The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS B

Thursday, June 19, 2008 — 1:15 to 4:15 p.m., only

Print Your Name: Steven Sibol

Print Your School's Name: www.jmap.org

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.

The formulas that you may need to answer some questions in this examination are found on page 23. This sheet is perforated so you may remove it from this booklet.

This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice. . .

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

MATHEMATICS B

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [40]

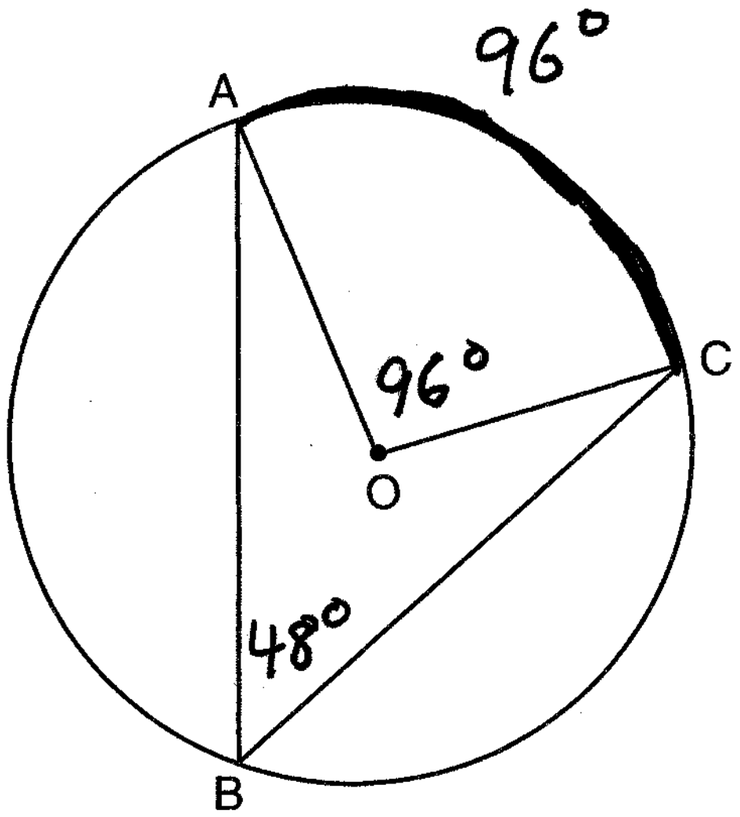
Use this space for computations.

1 For which value of x is $f(x) = \frac{1}{27 - 3^x}$ undefined?

(1) 1
(2) 0
(3) 3
(4) –3

$27 - 3^x = 0$
$27 = 3^x$
$x = 3$

2 In the accompanying diagram of circle O, $\overline{AB}$ and $\overline{BC}$ are chords and $m\angle AOC = 96$. What is $m\angle ABC$?



$\frac{1}{2} \cdot 96 = 48$

(1) 32
(2) 48
(3) 96
(4) 192

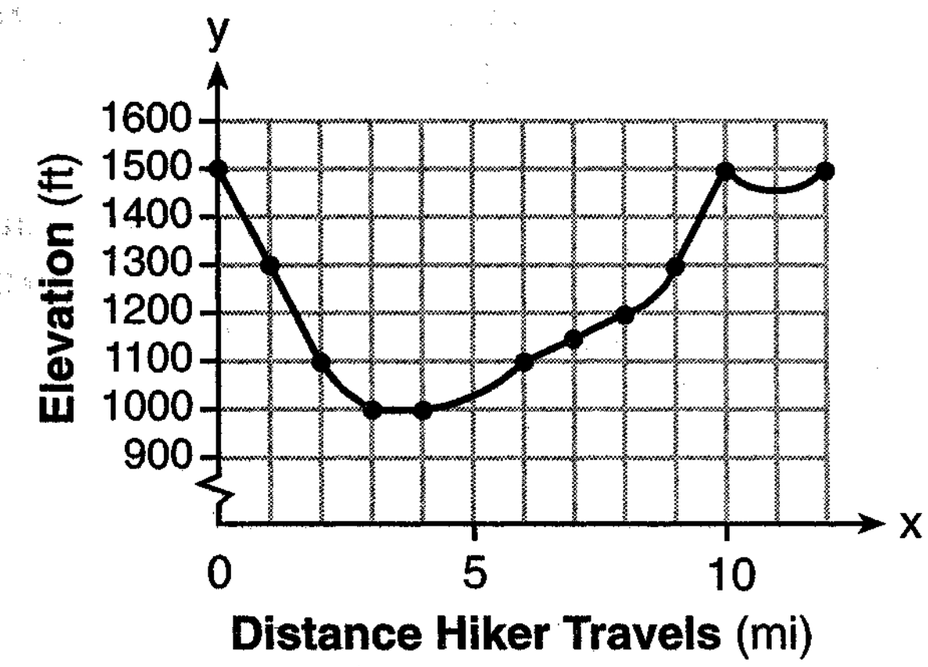
3 Kathy deposits \$25 into an investment account with an annual rate of 5%, compounded annually. The amount in her account can be determined by the formula $A = P(1 + R)^t$, where P is the amount deposited, R is the annual interest rate, and t is the number of years the money is invested. If she makes no other deposits or withdrawals, how much money will be in her account at the end of 15 years?

(1) \$25.75
(2) \$43.75
(3) \$51.97
(4) \$393.97

$A = P(1+R)^t$
$= 25(1+.05)^{15}$
$\approx \$51.97$

Use this space for computations.

4 The accompanying graph shows the elevation of a certain region in New York State as a hiker travels along a trail.



What is the domain of this function?

(1) $1{,}000 \le x \le 1{,}500$
(2) $1{,}000 \le y \le 1{,}500$
(3) $0 \le x \le 12$
(4) $0 \le y \le 12$

range

5 Which relation is a function?

(1) $x^2 + y^2 = 16$ circle
(2) $2x^2 + 6y^2 = 1$ ellipse
(3) $y^2 = x^2 + 3x - 4$ hyperbola
(4) $y = x^2 + 3x - 4$ parabola

6 If $f(x) = x^2 + 4$ and $g(x) = \sqrt{1 - x}$, what is the value of $f(g(-3))$?

(1) $2i\sqrt{3}$
(2) 2
(3) 8
(4) 13

$g(-3) = \sqrt{1-(-3)} = \sqrt{4} = 2$
$f(2) = 2^2 + 4 = 8$

Use this space for computations.

7 Which expression represents the sum of the sequence 3, 5, 7, 9, 11?

(1) $\sum_{n=0}^{5}(2n+1)$

(2) $\sum_{n=1}^{5}3n$

(3) $\sum_{n=1}^{5}(3n+1)$

(4) $\sum_{n=1}^{5}(2n+1)$

8 Which value of a does *not* satisfy the inequality $|a| > 2a - 3$?

(1) –1
(2) 0
(3) 3
(4) –5

$a > 2a-3$ $\quad$ $a < -2a+3$

$3 > a$ $\quad$ $3a < 3$

$a < 3$ $\quad$ $a < 1$

9 If point (5,2) is rotated counterclockwise 90° about the origin, its image will be point

(1) (2,5)
(2) (2,–5)
(3) (–2,5)
(4) (–5,–2)

$(x, y) \rightarrow (-y, x)$

10 What is the sum of $5 - 3i$ and the conjugate of $3 + 2i$?

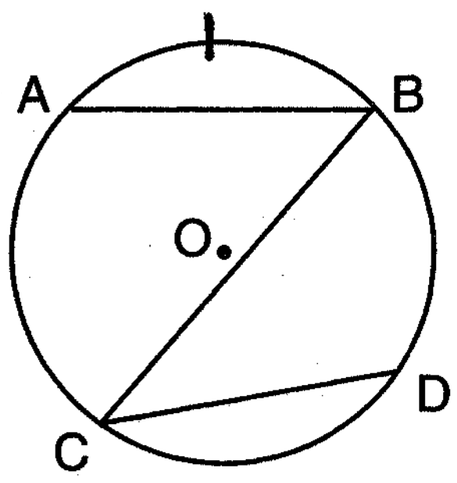
(1) $2 + 5i$
(2) $2 - 5i$
(3) $8 + 5i$
(4) $8 - 5i$

$3 - 2i$

$5 - 3i + 3 - 2i = 8 - 5i$

Use this space for computations.

11 In the accompanying diagram of circle O, $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.



Equal chords intercept equal arcs.

Which statement is true?

(1) $\overline{AB} \cong \overline{CD}$
(2) $\overset{\frown}{AC} \cong \overset{\frown}{BD}$
(3) $\overline{AB} \parallel \overline{CD}$
(4) $\angle ABC \cong \angle BCD$

12 The expression $\cos^2 4\theta + \sin^2 4\theta$ is equivalent to

(1) 1
(2) 2
(3) $\cos \theta$
(4) $\cos 8\theta$

13 The value of $\sqrt{x^2 - 9}$ is a real and irrational number when x is equal to

(1) 5
(2) 0
(3) −3
(4) 4

$\sqrt{x^2-9}$
$\sqrt{4^2-9}$
$\sqrt{7}$

14 If $2^{4x+1} = 8^{x+a}$, which expression is equivalent to x?

(1) $a - 1$
(2) $3a - 1$
(3) $\frac{a-1}{15}$
(4) $\frac{a-1}{3}$

$2^{4x+1} = 8^{x+a}$
$2^{4x+1} = (2^3)^{x+a}$
$2^{4x+1} = 2^{3x+3a}$

$4x+1 = 3x+3a$
$-3x \quad -3x$
$x+1 = 3a$
$-1 \quad -1$
$x = 3a-1$

Use this space for computations.

15 In 1995, the federal government paid off one-third of its debt. If the original amount of the debt was \$4,920,000,000,000, which expression represents the amount that was *not* paid off?

(1) 1.64×10^4
(2) 1.64×10^{12}
(3) 3.28×10^8
(4) 3.28×10^{12}

$\frac{2}{3} \times 4{,}920{,}000{,}000{,}000$

$3{,}280{,}000{,}000{,}000$

16 The expression $\frac{2}{\sin x} - \frac{5}{\sin x - 1}$ is equivalent to

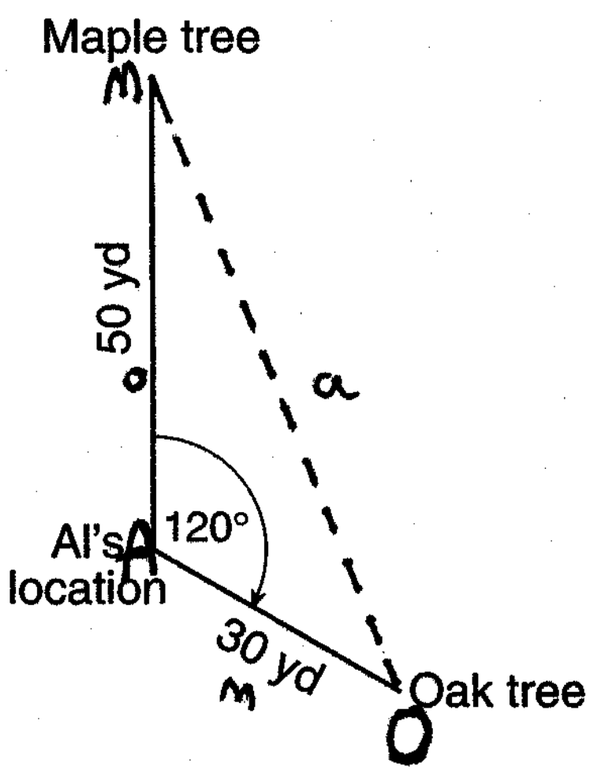
(1) $\frac{-3}{\sin x(\sin x - 1)}$
(2) $\frac{-3}{\sin x - 1}$
(3) $\frac{-3\sin x - 2}{\sin x(\sin x - 1)}$
(4) $\frac{-3\sin x - 2}{\sin x - 1}$

$\frac{2(\sin x - 1) - 5\sin x}{\sin x(\sin x - 1)}$

$\frac{2\sin x - 2 - 5\sin x}{\sin x(\sin x - 1)}$

$\frac{-3\sin x - 2}{\sin x(\sin x - 1)}$

17 Al is standing 50 yards from a maple tree and 30 yards from an oak tree in the park. His position is shown in the accompanying diagram. If he is looking at the maple tree, he needs to turn his head 120° to look at the oak tree.



$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = m^2 + o^2 - 2mo\cos A$

$a^2 = 30^2 + 50^2 - 2(30)(50)\cos 120°$

$a^2 = 4900$

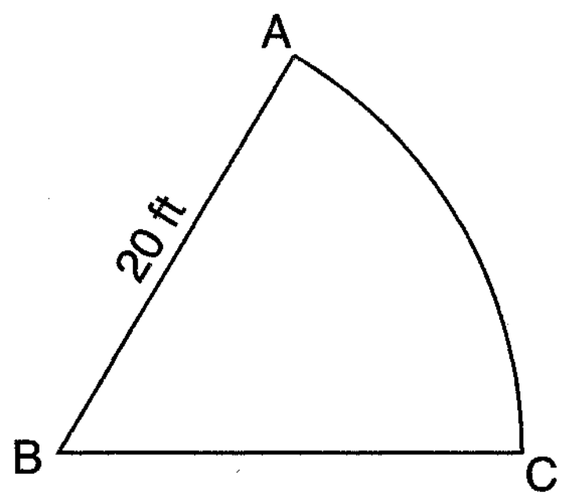
$a = 70$

How many yards apart are the two trees?

(1) 58.3
(2) 65.2
(3) 70
(4) 75

Use this space for computations.

18 A sprinkler system is set up to water the sector shown in the accompanying diagram, with angle ABC measuring 1 radian and radius $AB = 20$ feet.



What is the length of arc AC, in feet?

(1) 63
(2) 31
(3) 20
(4) 10

$\theta = \frac{s}{r}$

$1 = \frac{s}{20}$

$s = 20$

19 The expression $i^{100} + i^{101} + i^{102}$ equals

(1) 1
(2) −1
(3) $-i$
(4) i

$i^0 + i^1 + i^2$

$1 + i + (-1)$

i

20 Which equation has roots whose sum is 3 and whose product is −4?

(1) $x^2 + 3x - 4 = 0$
(2) $x^2 - 3x - 4 = 0$
(3) $x^2 + 4x - 3 = 0$
(4) $x^2 - 4x + 3 = 0$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4 \quad x = -1$

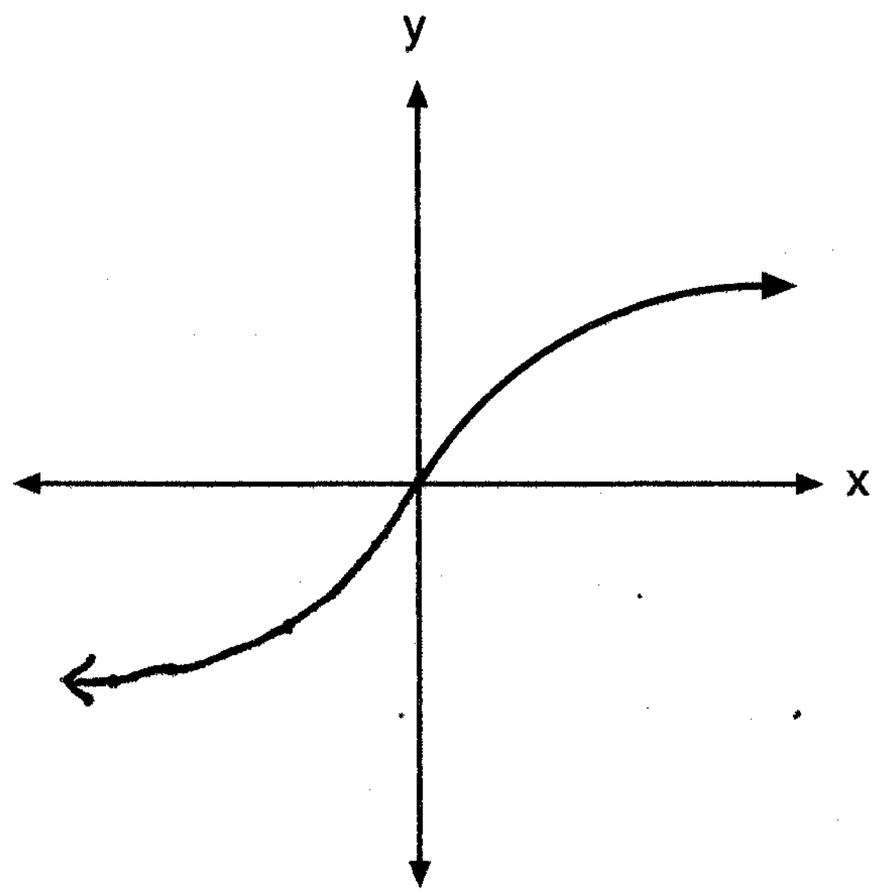
$4 + -1 = 3$

$4 \cdot -1 = -4$

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

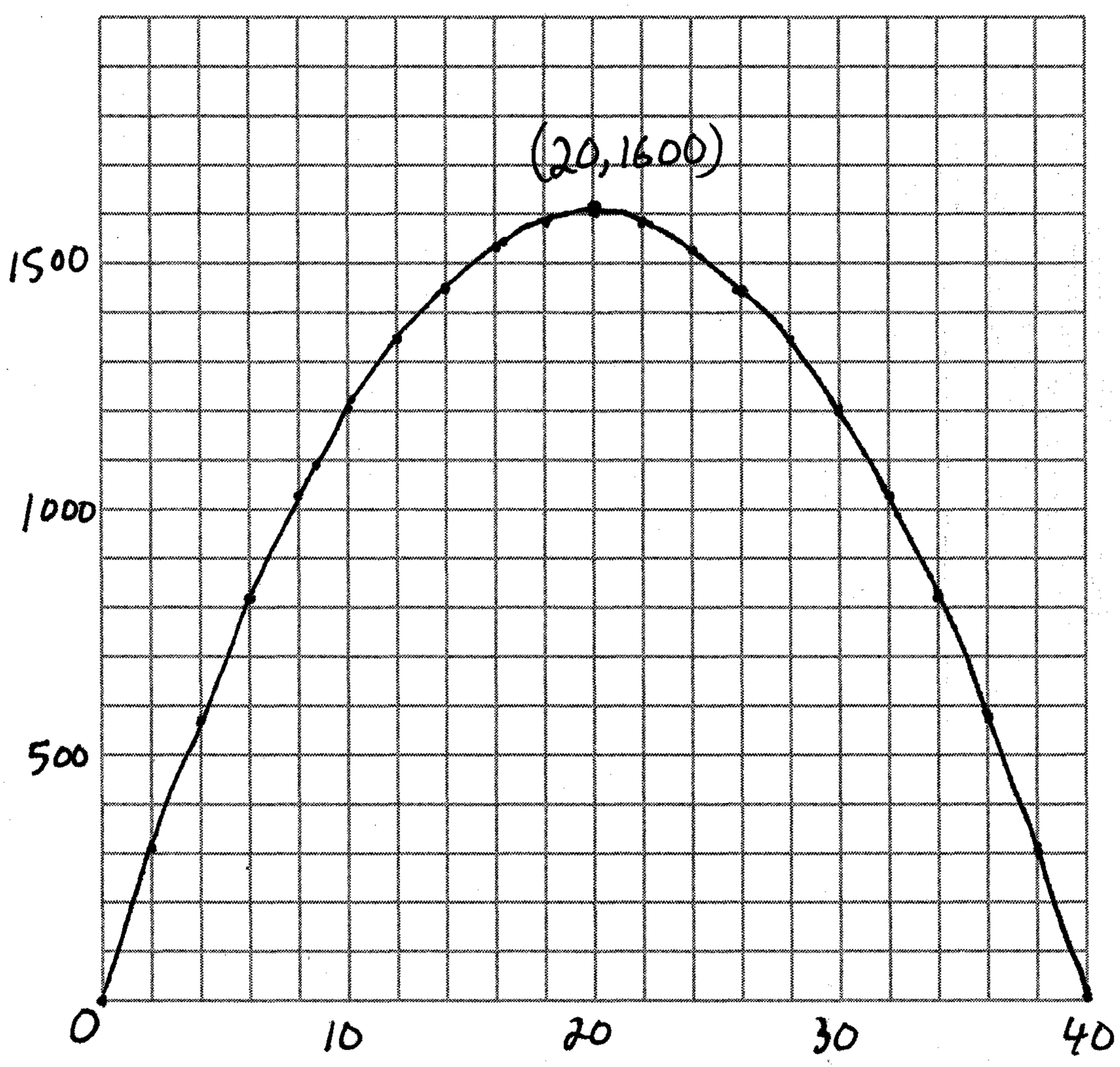
21 The entire graph of f(x) is symmetric with respect to the origin. If the accompanying graph represents f(x) for $x \geq 0$, sketch, on the same set of axes, the graph of f(x) for $x \leq 0$.



same as a 180° rotation

22 A laundry owner's estimate of her weekly profits, p, in dollars, is given by the equation $p = -4w^2 + 160w$, where w represents the number of workers she hires. What is the number of workers she should hire in order to earn the greatest profit? [The use of the accompanying grid is optional.]

$$w = \frac{-B}{2A} = \frac{-160}{2(-4)} = 20$$



23 Simplify: $\dfrac{\frac{x}{3}-\frac{3}{x}}{\frac{x-3}{x}}$

$$\left(\frac{x}{3}-\frac{3}{x}\right)\div\frac{x-3}{x}$$

$$\left(\frac{x}{3}-\frac{3}{x}\right)\times\frac{x}{x-3}$$

$$\frac{x^2-9}{3x}\times\frac{x}{x-3}$$

$$\frac{(x+3)\cancel{(x-3)}}{3\cancel{x}}\times\frac{\cancel{x}}{\cancel{x-3}}$$

$$\frac{x+3}{3}$$

24 The coordinates of quadrilateral $PRAT$ are $P(a,b)$, $R(a,b + 3)$, $A(a + 3,b + 4)$, and $T(a + 6,b + 2)$. Prove that $\overline{RA}$ is parallel to $\overline{PT}$.

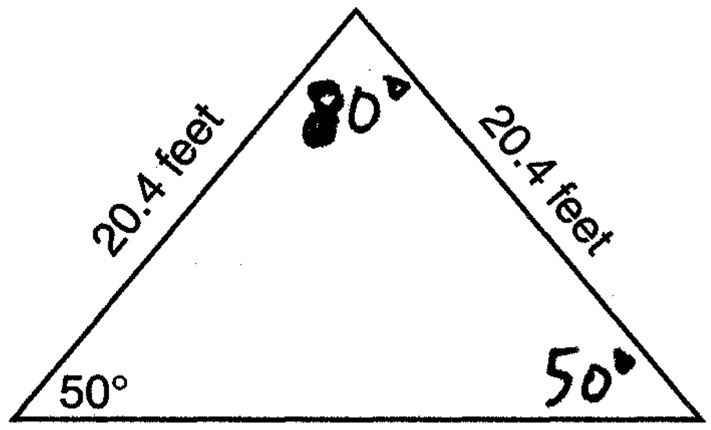
Parallel lines have equal slope.

$$m\overline{RA} = \frac{(b+3)-(b+4)}{a-(a+3)} = \frac{-1}{-3} = \frac{1}{3}$$

$$m\overline{PT} = \frac{b-(b+2)}{a-(a+6)} = \frac{-2}{-6} = \frac{1}{3}$$

Because $\overline{RA}$ and $\overline{PT}$ have equal slopes, they are parallel.

25 The accompanying diagram shows the peak of a roof that is in the shape of an isosceles triangle. A base angle of the triangle is 50° and each side of the roof is 20.4 feet. Determine, to the *nearest tenth of a square foot*, the area of this triangular region.

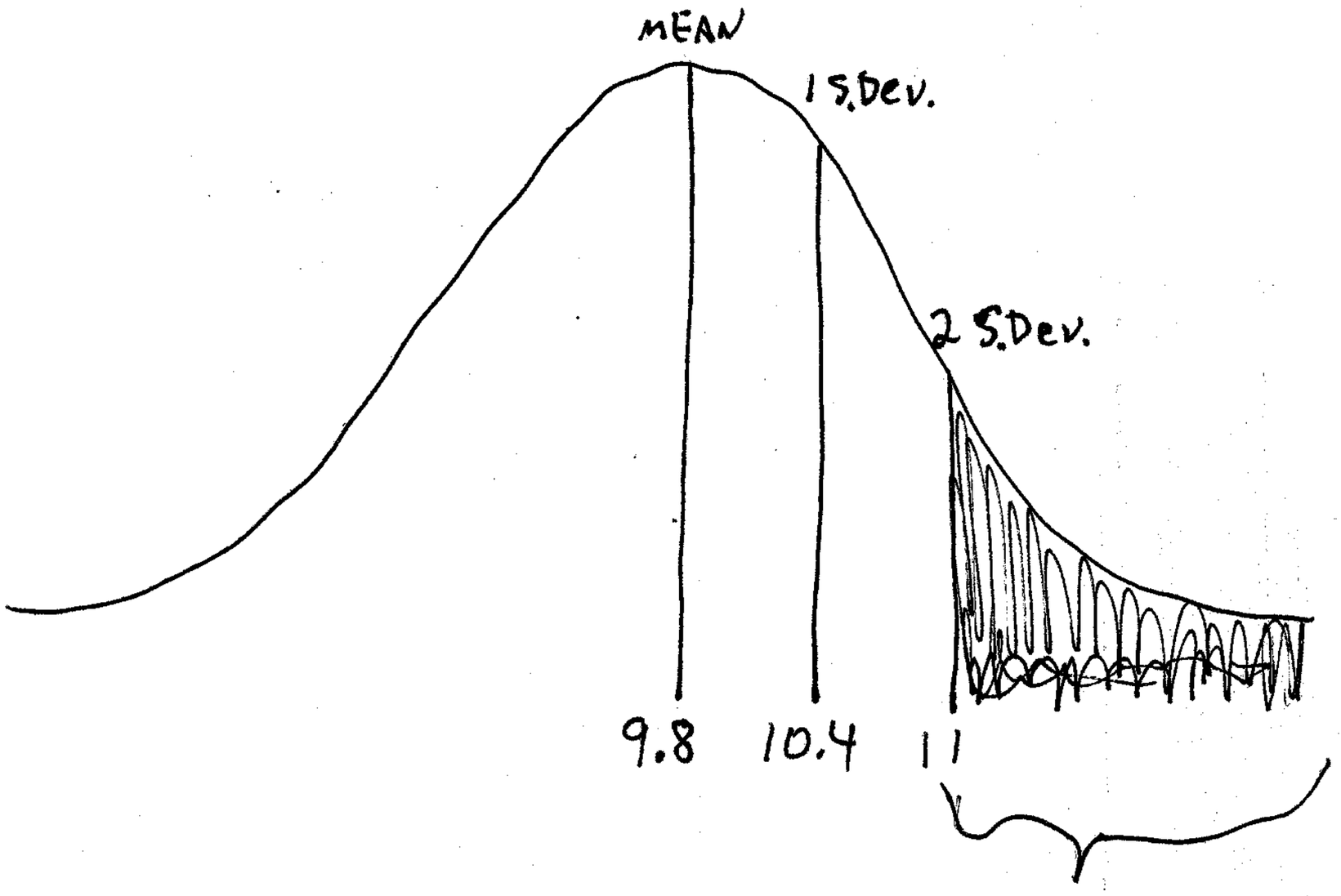


$$\text{Area} = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2}(20.4)(20.4)\sin 80$$

$$\approx 204.9$$

26 The weights of the boxes of animal crackers coming off an assembly line differ slightly and form a normal distribution whose mean is 9.8 ounces and whose standard deviation is 0.6 ounce. Determine the number of boxes of animal crackers in a shipment of 5,000 boxes that are expected to weigh *more than* 11 ounces.



1.7% + 0.5% + 0.1%

= 2.3%

2.3% × 5000 = 115

Part III

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [24]

27 The accompanying table shows the amount of water vapor, y, that will saturate 1 cubic meter of air at different temperatures, x.

Amount of Water Vapor That Will Saturate 1 Cubic Meter of Air at Different Temperatures

Air Temperature (x) (°C)	**Water Vapor (y)** (g)
–20	1
–10	2
0	5
10	9
20	17
30	29
40	50

Write an exponential regression equation for this set of data, rounding all values to the *nearest thousandth*.

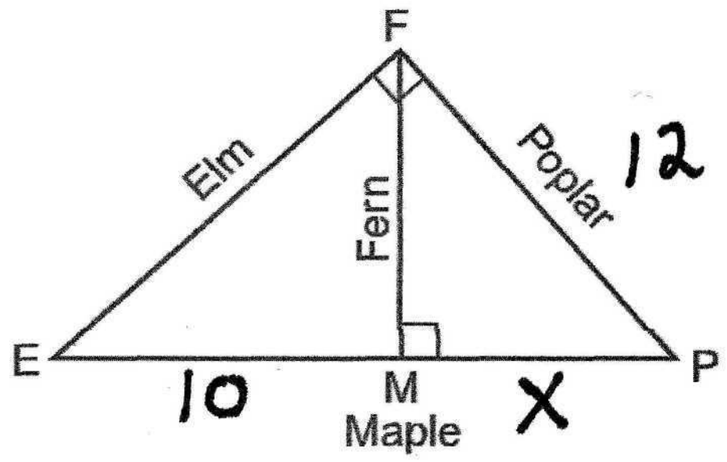
$y = 4.194(1.068)^x$

Using this equation, predict the amount of water vapor that will saturate 1 cubic meter of air at a temperature of 50°C, and round your answer to the *nearest tenth of a gram*.

$y = 4.194(1.068)^{50}$

≈ 112.5

28 Four streets in a town are illustrated in the accompanying diagram. If the distance on Poplar Street from F to P is 12 miles and the distance on Maple Street from E to M is 10 miles, find the distance on Maple Street, in miles, from M to P.



$$\frac{10+x}{12} = \frac{12}{x}$$

$$10x + x^2 = 144$$

$$x^2 + 10x - 144 = 0$$

$$(x+18)(x-8) = 0$$

$x = -18$ ~~$x = -18$~~ $\boxed{x = 8}$

29 Find all values of θ in the interval $0° \leq \theta < 360°$ that satisfy the equation $3\cos 2\theta + 2\sin\theta + 1 = 0$, and round all answers to the *nearest hundredth of a degree*. [Only an algebraic solution can receive full credit.]

$3\cos 2\theta + 2\sin\theta + 1 = 0$

$3(1 - 2\sin^2\theta) + 2\sin\theta + 1 = 0$

$3 - 6\sin^2\theta + 2\sin\theta + 1 = 0$

$-6\sin^2\theta + 2\sin\theta + 4 = 0$ divide by -2

$3\sin^2\theta - \sin\theta - 2 = 0$

let $\sin\theta = x$

$3x^2 - x - 2 = 0$

$(3x + 2)(x - 1) = 0$

$3x + 2 = 0$ $\quad$ $x - 1 = 0$

$x = -\frac{2}{3}$ $\quad$ $x = 1$

$\sin\theta = -\frac{2}{3}$ $\quad$ $\sin\theta = 1$

$\theta = \sin^{-1}\left(-\frac{2}{3}\right)$ $\quad$ $\theta = 90°$

$\theta = 318.19$

or

221.81

30 The probability of rain on the last day of July is 90%. If the probability remains constant for the first seven days of August, what is the probability that it will rain *at least* six of those seven days in August?

$$P = {}_nC_r\, p^r q^{n-r}$$

$n = 7$

$r = 6, 7$

$p = .9$

$q = .1$

$$P(6) = {}_7C_6 \cdot .9^6 \cdot .1^1 = .372$$

$$P(7) = {}_7C_7 \cdot .9^7 \cdot .1^0 = .478$$

$$.850$$

85%

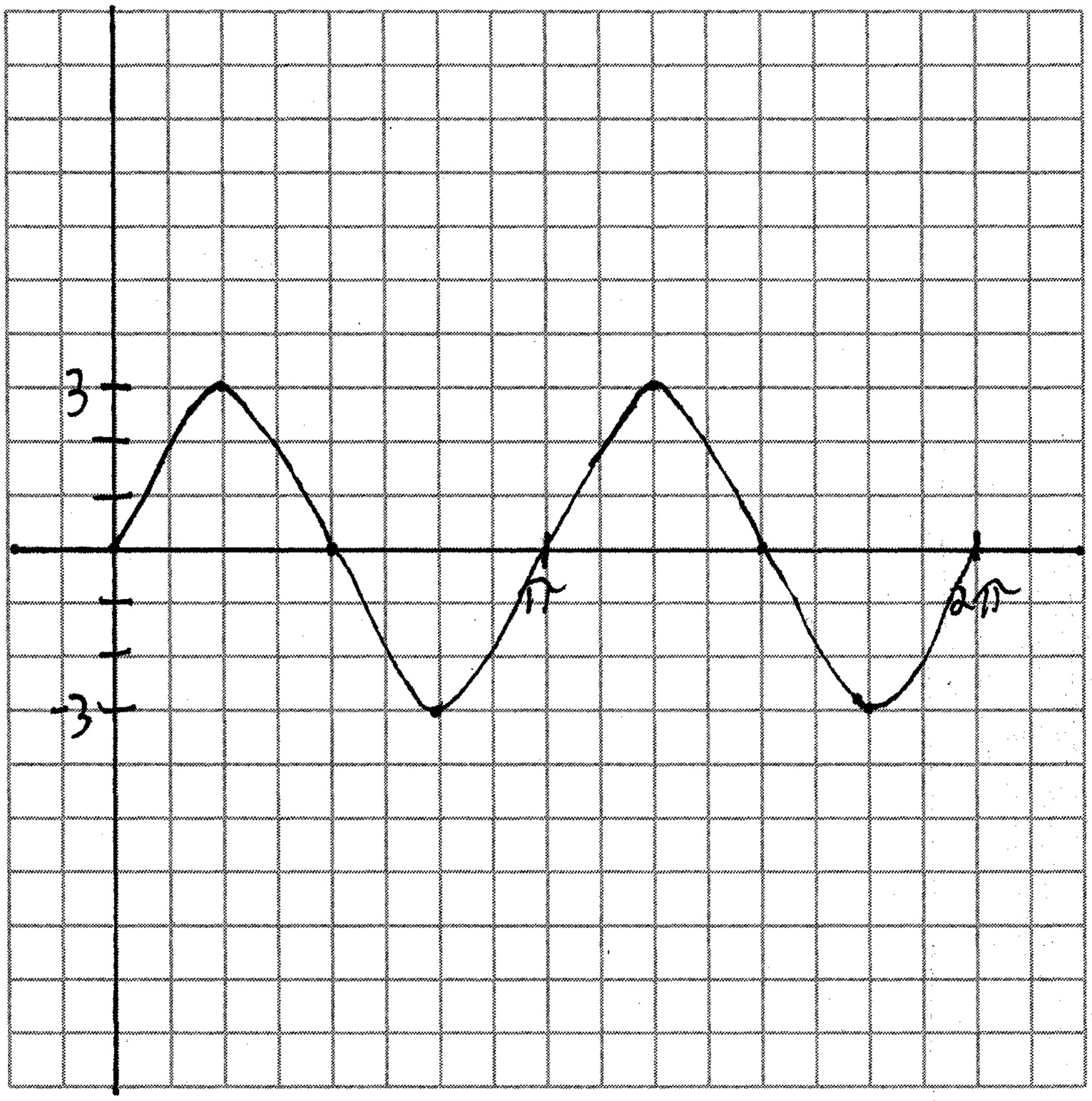
31 The engineering office in the village of Whitesboro has a map of the village that is laid out on a rectangular coordinate system. A traffic circle located on the map is represented by the equation $(x + 4)^2 + (y - 2)^2 = 81$. The village planning commission asks that the transformation D_2 be applied to produce a new traffic circle, where the center of dilation is at the origin.

Find the coordinates of the center of the new traffic circle.

Find the length of the radius of the new traffic circle.

	Center	Radius
Before D_2	$(-4, 2)$	$\sqrt{81} = 9$
After D_2	$(-8, 4)$	18

32 A radio wave has an amplitude of 3 and a wavelength (period) of π meters. On the accompanying grid, using the interval 0 to 2π, draw a possible sine curve for this wave that passes through the origin.



Part IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

33 Solve for x: $\log_3 (x^2 - 4) - \log_3 (x + 2) = 2$

$$\log_3 \frac{x^2-4}{x+2} = 2$$

$$\frac{x^2-4}{x+2} = 3^2$$

$$\frac{(x+2)(x-2)}{x+2} = 9$$

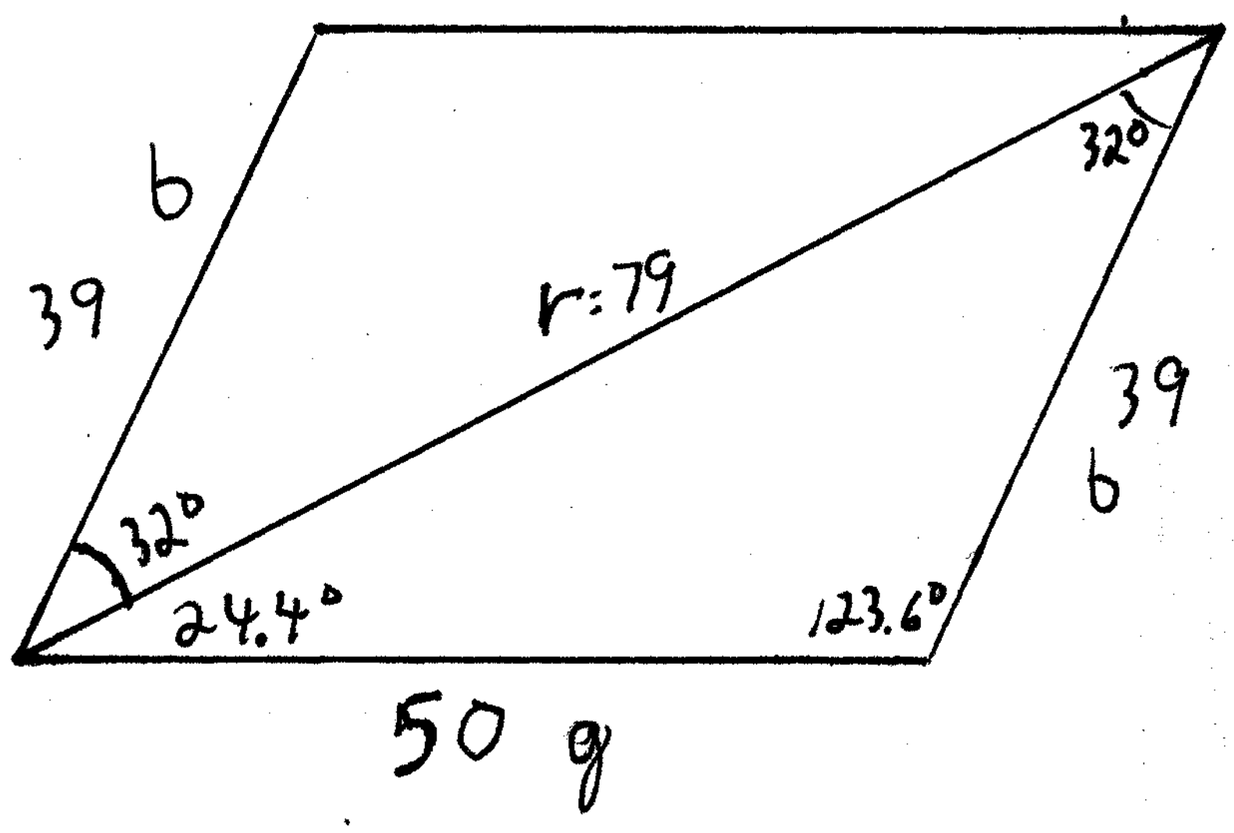
$$x - 2 = 9$$

$$x = 11$$

34 Gerardo and Bennie are pushing a box. Gerardo pushes with a force of 50 pounds in an easterly direction, and Bennie pushes with a force of 39 pounds in a northeasterly direction. The resultant force forms an angle of 32° with the 39-pound force.

Find the angle between the 50-pound force and the 39-pound force, to the *nearest tenth of a degree*.

Find the magnitude of the resultant force, to the *nearest pound*.



$$\frac{g}{\sin G} = \frac{b}{\sin B}$$

$$\frac{50}{\sin 32} = \frac{39}{\sin B}$$

$$B \approx 24.4°$$

$$32 + 24.4 \approx 56.4°$$

$$\frac{r}{\sin R} = \frac{g}{\sin G}$$

$$\frac{r}{\sin 123.6} = \frac{50}{\sin 32}$$

$$r \approx 79$$