The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS B

Thursday, August 16, 2001 — 8:30 to 11:30 a.m., only

Print Your Name: Steve Sibol

Print Your School's Name: HSCR

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will *not* be scored. All work should be written in pen, except graphs and drawings, which should be done in pencil.

This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. The formulas that you may need to answer some questions in this examination are found on page 2.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator, a straightedge (ruler), and a compass must be available for your use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Record your answers in the spaces provided on the separate answer sheet. [40]

1 Which relation is *not* a function?

(1) $y = 2x + 4$
(2) $y = x^2 - 4x + 3$
(3) $x = 3y - 2$
(4) $x = y^2 + 2x - 3$

Use this space for computations.

2 The solution set of $|3x + 2| < 1$ contains

(1) only negative real numbers
(2) only positive real numbers
(3) both positive and negative real numbers
(4) no real numbers

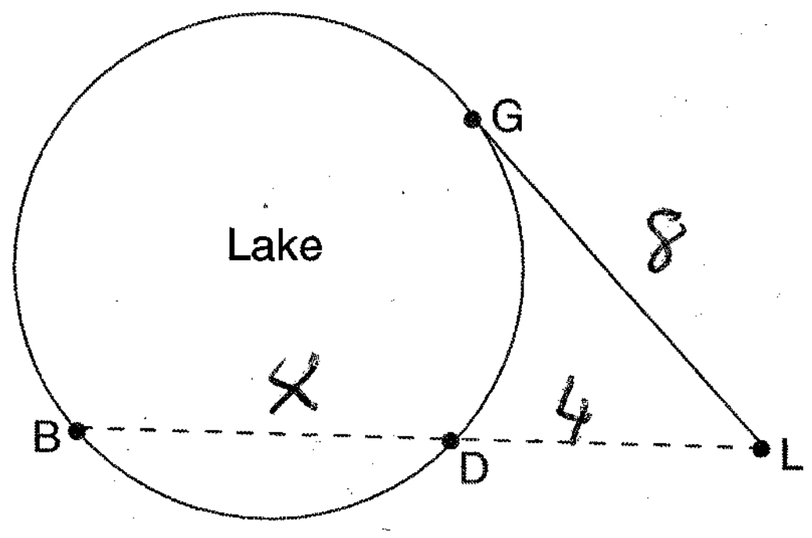
$3x+2<1$ $\quad -2 \quad -2$ $\quad \frac{3x}{3} < \frac{-1}{3}$

$3x+2>-1$ $\quad -2 \quad -2$ $\quad \frac{3x}{3} > \frac{-3}{3}$

$x > -1$

$-1 < x < -\frac{1}{3}$

3 In the accompanying diagram, cabins B and G are located on the shore of a circular lake, and cabin L is located near the lake. Point D is a dock on the lake shore and is collinear with cabins B and L. The road between cabins G and L is 8 miles long and is tangent to the lake. The path between cabin L and dock D is 4 miles long.



(Not drawn to scale)

What is the length, in miles, of $\overline{BD}$?

(1) 24
(2) 12
(3) 8
(4) 4

$4(x+4) = 8^2$

$4x+16 = 64$

$x = 12$

4 The solution set of the equation $\sqrt{x + 6} = x$ is

(1) $\{-2,3\}$
(2) $\{-2\}$
(3) $\{3\}$
(4) $\{\ \}$

$\sqrt{x+6} = x$

$x+6 = x^2$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$3 \quad -2$

5 Which transformation is a direct isometry?

(1) D_2
(2) D_{-2}
(3) $r_{y\text{-axis}}$
(4) $T_{2,5}$

Use this space for computations.

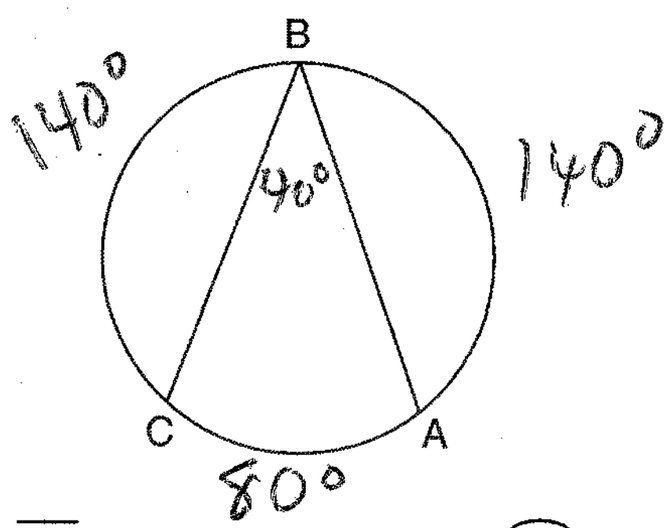
6 The roots of the equation $x^2 - 3x - 2 = 0$ are

(1) real, rational, and equal
(2) real, rational, and unequal
(3) real, irrational, and unequal
(4) imaginary

$a = 1$
$b = -3$
$c = -2$

$b^2 - 4ac$
$(-3)^2 - 4(1)(-2)$
$9 + 8$
17

7 The new corporate logo created by the design engineers at Magic Motors is shown in the accompanying diagram.



If chords $\overline{BA}$ and $\overline{BC}$ are congruent and $m\overset{\frown}{BC} = 140$, what is $m\angle B$?

(1) 40
(2) 80
(3) 140
(4) 280

8 At Mogul's Ski Resort, the beginner's slope is inclined at an angle of 12.3°, while the advanced slope is inclined at an angle of 26.4°. If Rudy skis 1,000 meters down the advanced slope while Valerie skis the same distance on the beginner's slope, how much longer was the horizontal distance that Valerie covered?

(1) 81.3 m
(2) 231.6 m
(3) 895.7 m
(4) 977.0 m

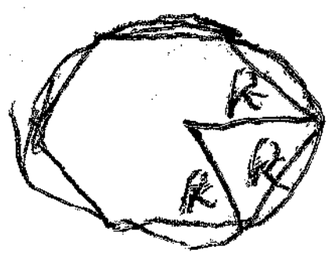
advanced slope
$\cos 26.4 = \frac{\text{adjacent}}{1000}$
$\text{adjacent} \approx 895.7$

beginner's slope
$\cos 12.3 = \frac{\text{adjacent}}{1000}$
[4] $\text{adjacent} \approx 977$

$977 - 895.7 \approx 81.3$

9 A regular hexagon is inscribed in a circle. What is the ratio of the length of a side of the hexagon to the minor arc that it intercepts?

(1) $\frac{\pi}{6}$
(2) $\frac{3}{6}$
(3) $\frac{3}{\pi}$
(4) $\frac{6}{\pi}$



The hexagon divides the circle into 6 equal arcs.
$\theta = \frac{s}{R}$ $\frac{\pi}{3} = \frac{s}{R}$ $s = \frac{\pi R}{3}$

Use this space for computations.

A regular hexagon may be divided into 6 equal equilateral triangles. A side of a hexagon is equal to the radius.

ratio $\frac{R}{R(\frac{\pi}{3})}$

10 If $\log 5 = a$, then $\log 250$ can be expressed as

(1) $50a$
(2) $2a + 1$
(3) $10 + 2a$
(4) $25a$

$\log 250 = \log 10 \cdot 5^2 = \log 10 + \log 5^2$
$\log 10 + 2\log 5$
$1 + 2a$

11 On a trip, a student drove 40 miles per hour for 2 hours and then drove 30 miles per hour for 3 hours. What is the student's average rate of speed, in miles per hour, for the whole trip?

(1) 34
(2) 35
(3) 36
(4) 37

The student drove 80 miles and then 90 miles in 5 hours

$\frac{170}{5} = 34$

12 A ball is thrown straight up at an initial velocity of 54 feet per second. The height of the ball t seconds after it is thrown is given by the formula $h(t) = 54t - 12t^2$. How many seconds after the ball is thrown will it return to the ground? $h = 0$

(1) 9.2
(2) 6
(3) 4.5
(4) 4

$54t - 12t^2 = 0$
$6t(9 - 2t) = 0$
$6t = 0$ $\quad 9 - 2t = 0$
$t = 0$ $\quad t = 4.5$

13 What is the period of the function $y = 5 \sin 3x$?

(1) 5
(2) $\frac{2\pi}{5}$
(3) 3
(4) $\frac{2\pi}{3}$

period $= \frac{2\pi}{b} = \frac{2\pi}{3}$

14 A cellular telephone company has two plans. Plan *A* charges \$11 a month and \$0.21 per minute. Plan *B* charges \$20 a month and \$0.10 per minute. After how much time, to the *nearest minute*, will the cost of plan *A* be equal to the cost of plan *B*?

(1) 1 hr 22 min
(2) 1 hr 36 min
(3) 81 hr 8 min
(4) 81 hr 48 min

Use this space for computations.

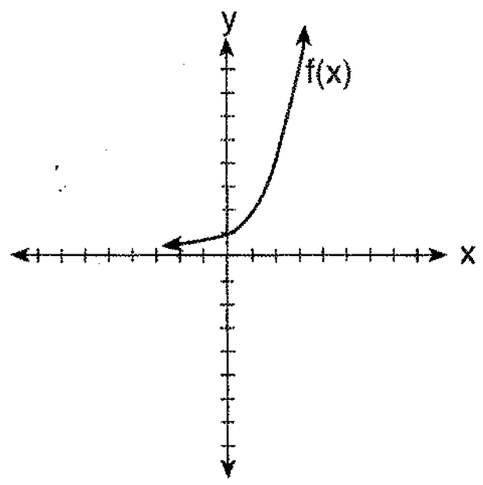
$0.21m + 11 = .10m + 20$

$.11m = 9$

$m \approx 81.82$ min

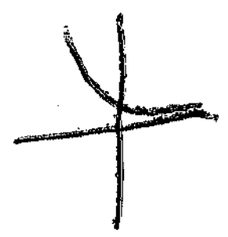
$m \approx 1$ hr. 22 min

15 The graph of f(x) is shown in the accompanying diagram.



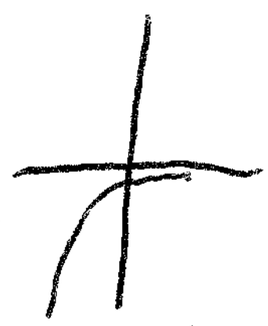
f(x) ≠

After $r_{y\text{-axis}}$

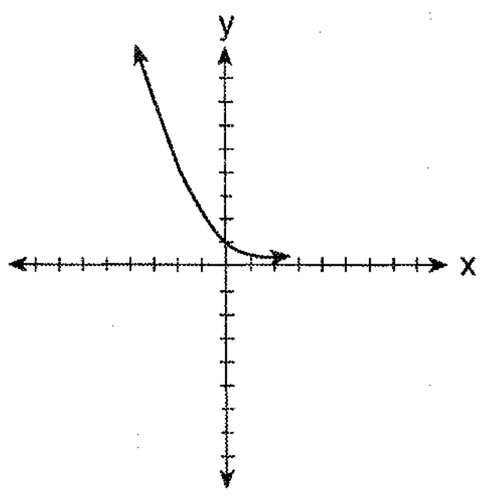


Which graph represents $f(x)_{r_{x\text{-axis}} \circ r_{y\text{-axis}}}$?

After $r_{x\text{-axis}}$



(1)



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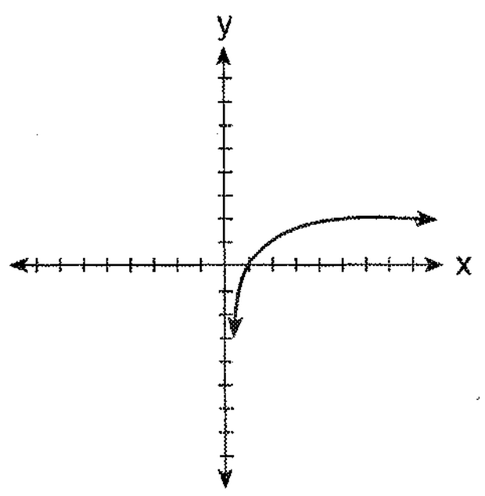

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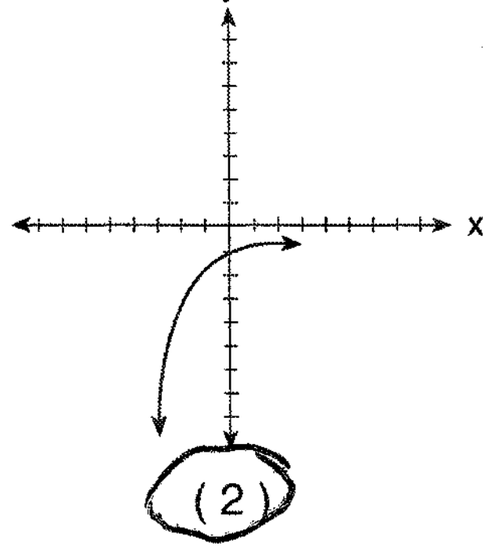
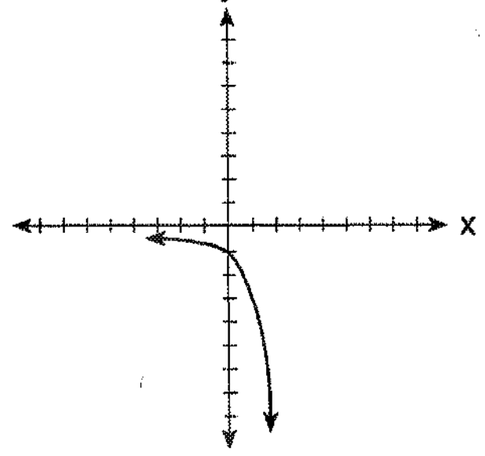


(4)



16 A wedge-shaped piece is cut from a circular pizza. The radius of the pizza is 6 inches. The rounded edge of the crust of the piece measures 4.2 inches. To the *nearest tenth,* the angle of the pointed end of the piece of pizza, in radians, is

(1) 0.7
(2) 1.4
(3) 7.0
(4) 25.2

Use this space for computations.

$\theta = \frac{s}{r} = \frac{4.2}{6} = .7$

17 If the length of a rectangular garden is represented by $\frac{x^2 + 2x}{x^2 + 2x - 15}$ and its width is represented by $\frac{2x - 6}{2x + 4}$, which expression represents the area of the garden?

(1) x
(2) $x + 5$
(3) $\frac{x^2 + 2x}{2(x + 5)}$
(4) $\frac{x}{x + 5}$

$\frac{x(x+2)}{(x+5)(x-3)} \times \frac{2(x-3)}{2(x+2)}$

18 Determine the value of x and y if $2^y = 8^x$ and $3^y = 3^{x+4}$.

(1) $x = 6, y = 2$
(2) $x = -2, y = -6$
(3) $x = 2, y = 6$
(4) $x = y$

$2^y = (2^3)^x$

$y = 3x \quad y = x + 4$

$3x = x + 4$

$2x = 4$

$x = 2$

$y = 3x$

$= 3(2)$

$= 6$

19 If Jamar can run $\frac{3}{5}$ of a mile in 2 minutes 30 seconds, what is his rate in miles per minute?

(1) $\frac{4}{5}$
(2) $\frac{6}{25}$
(3) $3\frac{1}{10}$
(4) $4\frac{1}{6}$

$\frac{\frac{3}{5}}{2.5} = \frac{6}{25}$

20 A box contains one 2-inch rod, one 3-inch rod, one 4-inch rod, and one 5-inch rod. What is the maximum number of different triangles that can be made using these rods as sides?

(1) 1
(2) 2
(3) 3
(4) 4

$2+3>4 \quad 2+4>5 \quad 3+4>5$

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

21 If the sine of an angle is $\frac{3}{5}$ and the angle is *not* in Quadrant I, what is the value of the cosine of the angle?

$\sin^2 x + \cos^2 x = 1$

$\left(\frac{3}{5}\right)^2 + \cos^2 x = 1$

$\frac{9}{25} + \cos^2 x = 1$

$\cos^2 x = \frac{16}{25}$

$\cos x = \pm \frac{4}{5}$

Since the angle is not in Quadrant I

$\cos x = -\frac{4}{5}$

22 Show that the product of $a + bi$ and its conjugate is a real number.

$(a+bi)(a-bi)$

$a^2 - abi + abi - b^2 i^2$

$a^2 - b^2(-1)$

$a^2 + b^2$

If a and b are real, a^2 and b^2 are real and their sum is real

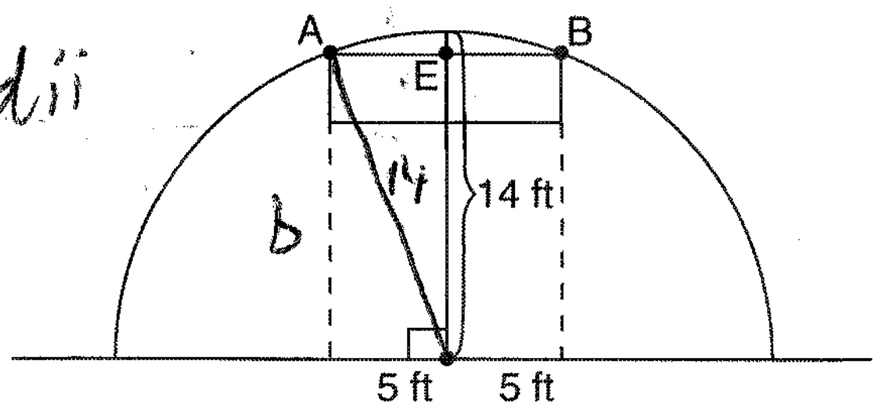
23 The price per person to rent a limousine for a prom varies inversely as the number of passengers. If five people rent the limousine, the cost is \$70 each. How many people are renting the limousine when the cost *per couple* is \$87.50?

$$5 \times 70 = x \cdot \frac{87.50}{2}$$

$$x = 8$$

24 The accompanying diagram shows a semicircular arch over a street that has a radius of 14 feet. A banner is attached to the arch at points A and B, such that $AE = EB = 5$ feet. How many feet above the ground are these points of attachment for the banner?

All radii are equal.



$$a^2 + b^2 = c^2$$

$$5^2 + b^2 = 14^2$$

$$b^2 = 171$$

$$b = \sqrt{171}$$

25 Working by herself, Mary requires 16 minutes more than Antoine to solve a mathematics problem. Working together, Mary and Antoine can solve the problem in 6 minutes. If this situation is represented by the equation $\frac{6}{t} + \frac{6}{t+16} = 1$, where t represents the number of minutes Antoine works alone to solve the problem, how many minutes will it take Antoine to solve the problem if he works by himself?

$$\frac{6(t+16)+6t}{t(t+16)} = 1$$

$6t+96+6t = t^2+16t$

$t^2+4t-96=0$

$(t+12)(t-8)=0$

$t=-12 \quad t=8$

extraneous

26 If $\sin x = \frac{4}{5}$, where $0° < x < 90°$, find the value of $\cos(x + 180°)$.

If $\sin x = \frac{4}{5}$, then $\cos x = \frac{3}{5}$

$\cos x+180 = (\cos x)(\cos 180) - (\sin x)(\sin 180)$

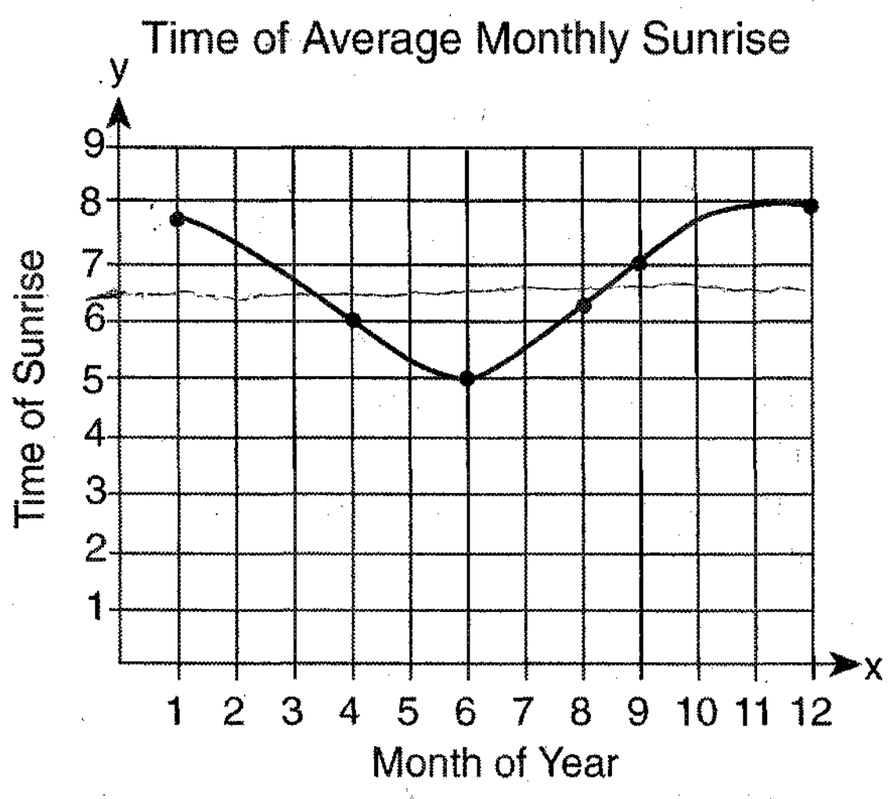
$= \left(\frac{3}{5}\right)(-1) - \left(\frac{4}{5}\right)(0)$

$= -\frac{3}{5}$

Part III

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [24]

27 The times of average monthly sunrise, as shown in the accompanying diagram, over the course of a 12-month interval can be modeled by the equation $y = A \cos (Bx) + D$. Determine the values of A, B, and D, and explain how you arrived at your values.



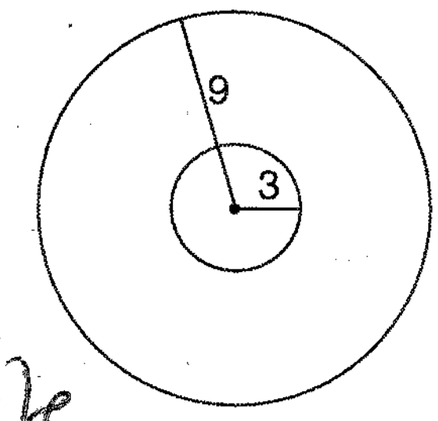
The midpoint of the graph's maximum and minimum is 6.5, so D = 6.5. The maximum, 8, and the minimum, 5, is 1.5 From the midpoint, so A = 1.5

period = $\frac{2\pi}{a}$ $\quad 12 = \frac{2\pi}{a}$

~~Period~~ = ~~2π~~ $\quad a \approx .5$

$y = 1.5 \cos(.5x) + 6.5$

28 As shown in the accompanying diagram, a circular target with a radius of 9 inches has a bull's-eye that has a radius of 3 inches. If five arrows randomly hit the target, what is the probability that *at least* four hit the bull's-eye?



Area of bull's-eye is 9π.

Area of the entire target is 81π.

$n = 5$

$r = 4, 5$

$p = \frac{9\pi}{81\pi} = \frac{1}{9}$

$q = \frac{8}{9}$

$P(4) = {}_5C_4\left(\frac{1}{9}\right)^4\left(\frac{8}{9}\right)^1 = \frac{40}{59049}$

$P(5) = {}_5C_5\left(\frac{1}{9}\right)^5\left(\frac{8}{9}\right)^0 = \frac{1}{59049}$

$\frac{41}{59049}$

29 Twenty high school students took an examination and received the following scores:

70, 60, 75, 68, 85, 86, 78, 72, 82, 88, 88, 73, 74, 79, 86, 82, 90, 92, 93, 73

Determine what percent of the students scored within one standard deviation of the mean. Do the results of the examination approximate a normal distribution? Justify your answer.

$\bar{x} = 79.7$

$\sigma_x = 8.7$

The relevant range is

$71 \leq x \leq 88.4$

$\frac{14}{20}$ scores (70%) are within one standard deviation of the mean. The results approximate a normal distribution of 68.2%

30 A small, open-top packing box, similar to a shoebox without a lid, is three times as long as it is wide, and half as high as it is long. Each square inch of the bottom of the box costs \$0.008 to produce, while each square inch of any side costs \$0.003 to produce.

Write a function for the cost of the box described above.

The area of the bottom is $w \times 3w = 3w^2$, which at \$.008/sq in costs $.024w^2$. The area of the two smaller sides is $2 \times w \times \frac{3}{2}w = 3w^2$. The area of the two larger sides is $2 \times 3w \times \frac{3}{2}w = 9w^2$. The total area of the sides is $12w^2$, which at \$.003/sq in costs \$$.036w^2$.

A function for the total cost is $f(w) = .06w^2$

Using this function, determine the dimensions of a box that would cost \$0.69 to produce.

$\frac{.06w^2}{.06} = \frac{.69}{.06}$

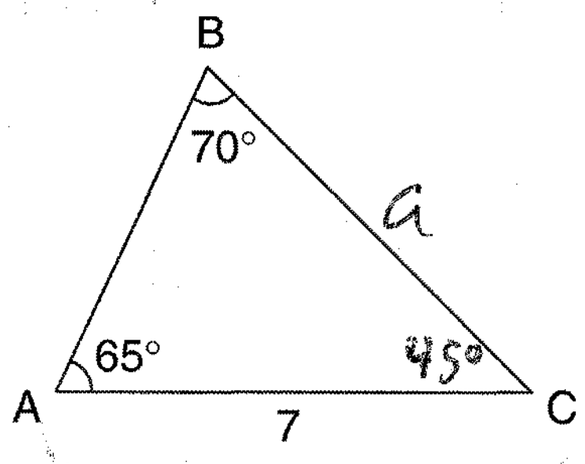
$w^2 = 11.5$

$w = \sqrt{11.5}$

$l = 3\sqrt{11.5}$

$h = \frac{3}{2}\sqrt{11.5}$

31 In the accompanying diagram of $\triangle ABC$, $m\angle A = 65$, $m\angle B = 70$, and the side opposite vertex B is 7. Find the length of the side opposite vertex A, and find the area of $\triangle ABC$.



$$\frac{a}{\sin 65} = \frac{7}{\sin 70}$$

$$a \approx 6.75$$

$$\text{AREA} = ab\sin C$$
$$= (6.75)(7)\sin 45$$
$$\approx 16.71$$

32 The amount A, in milligrams, of a 10-milligram dose of a drug remaining in the body after t hours is given by the formula $A = 10(0.8)^t$. Find, to the *nearest tenth of an hour*, how long it takes for half of the drug dose to be left in the body.

$$\frac{5}{10} = \frac{10(0.8)^t}{10}$$

$$\log .5 = \log .8^t$$

$$\frac{\log .5}{\log .8} = \frac{t \log .8}{\log .8}$$

$$3.1 \approx t$$

Part IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

33 The availability of leaded gasoline in New York State is decreasing, as shown in the accompanying table.

Year	1984	1988	1992	1996	2000
Gallons Available (in thousands)	150	124	104	76	50

Determine a linear relationship for x (years) versus y (gallons available), based on the data given. The data should be entered using the year and gallons available (in thousands), such as (1984,150).

$y = -6.2x + 12,451.2$

If this relationship continues, determine the number of gallons of leaded gasoline available in New York State in the year 2005.

$y = -6.2(2005) + 12451.2$

$\approx 20,200$

If this relationship continues, during what year will leaded gasoline first become unavailable in New York State?

$0 = -6.2x + 12,451.2$

$x = 2008$

34 Given: $A(1,6)$, $B(7,9)$, $C(13,6)$, and $D(3,1)$

Prove: $ABCD$ is a trapezoid. [*The use of the accompanying grid is optional.*]

	STATEMENT	REASON
(1)	Quadrilateral A(1,6), B(7,9), C(13,6), D(3,1)	(1) Given
(2)	$m_{\overline{AB}} = \frac{9-6}{7-1} = \frac{3}{6} = \frac{1}{2}$ $m_{\overline{CD}} = \frac{6-1}{13-3} = \frac{5}{10} = \frac{1}{2}$ $m_{\overline{AD}} = \frac{6-1}{1-3} = -\frac{5}{2}$ $m_{\overline{BC}} = \frac{9-6}{7-13} = \frac{-3}{6} = -\frac{1}{2}$	(2) Definition of slope
(3)	ABCD is a trapezoid	(3) A trapezoid is a quadrilateral with one and only one pair of parallel sides (have the same slope)