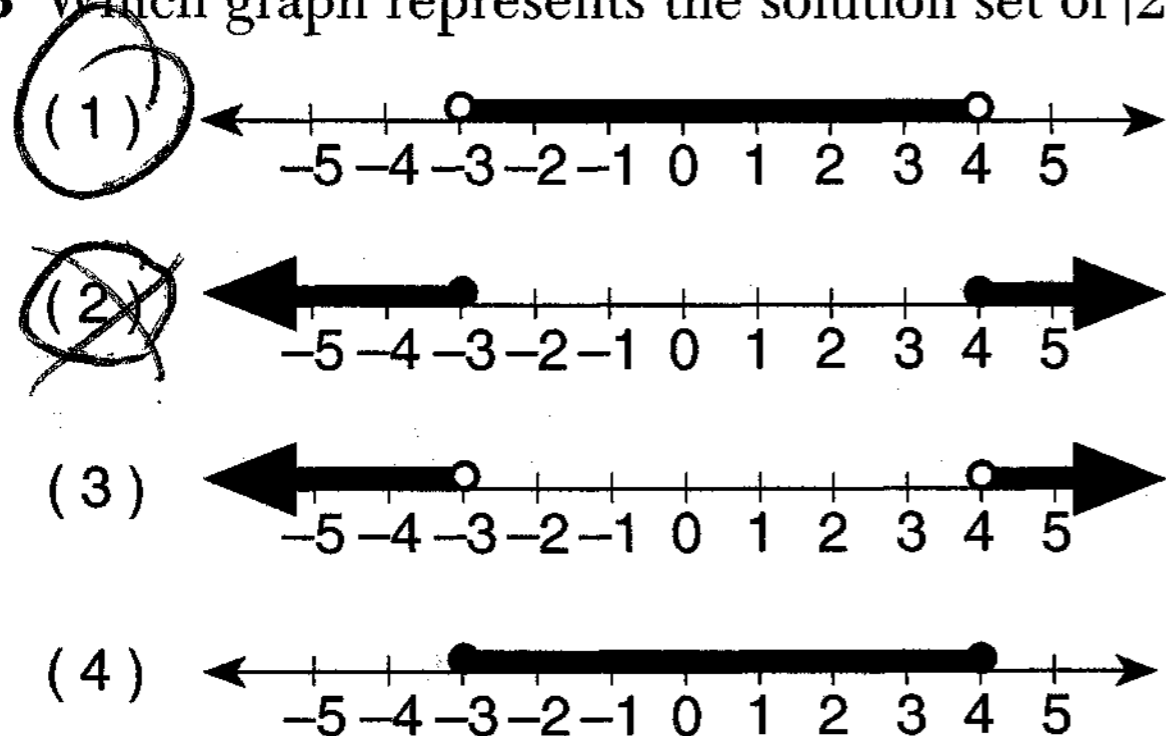**(1)** — vertical line, y-axis: Chance of Human Survival, x-axis: Pollution *(circled)*

(3) — decreasing line, y-axis: Chance of Human Survival, x-axis: Pollution

(2) — horizontal line, y-axis: Chance of Human Survival, x-axis: Pollution

(4) — increasing line, y-axis: Chance of Human Survival, x-axis: Pollution

2 What is the value of x in the equation $\sqrt{5 - 2x} = 3i$?

(1) 1
(2) 7 *(circled)*
(3) −2
(4) 4

(handwritten computations)
$$\sqrt{5 - 2x} = 3i$$
$$5 - 2x = 9i^2$$
$$5 - 2x = 9(-1)$$
$$5 - 2x = -9$$
$$-2x = -14$$
$$x = 7$$

3 Which graph represents the solution set of $|2x - 1| < 7$?

(1) *(circled)* — open circles at −3 and 4, shaded between
    −5 −4 −3 −2 −1 0 1 2 3 4 5

(2) *(circled and crossed out)* — arrows outward from −3 and 4
    −5 −4 −3 −2 −1 0 1 2 3 4 5

(3) — open circles at −3 and 4, arrows outward
    −5 −4 −3 −2 −1 0 1 2 3 4 5

(4) — closed circles at −3 and 4, shaded between
    −5 −4 −3 −2 −1 0 1 2 3 4 5

(handwritten computations)
$$2x - 1 < 7 \qquad 2x - 1 > -7$$
$$2x < 8 \qquad 2x > -6$$
$$x < 4 \qquad x > -3$$

4 The strength of a medication over time is represented by the equation $y = 200(1.5)^{-x}$, where x represents the number of hours since the medication was taken and y represents the number of micrograms per millimeter left in the blood. Which graph best represents this relationship?

exponential (1)

log (3)

linear (2)

(4)
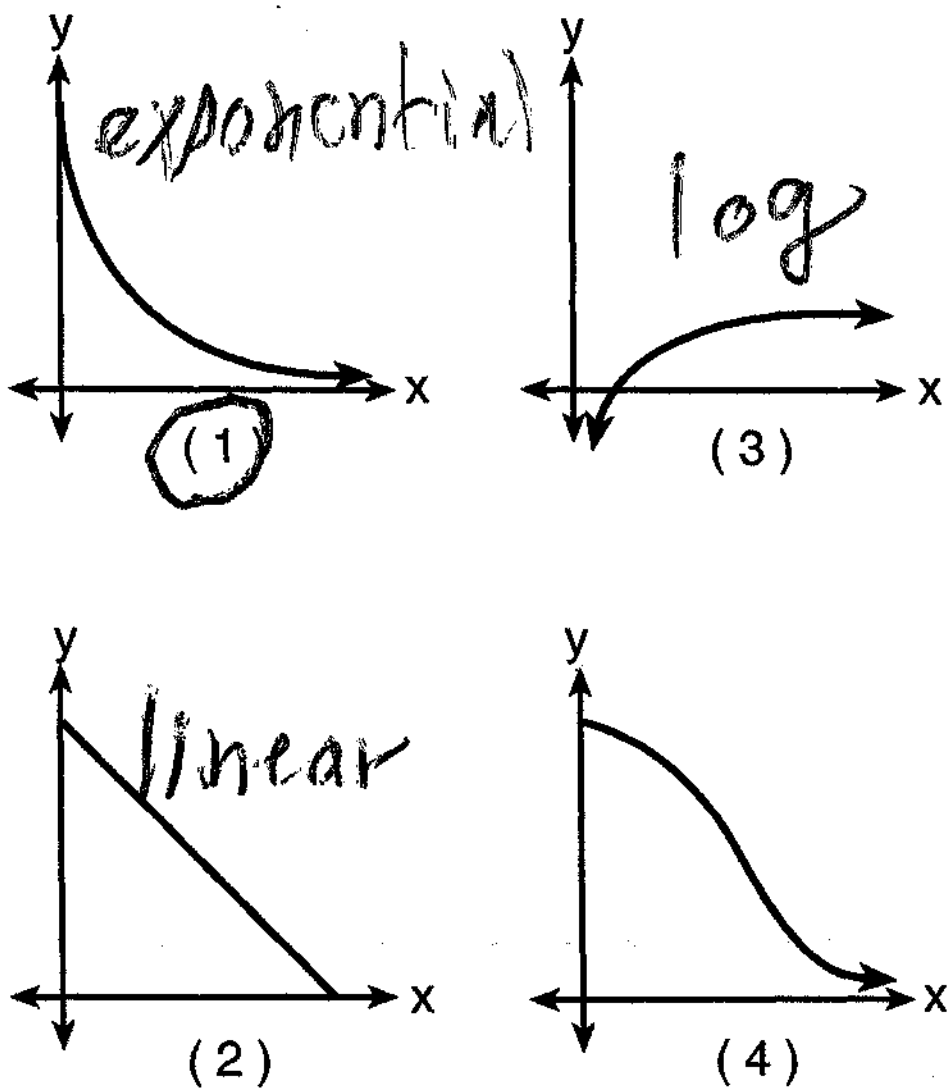
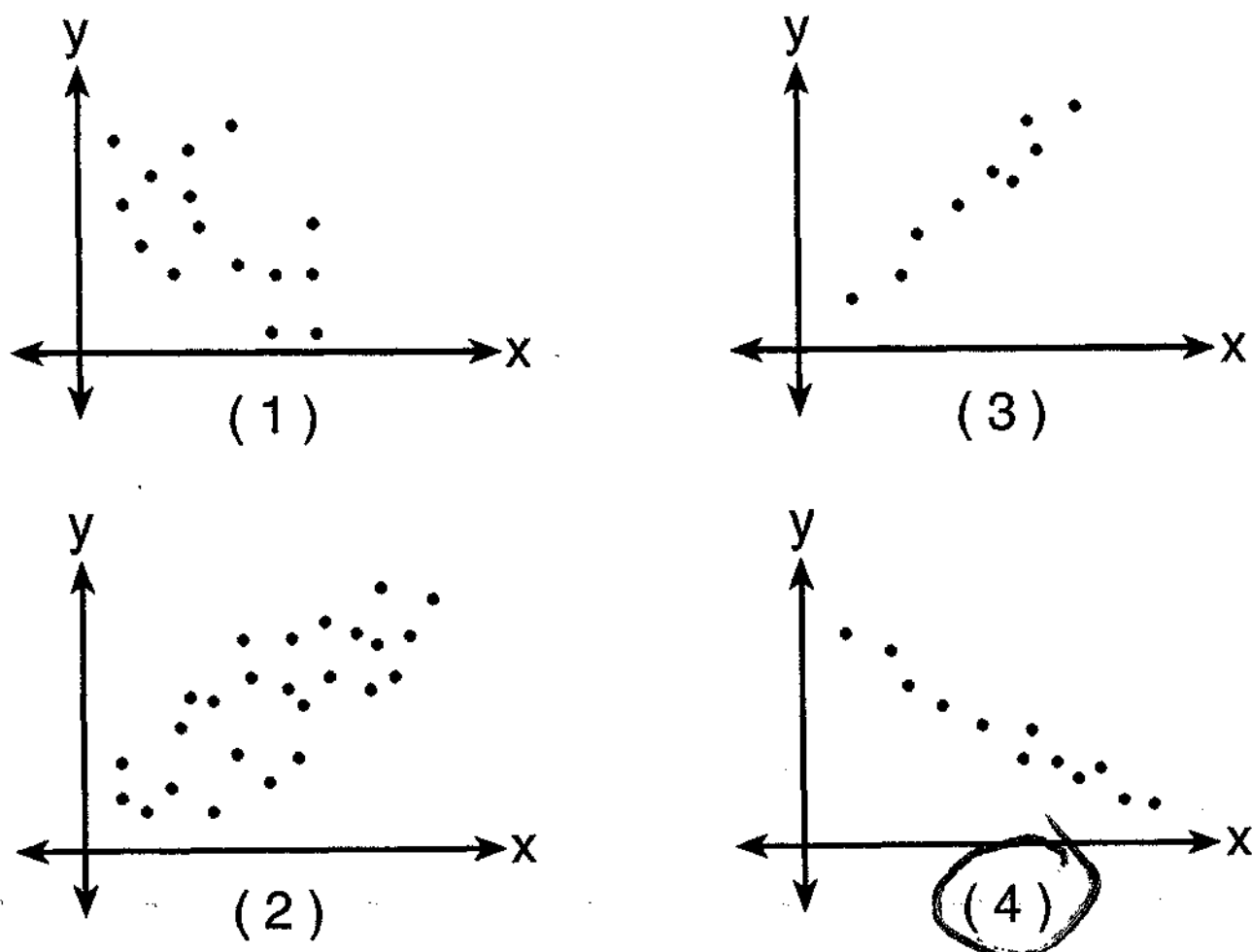$$y = 200(1.5)^{-x}$$
$$y = 200\left(\frac{3}{2}\right)^{-x}$$
$$y = 200\left(\frac{2}{3}\right)^{x}$$

5 Written in simplest form, the expression $\dfrac{x^2y^2 - 9}{3 - xy}$ is equivalent to

(1) -1

(2) $\dfrac{1}{3 + xy}$

(3) $-(3 + xy)$

(4) $3 + xy$

$$\frac{(xy - 3)(xy + 3)}{(3 - xy)}$$

6 Which graph represents data used in a linear regression that produces a correlation coefficient closest to -1?

(1)

(3)

(2)

(4)

7 Which expression is equal to $\frac{2+\sqrt{3}}{2-\sqrt{3}}$?

$\frac{(2+\sqrt{3})}{(2+\sqrt{3})} = \frac{4+2\sqrt{3}+2\sqrt{3}+3}{4+2\sqrt{3}-2\sqrt{3}-3} = \frac{7+4\sqrt{3}}{1}$

(1) $\frac{1-4\sqrt{3}}{7}$ (3) $1-4\sqrt{3}$

(2) $\frac{7+4\sqrt{3}}{7}$ (4) $7+4\sqrt{3}$

8 Which transformation is *not* an isometry?

(1) rotation   (3) dilation
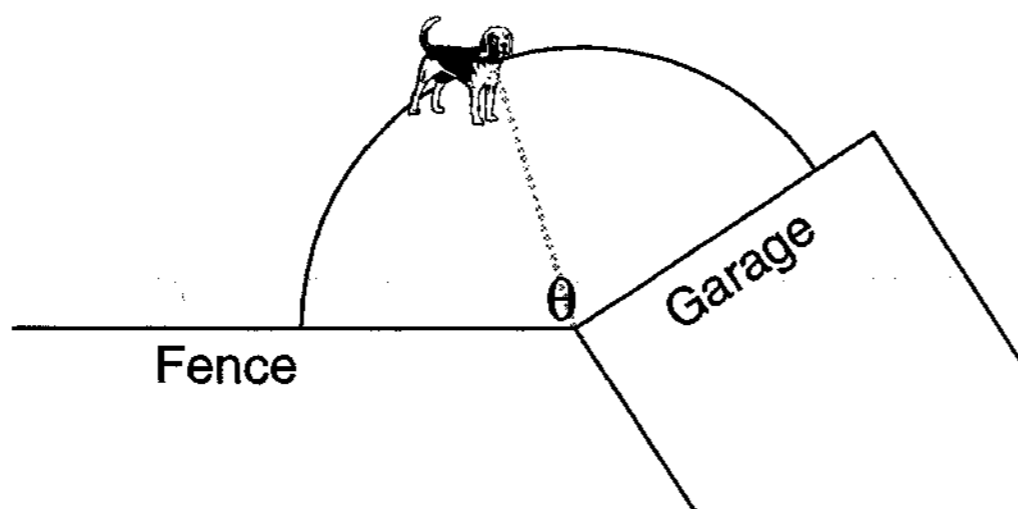(2) line reflection   (4) translation

9 A dog has a 20-foot leash attached to the corner where a garage and a fence meet, as shown in the accompanying diagram. When the dog pulls the leash tight and walks from the fence to the garage, the arc the leash makes is 55.8 feet.

(Not drawn to scale)

$\theta = \frac{S}{r}$

$= \frac{55.8}{20}$

$= 2.79$

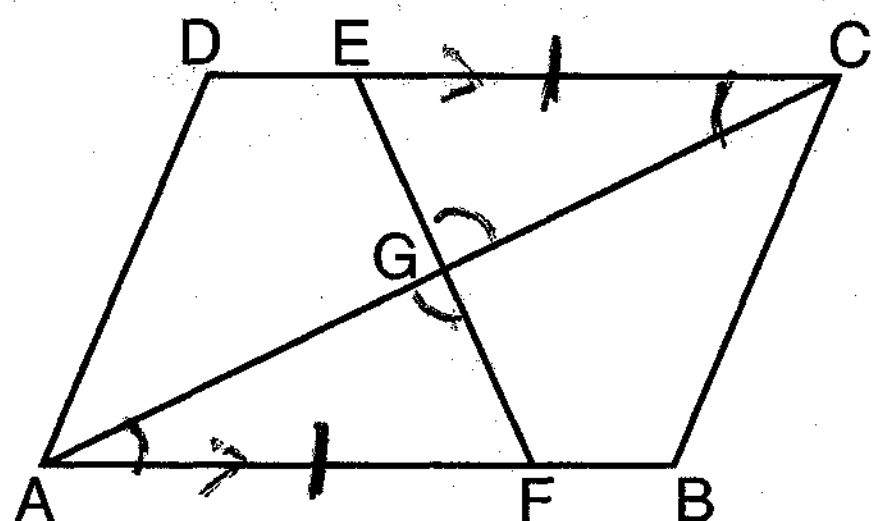What is the measure of angle θ between the garage and the fence, in radians?

(1) 0.36   (3) 3.14
(2) 2.79   (4) 160

10 In the accompanying diagram of parallelogram $ABCD$, $\overline{DE} \cong \overline{BF}$.

Triangle EGC can be proved congruent to triangle FGA by

(1) HL $\cong$ HL
(3) AAS $\cong$ AAS
(2) AAA $\cong$ AAA
(4) SSA $\cong$ SSA

$\angle EGC \cong \angle FGA$ because they are vertical angles
$\overline{CD} \cong \overline{AB}$ because opposite sides of a parallelogram are congruent.

Because $\overline{DE} \cong \overline{BF}$, $\overline{EC} \cong \overline{FA}$
$\overline{CD} \parallel \overline{AB}$ because opposite sides of a parallelogram are parallel.

Since $\overline{AGC}$ is a transversal, $\angle ECG \cong \angle FAG$ because they are alternate interior angles

11 An architect commissions a contractor to produce a triangular window. The architect describes the window as $\triangle ABC$, where $m\angle A = 50$, $BC = 10$ inches, and $AB = 12$ inches. How many distinct triangles can the contractor construct using these dimensions?
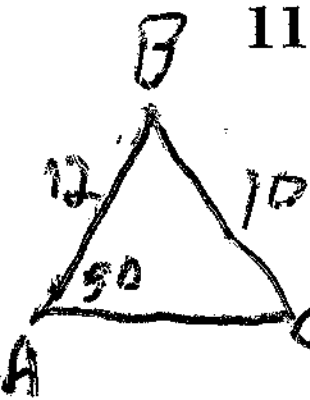
(1) 1
(3) more than 2
(2) 2
(4) 0

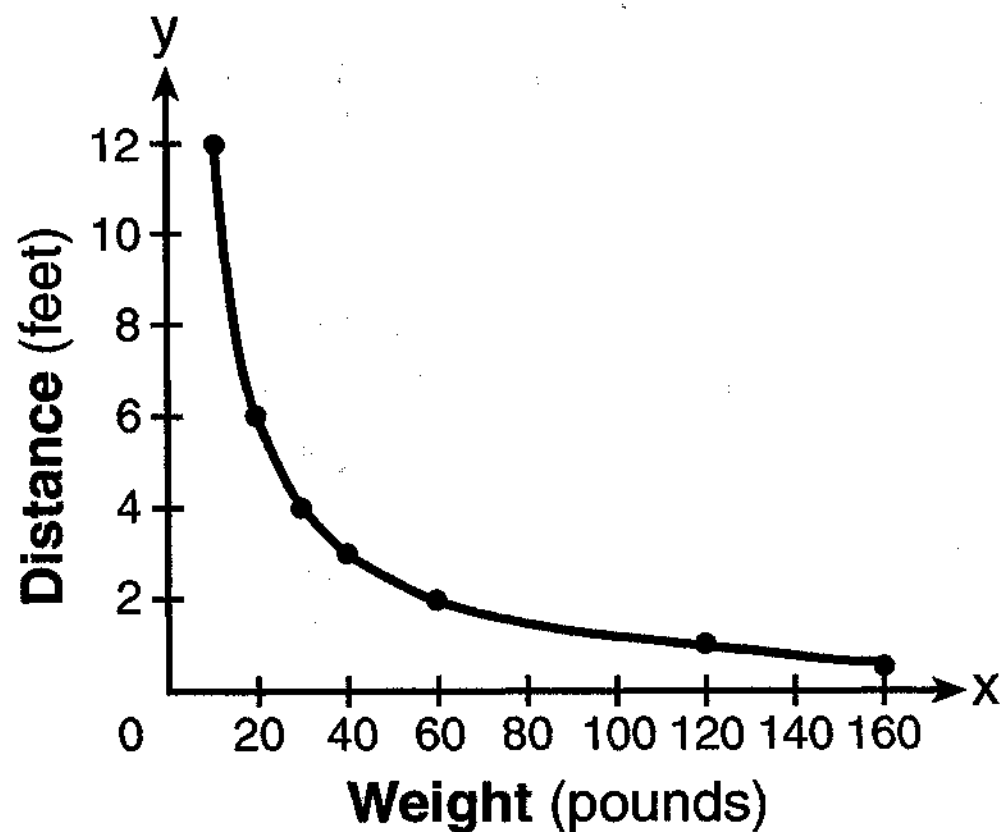$\dfrac{10}{\sin 50} < \dfrac{12}{\sin C}$

$C = 67$ or 113

$67 + 50 < 180$ ✓
$113 + 50 < 180$ ✓

12 The accompanying graph shows the relationship between a person's weight and the distance that the person must sit from the center of a seesaw to make it balanced.

Which equation best represents this graph?

(1) $y = 12x^2$ quadratic
(3) $y = 2 \log x$ log
(2) $y = -120x$ linear
(4) $y = \dfrac{120}{x}$

13 If f and g are two functions defined by $f(x) = 3x + 5$ and $g(x) = x^2 + 1$, then $g(f(x))$ is

(1) $x^2 + 3x + 6$ (3) $3x^2 + 8$
(2) $9x^2 + 30x + 26$ (4) $9x^2 + 26$

Handwritten work:
$g(3x+5) = (3x+5)^2 + 1$
$= 9x^2 + 15x + 15x + 25 + 1$
$= 9x^2 + 30x + 26$

14 What is the product of $5 + \sqrt{-36}$ and $1 - \sqrt{-49}$, expressed in simplest $a + bi$ form?

(1) $-37 + 41i$ (3) $47 + 41i$
(2) $5 - 71i$ (4) $47 - 29i$

Handwritten work:
$(5 + \sqrt{-36})(1 - \sqrt{-49})$
$(5 + 6i)(1 - 7i)$
$5 - 35i + 6i - 42i^2$
$5 - 29i - 42(-1) = 47 - 29i$

15 The expression $\dfrac{2\cos\theta}{\sin 2\theta}$ is equivalent to

(1) $\csc\theta$ (3) $\cot\theta$
(2) $\sec\theta$ (4) $\sin\theta$

Handwritten work:
$\dfrac{2\cos\theta}{\sin 2\theta} = \dfrac{2\cos\theta}{2\sin\theta\cos\theta} = \dfrac{1}{\sin\theta} = \csc\theta$

16 If $\sin x = \frac{12}{13}$, $\cos y = \frac{3}{5}$, and x and y are acute angles, the value of $\cos(x - y)$ is

(1) $\frac{21}{65}$ (3) $-\frac{14}{65}$
(2) $\frac{63}{65}$ (4) $-\frac{33}{65}$

Handwritten work:
$\sin x = \frac{12}{13}$
$\cos x = 5/13$
$\cos y = \frac{3}{5}$
$\sin y = 4/5$
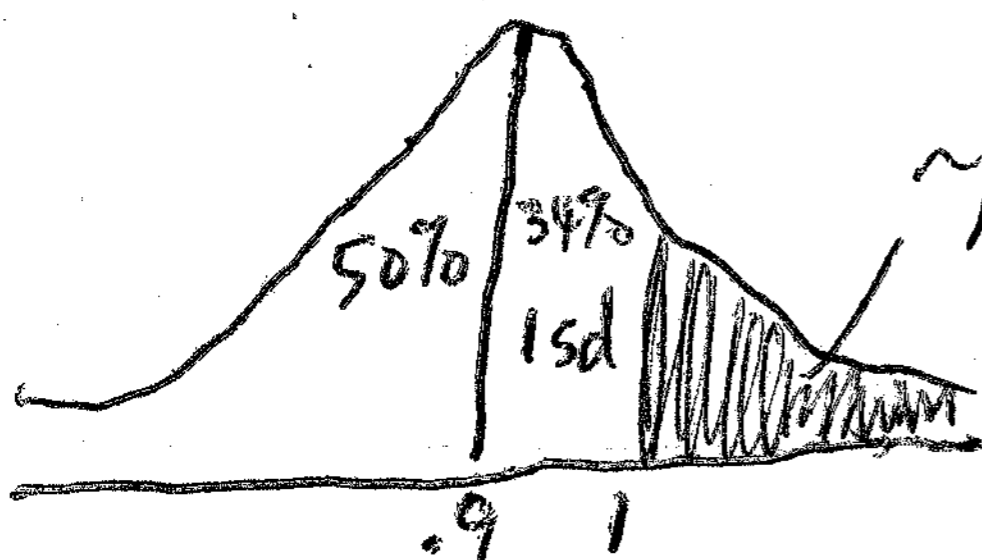Use $\cos^2 x + \sin^2 x = 1$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$
$\left(\frac{5}{13}\right)\left(\frac{3}{5}\right) + \left(\frac{12}{13}\right)\left(\frac{4}{5}\right)$
$\frac{15}{65} + \frac{48}{65}$
$\frac{63}{65}$

17 The amount of ketchup dispensed from a machine at Hamburger Palace is normally distributed with a mean of 0.9 ounce and a standard deviation of 0.1 ounce. If the machine is used 500 times, approximately how many times will it be expected to dispense 1 or more ounces of ketchup?

(1) 5   (3) 80
(2) 16   (4) 100

Handwritten work:
$\sim 16\% \times 500 \approx 80$

Handwritten bell curve diagram labeled 50%, 34%, 1sd, .9, 1

18 A commercial artist plans to include an ellipse in a design and wants the length of the horizontal axis to equal 10 and the length of the vertical axis to equal 6. Which equation could represent this ellipse?

(1) $9x^2 + 25y^2 = 225$ (3) $x^2 + y^2 = 100$ *circle*

(2) $9x^2 - 25y^2 = 225$ (4) $3y = 20x^2$

hyperbola   *quadratic*

19 A function is defined by the equation $y = \frac{1}{2}x - \frac{3}{2}$. Which equation defines the inverse of this function?

(1) $y = 2x + 3$ (3) $y = 2x + \frac{3}{2}$

(2) $y = 2x - 3$ (4) $y = 2x - \frac{3}{2}$

$$x = \frac{1}{2}y - \frac{3}{2}$$

$$2x = y - 3$$

$$2x + 3 = y$$

20 In the equation $ax^2 + 6x - 9 = 0$, imaginary roots will be generated if

(1) $-1 < a < 1$ (3) $a > -1$, only

(2) $a < 1$, only   (4) $a < -1$

$a = a \quad b = 6 \quad c = -9$

$$b^2 - 4ac < 0$$

$$6^2 - 4a(-9) < 0$$

$$36 + 36a < 0$$

$$\frac{36a}{36} < \frac{-36}{36}$$

$$a < -1$$

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

21 The height, h, in feet, a ball will reach when thrown in the air is a function of time, t, in seconds, given by the equation $h(t) = -16t^2 + 30t + 6$. Find, to the *nearest tenth*, the maximum height, in feet, the ball will reach.

$a = -16 \quad b = 30 \quad c = 6$

Maximum height occurs at the axis of symmetry

$$t = x = \frac{-b}{2a} = \frac{-30}{2(-16)} = \frac{15}{16} \text{ seconds}$$

$$h\left(\frac{15}{16}\right) = -16\left(\frac{15}{16}\right)^2 + 30\left(\frac{15}{16}\right) + 6$$

$$h \approx 20.1 \text{ seconds}$$

22 Find the value of $(x + 2)^0 + (x + 1)^{-\frac{2}{3}}$ when $x = 7$.

$$(7+2)^0 + (7+1)^{-\frac{2}{3}}$$

$$9^0 + 8^{-2/3}$$

$$1 + \frac{1}{8^{2/3}}$$

$$1 + \frac{1}{\sqrt[3]{64}} = 1 + \frac{1}{4} = 1\frac{1}{4}$$

23 Express in simplest form: $\dfrac{\frac{x}{4} - \frac{4}{x}}{1 - \frac{4}{x}}$

$$\frac{\frac{x^2-16}{4x}}{\frac{x-4}{x}} = \frac{x^2-16}{4\cancel{x}} \times \frac{\cancel{x}}{x-4}$$

$$= \frac{(x+4)\cancel{(x-4)}}{4} \times \frac{1}{\cancel{x-4}}$$

$$= \frac{x+4}{4}$$

24 The triangular top of a table has two sides of 14 inches and 16 inches, and the angle between the sides is 30°. Find the area of the tabletop, in square inches.

$$K = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2}(14)(16)\sin 30$$

$$= 56$$

25 Meteorologists can determine how long a storm lasts by using the function $t(d) = 0.07d^{\frac{3}{2}}$, where d is the diameter of the storm, in miles, and t is the time, in hours. If the storm lasts 4.75 hours, find its diameter, to the *nearest tenth of a mile*.

$$\frac{4.75}{.07} = \frac{.07d^{3/2}}{.07}$$

$$\left(\frac{4.75}{.07}\right)^{2/3} = \left(d^{3/2}\right)^{2/3}$$

$$16.6 \approx d$$

26 Tom scored 23 points in a basketball game. He attempted 15 field goals and 6 free throws. If each successful field goal is 2 points and each successful free throw is 1 point, is it possible he successfully made all 6 of his free throws? Justify your answer.

No. Tom scored an odd number of total points, Since field goals are worth an even number of points, he must have scored an odd number of free throws.

Even + Odd = Odd
2pts + 1pts = 23

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [24]

27 On the accompanying grid, graph and label $\overline{AB}$, where A is $(0,5)$ and B is $(2,0)$. Under the transformation $r_{x\text{-axis}} \circ r_{y\text{-axis}}(\overline{AB})$, A maps to A'', and B maps to B''. Graph and label $\overline{A''B''}$. What single transformation would map $\overline{AB}$ to $\overline{A''B''}$?
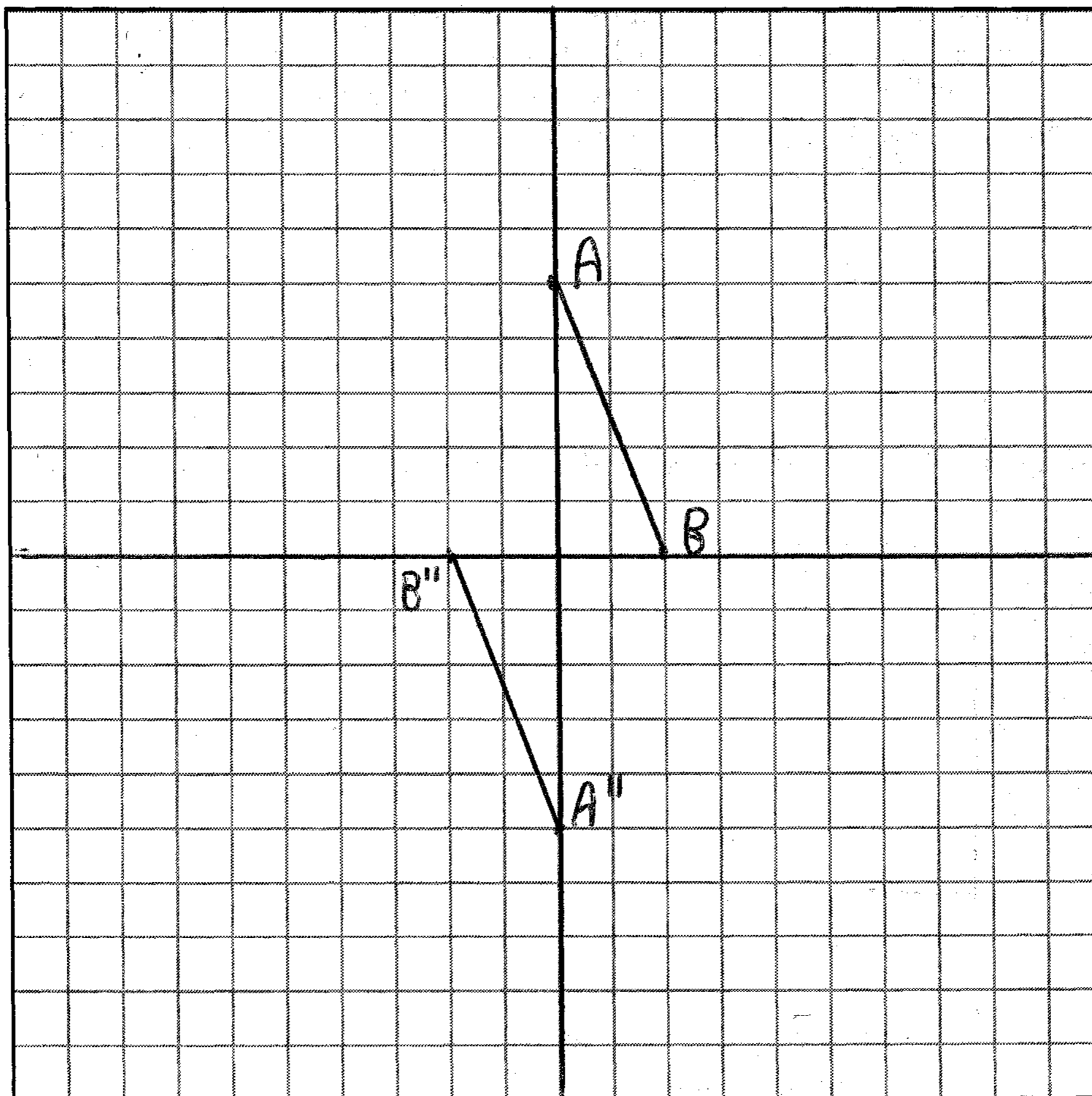
Single transformations include R_{180}, R_{-180} or $r_{(0,0)}$

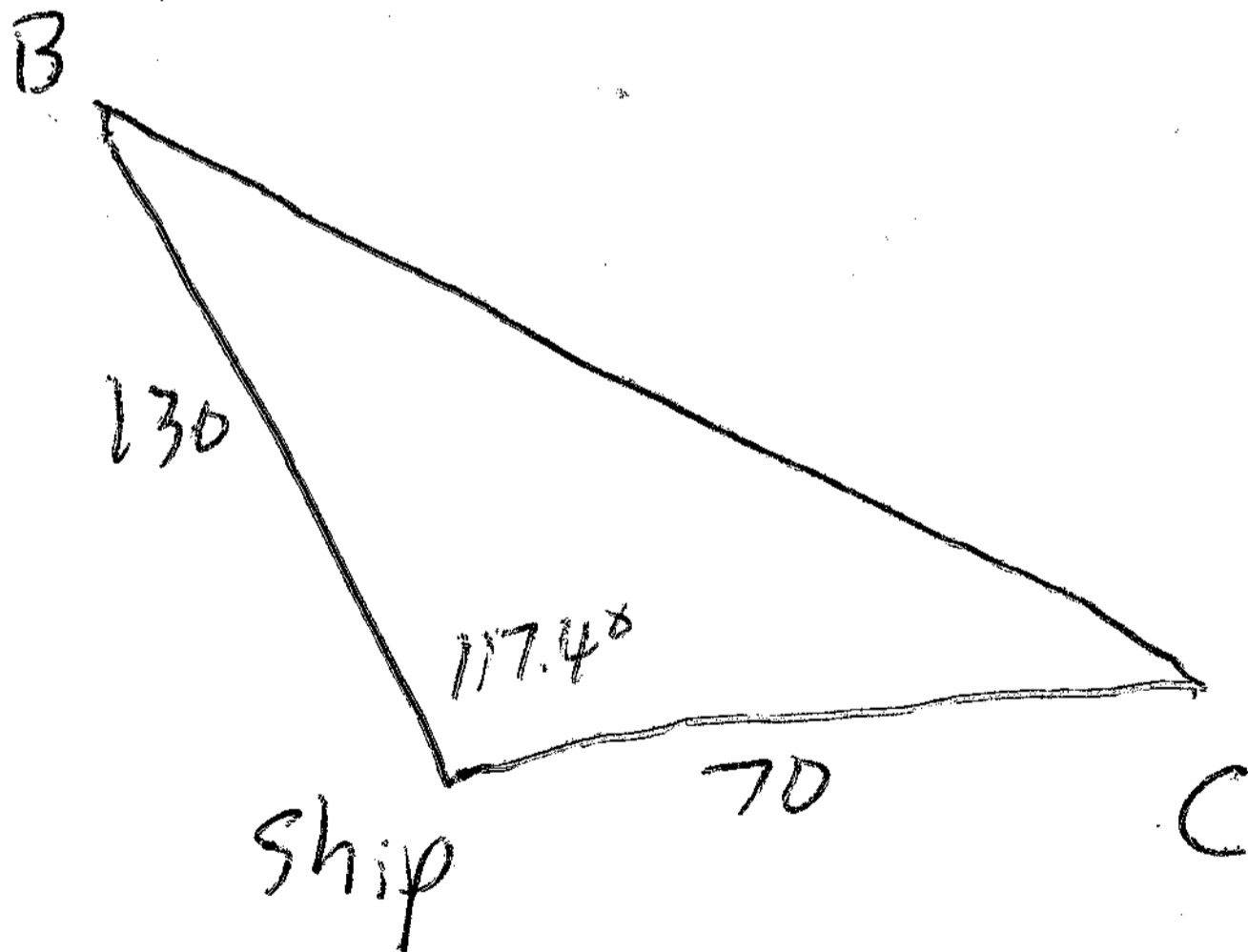**28** Express, in simplest $a + bi$ form, the roots of the equation $x^2 + 5 = 4x$.

$x^2 - 4x + 4 = -5 + 4$ Complete the square.

$(x-2)^2 = -1$

$x - 2 = \pm\sqrt{-1}$

$x = 2 \pm i$

29 A ship at sea is 70 miles from one radio transmitter and 130 miles from another. The angle between the signals sent to the ship by the transmitters is 117.4°. Find the distance between the two transmitters, to the *nearest mile*.

$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 70^2 + 130^2 - 2(70)(130) \cos 117.4$

$a^2 \approx 30,175.6$

$a \approx 174$

30 A student attaches one end of a rope to a wall at a fixed point 3 feet above the ground, as shown in the accompanying diagram, and moves the other end of the rope up and down, producing a wave described by the equation $y = a \sin bx + c$. The range of the rope's height above the ground is between 1 and 5 feet. The period of the wave is 4π. Write the equation that represents this wave.

3 ft   3 ft
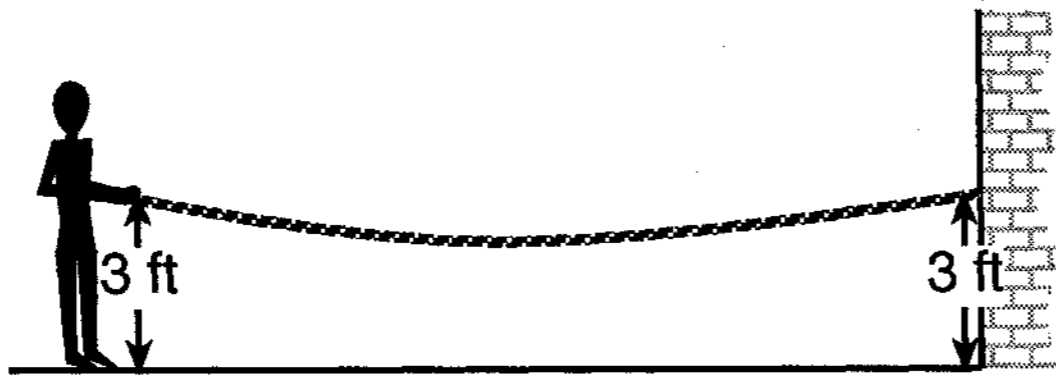
$$\text{Period} = \frac{2\pi}{b}$$

$$4\pi = \frac{2\pi}{b}$$

$$b = \frac{1}{2}$$

The sine wave starts at (0,0). To intersect at (0,3), c = 3. Since the minimum and maximum are 1 and 5, a = 2

$$y = 2\sin\left(\tfrac{1}{2}x\right) + 3$$

31 The table below shows the results of an experiment that relates the height at which a ball is dropped, x, to the height of its first bounce, y.

Drop Height (x) (cm)	Bounce Height (y) (cm)
100	26
90	23
80	21
70	18
60	16

Find $\bar{x}$, the mean of the drop heights.

Find $\bar{y}$, the mean of the bounce heights.

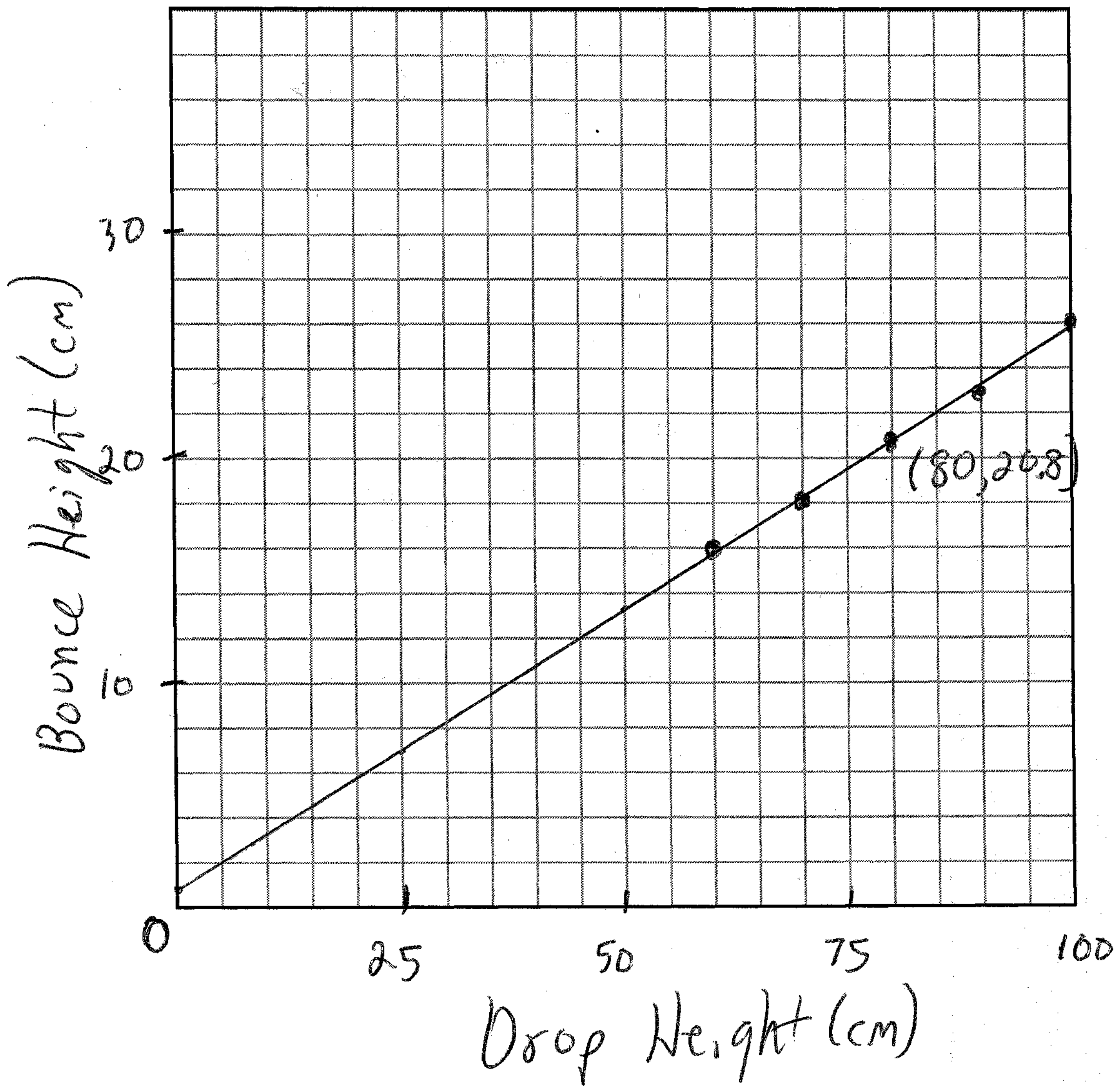Find the linear regression equation that best fits the data.

Show that $(\bar{x}, \bar{y})$ is a point on the line of regression. [The use of the grid on the next page is optional.]

$$\overline{X} = 80$$

$$\overline{Y} = 20.8$$

$$y = 0.25x + 0.8$$
$$= 0.25(80) + 0.8$$
$$= 20.8$$

Bounce Height (cm) vs Drop Height (cm)

(80, 20.8)

32 A company calculates its profit by finding the difference between revenue and cost. The cost function of producing x hammers is $C(x) = 4x + 170$. If each hammer is sold for $10, the revenue function for selling x hammers is $R(x) = 10x$.

How many hammers must be sold to make a profit?

How many hammers must be sold to make a profit of $100?

To make a profit:

$$10x > 4x + 170$$

$$\frac{6x}{6} > \frac{170}{6}$$

$$x > 28.3 \qquad 29 \text{ hammers must be sold}$$

To make a profit of $100:

$$10x - (4x + 170) = 100$$

$$6x - 170 = 100$$

$$\frac{6x}{6} = \frac{270}{6}$$

$$x = 45 \text{ hammers must be sold}$$

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

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33 Given circle O with diameter $\overline{GOAL}$; secants $\overline{HUG}$ and $\overline{HTAM}$ intersect at point H; $\text{m}\widehat{GM}:\text{m}\widehat{ML}:\text{m}\widehat{LT} = 7{:}3{:}2$; and chord $\overline{GU} \cong$ chord $\overline{UT}$.
Find the ratio of $\text{m}\angle UGL$ to $\text{m}\angle H$.
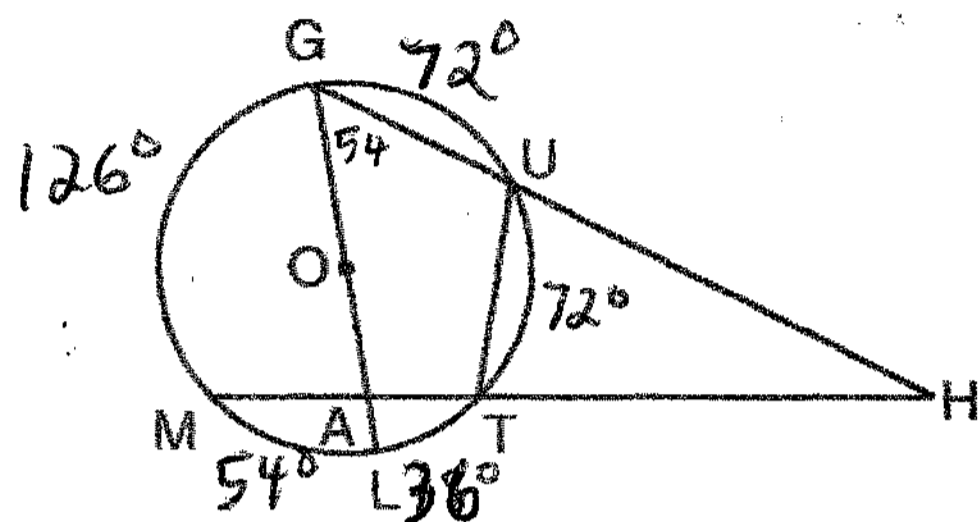
$$180° \begin{cases} \text{m}\,\widehat{GM} & \frac{7}{10} \times 180 = 126° \\ \text{m}\,\widehat{ML} & \frac{3}{10} \times 180 = 54° \end{cases}$$

$$\text{m}\,\widehat{LT} \quad \frac{2}{10} \times 180 = 36°$$

$$\widehat{GU} + \widehat{UT} + 36 = 180$$
$$x + x + 36 = 180$$
$$2x = 144$$
$$x = 72$$

$$\text{m}\angle UGL = \frac{72+36}{2} = 54°$$

$$\text{m}\angle H = \frac{126-72}{2} = 27$$

ratio is $2{:}1$

34 When Joe bowls, he can get a strike (knock down all the pins) 60% of the time. How many times more likely is it for Joe to bowl *at least* three strikes out of four tries as it is for him to bowl zero strikes out of four tries? Round your answer to the *nearest whole number*.

$$n = 4$$

$$r = 3, 4, 0$$

$$P = \frac{6}{10}$$

$$q = \frac{4}{10}$$

$$P(3) = {}_4C_3 \left(\frac{6}{10}\right)^3 \left(\frac{4}{10}\right)^1 = \frac{3456}{10,000}$$

$$P(4) = {}_4C_4 \left(\frac{6}{10}\right)^4 \left(\frac{4}{10}\right)^0 = \frac{1296}{10,000}$$

$$P(\geq 3) = \frac{4752}{10,000}$$

$$P(0) = {}_4C_0 \left(\frac{6}{10}\right)^0 \left(\frac{4}{10}\right)^4 = \frac{256}{10,000}$$

$$\frac{\frac{4752}{10000}}{\frac{256}{10000}} = 18.5625 \approx 19$$