The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS B

Thursday, June 15, 2006 — 1:15 to 4:15 p.m., only

Print Your Name: Steve Sibol

Print Your School's Name: HSCR

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.

The formulas that you may need to answer some questions in this examination are found on page 19. This sheet is perforated so you may remove it from this booklet.

This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice. . .

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

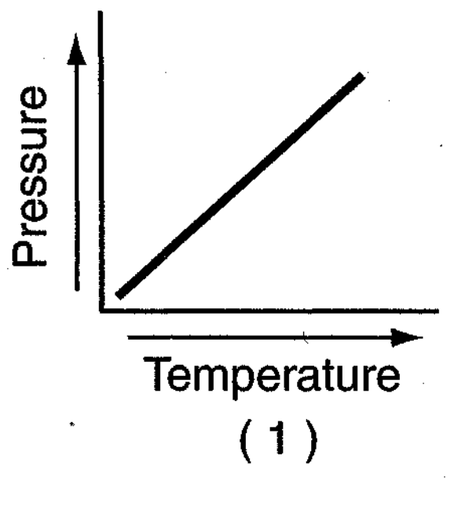
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

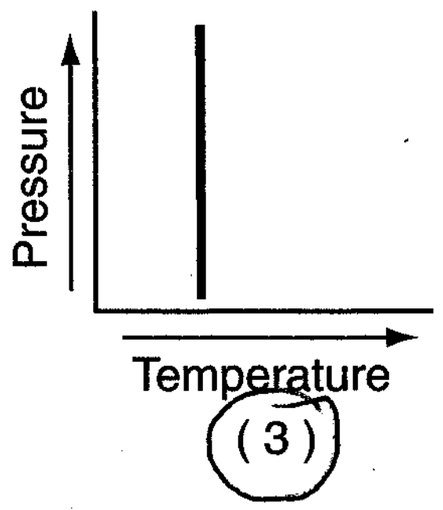
Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [40]

1 Each graph below represents a possible relationship between temperature and pressure. Which graph does *not* represent a function?

Use this space for computations.

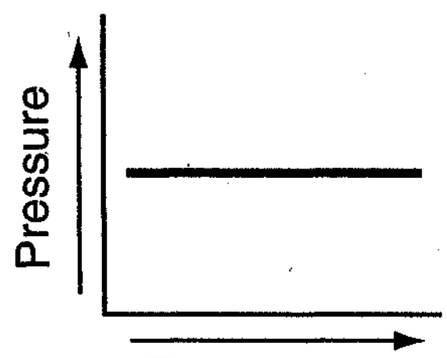


(1)

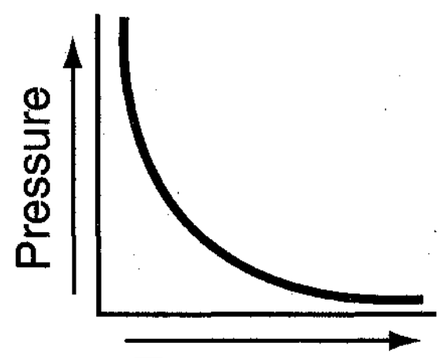


(3)

Does not pass the vertical line test



(2)



(4)

2 If $f(x) = x^{-\frac{3}{2}}$, then $f\left(\frac{1}{4}\right)$ is equal to

(1) 8

(2) -2

(3) $-\frac{1}{8}$

(4) -4

$F\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)^{-\frac{3}{2}}$

$= 4^{3/2}$

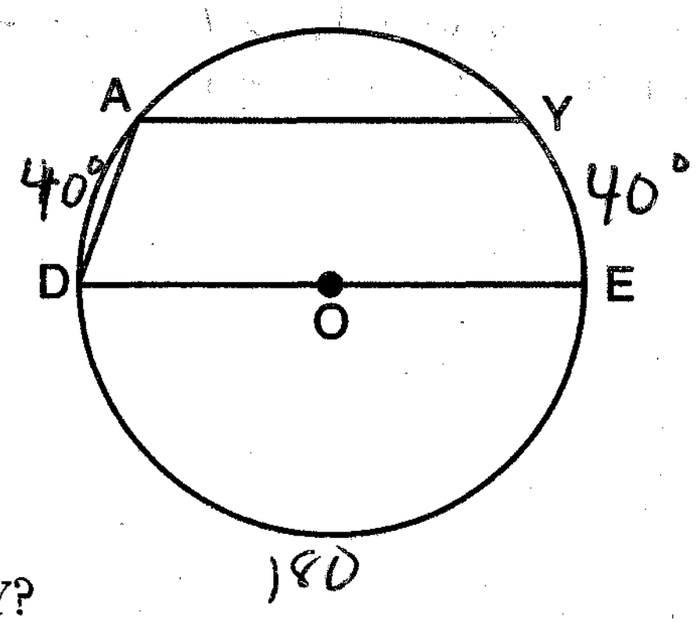
$= \sqrt{4^3}$

$= \sqrt{64}$

$= 8$

Use this space for computations.

3 In the accompanying diagram of circle O, chord $\overline{AY}$ is parallel to diameter $\overline{DOE}$, $\overline{AD}$ is drawn, and $m\overset{\frown}{AD} = 40$.



∠DAY intercepts major arc DY, which is 220° (180°+40°). The inscribed angle is $\frac{1}{2} \times 220 = 110$

What is $m\angle DAY$?

(1) 90
(2) 110
(3) 130
(4) 150

4 If x is a positive acute angle and $\sin x = \frac{1}{2}$, what is $\sin 2x$?

(1) $-\frac{1}{2}$
(2) $\frac{1}{2}$
(3) $-\frac{\sqrt{3}}{2}$
(4) $\frac{\sqrt{3}}{2}$

IDENTITY
$\sin 2x = 2\sin x \cos x$
$= 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$
$= \frac{\sqrt{3}}{2}$

$\cos^2 x + \sin^2 x = 1$
$\cos^2 x + \left(\frac{1}{2}\right)^2 = 1$
$\cos^2 x = 3/4$
$\cos x = \frac{\sqrt{3}}{2}$

OR

$\sin x = \frac{1}{2}$
$x = \sin^{-1}\left(\frac{1}{2}\right)$
$x = 30°$ (not 150° since x is acute)
$2x = 60°$
$\sin 60° = \frac{\sqrt{3}}{2}$

5 The temperature generated by an electrical circuit is represented by $t = f(m) = 0.3m^2$, where m is the number of moving parts. The resistance of the same circuit is represented by $r = g(t) = 150 + 5t$, where t is the temperature. What is the resistance in a circuit that has four moving parts?

(1) 51
(2) 156
(3) 174
(4) 8,670

$f(4) = 0.3(4)^2 = 4.8$
$g(4.8) = 150 + 5(4.8)$
$= 174$

6 If the equation $x^2 - kx - 36 = 0$ has $x = 12$ as one root, what is the value of k?

(1) 9
(2) 12
(3) 3
(4) –3

$x^2 - kx - 36 = 0$
$(12)^2 - 12k - 36 = 0$
$108 - 12k = 0$
$-12k = -108$
$k = +9$

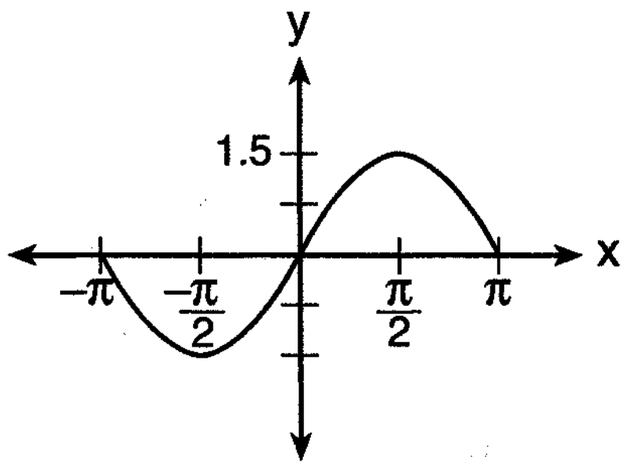
7 The height, f(x), of a bouncing ball after x bounces is represented by f(x) = 80(0.5)^x. How many times higher is the first bounce than the fourth bounce?

(1) 8
(2) 2
(3) 16
(4) 4

Use this space for computations.

$F(1) = 80(0.5)^1 = \frac{40}{5} = 8$
$F(4) = 80(0.5)^4 =$

8 A radio transmitter sends a radio wave from the top of a 50-foot tower. The wave is represented by the accompanying graph.



Because the minimum and maximum are ±1.5, the amplitude is 1.5. The period of the function is 2π, so the coefficient of x is 1.

What is the equation of this radio wave?

(1) $y = \sin x$
(2) $y = 1.5 \sin x$
(3) $y = \sin 1.5x$
(4) $y = 2 \sin x$

9 If tan θ = 2.7 and csc θ < 0, in which quadrant does θ lie?

(1) I
(2) II
(3) III
(4) IV

If the tangent of an angle is positive, it lies in quadrant I ~~or II~~ III

If the csc is negative, the sin of an angle is negative, and lies in quadrant III or IV.

10 The expression $\frac{1 - \cos^2 x}{\sin^2 x}$ is equivalent to

(1) 1
(2) –1
(3) sin x
(4) cos x

$\sin^2 x + \cos^2 x = 1$
$\sin^2 x = 1 - \cos^2 x$

$\frac{1 - \cos^2 x}{\sin^2 x} = \frac{\sin^2 x}{\sin^2 x} = 1$

11 The graph of $y = (x - 3)^2$ is shifted left 4 units and down 2 units. What is the axis of symmetry of the transformed graph?

(1) $x = -2$
(2) $x = -1$
(3) $x = 1$
(4) $x = 7$

The axis of symmetry of $y = (x-3)^2$ is $x = 3$
If shifted left 4 units, the axis is $x = -1$

[4]

12 The solution set of $2^{x^2+2x} = 2^{-1}$ is

(1) {1}
(2) {−1}
(3) {−1,1}
(4) { }

Because 2 is the common base:

$x^2+2x = -1$
$x^2+2x+1 = 0$
$(x+1)^2 = 0$
$x = -1$

Use this space for computations.

13 Which transformation best describes the relationship between the functions $f(x) = 2^x$ and $g(x) = \left(\frac{1}{2}\right)^x$?

(1) reflection in the line $y = x$
(2) reflection in the origin
(3) reflection in the x-axis
(4) reflection in the y-axis

$2^{-x} = \left(\frac{1}{2}\right)^x$ $\left(\frac{1}{2}\right)^{-x} = 2^x$

14 What is the multiplicative inverse of $3i$?

(1) $-3i$
(2) -3
(3) $\frac{1}{3}$
(4) $-\frac{i}{3}$

$\frac{1}{3i} \cdot \frac{i}{i} = \frac{i}{3i^2} \Rightarrow \frac{i}{3(-1)} = -\frac{i}{3}$

15 Mrs. Donahue made up a game to help her class learn about imaginary numbers. The winner will be the student whose expression is equivalent to $-i$. Which expression will win the game?

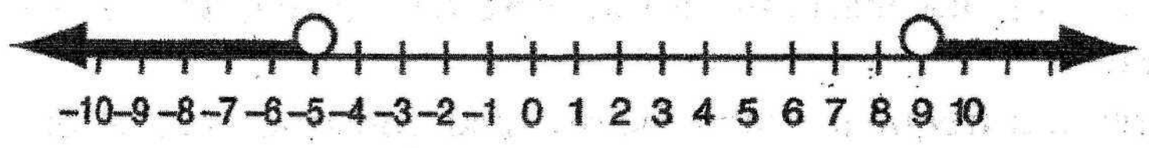
(1) i^{46}
(2) i^{47}
(3) i^{48}
(4) i^{49}

$-i = i^{4n+3}$

16 Which equation represents a hyperbola?

(1) $y^2 = 16 - x^2$ circle
(2) $y = 16 - x^2$ semi-parabola
(3) $y = 16x^2$ parabola
(4) $y = \frac{16}{x}$ hyperbola

17 The solution set of which inequality is represented by the accompanying graph?



(1) $|x-2|>7$
(2) $|x-2|<7$
(3) $|2-x|>-7$
(4) $|2-x|<-7$

Use this space for computations.

$|x-2|>7$

$x-2>7$ or $x-2<-7$

$x>9$ $x<-5$

18 According to Boyle's Law, the pressure, p, of a compressed gas is inversely proportional to the volume, v. If a pressure of 20 pounds per square inch exists when the volume of the gas is 500 cubic inches, what is the pressure when the gas is compressed to 400 cubic inches?

(1) 16 lb/in^2
(2) 25 lb/in^2
(3) 40 lb/in^2
(4) 50 lb/in^2

$p_1V_1 = p_2V_2$

$20 \times 500 = p_2 \times 400$

$25 = p_2$

19 What is the fourth term in the expansion of $(y-1)^7$?

(1) $35y^4$
(2) $35y^3$
(3) $-35y^4$
(4) $-35y^3$

$n = 7$

$r-1 = 3$

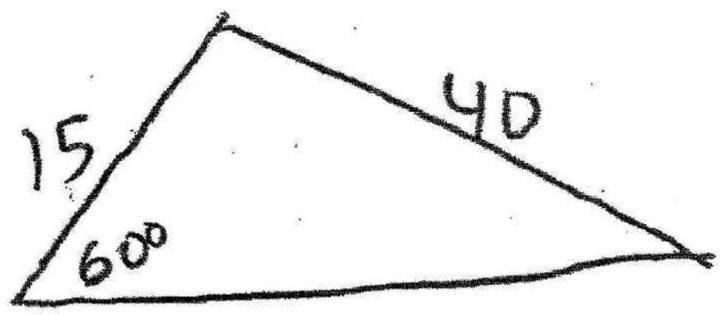
$nC_{r-1}\, x^{n-(r-1)}\, y^{r-1}$

$7C_3 (y)^{7-3} (-1)^3$

$-35y^4$

20 Sam needs to cut a triangle out of a sheet of paper. The only requirements that Sam must follow are that one of the angles must be 60°, the side opposite the 60° angle must be 40 centimeters, and one of the other sides must be 15 centimeters. How many different triangles can Sam make?

(1) 1
(2) 2
(3) 3
(4) 0



$\frac{40}{\sin 60} = \frac{15}{\sin C}$

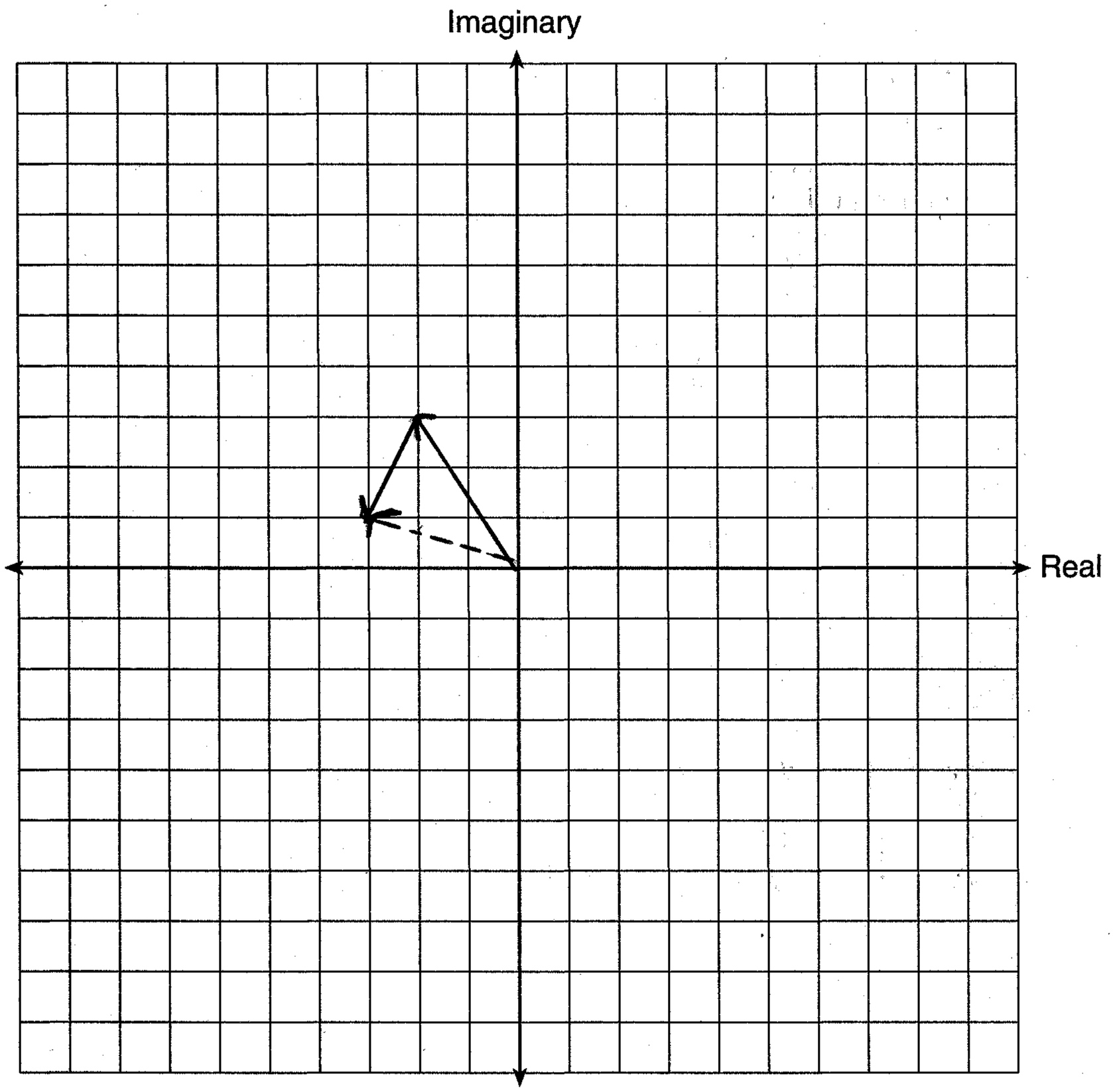
$C = 19° + 60° < 180°$

$C = 180 - 19 = 161 + 60 \not< 180$

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

21 Find the sum of $-2 + 3i$ and $-1 - 2i$. $= -3 + i$
Graph the resultant on the accompanying set of axes.



22 In $\triangle ABC$, $m\angle A = 53$, $m\angle B = 14$, and $a = 10$. Find b to the *nearest integer.*

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{10}{\sin 53} = \frac{b}{\sin 14}$$

$$b = \frac{10 \sin 14}{\sin 53}$$

$$b = 3$$

23 Solve for x: $\log_2 (x + 1) = 3$

$$x + 1 = 2^3$$

$$x + 1 = 8$$

$$x = 7$$

24 Evaluate: $\sum_{k=1}^{2} \frac{(-1)^{k-1}}{(2k-1)!}$

k	$\frac{(-1)^{k-1}}{(2k-1)!}$
1	$\frac{(-1)^{1-1}}{(2(1)-1)!} = \frac{-1^0}{1!} = \frac{1}{1} = 1$
2	$\frac{(-1)^{2-1}}{(2(2)-1)!} = \frac{-1^1}{3!} = \frac{-1}{6}$

$$\frac{5}{6}$$

25 Ginger and Mary Anne are planning a vacation trip to the island of Capri, where the probability of rain on any day is 0.3. What is the probability that during their five days on the island, they have *no* rain on *exactly* three of the five days?

$p(\text{no rain}) = .7$

$q = .3$

$n = 5$

$r = 3$

$${}_nC_r\, p^r q^{n-r}$$

$${}_5C_3 (.7)^3 (.3)^2$$

$$10 \left(\frac{343}{1000}\right)\left(\frac{9}{100}\right) = \frac{30870}{100000}$$

26 The pendulum of a clock swings through an angle of 2.5 radians as its tip travels through an arc of 50 centimeters. Find the length of the pendulum, in centimeters.

$$\theta = \frac{s}{r}$$

$$2.5 = \frac{50}{r}$$

$$r = 20$$

Part III

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [24]

27 Solve the following system of equations algebraically:

$$9x^2 + y^2 = 9$$
$$3x - y = 3$$

$3x - y = 3$

$y = 3x - 3$

$9x^2 + y^2 = 9$

$9x^2 + (3x-3)^2 = 9$

$9x^2 + 9x^2 - 18x + 9 = 9$

$18x^2 - 18x = 0$

$18x(x-1) = 0$

$18x = 0 \qquad x - 1 = 0$

$x = 0 \qquad x = 1$

$3x - y = 3$

$3(0) - y = 3$

$-y = 3$

$y = -3$

$(0, -3)$

$3x - y = 3$

$3(1) - y = 3$

$3 - y = 3$

$y = 0$

$(1, 0)$

28 Simplify for all values of a for which the expression is defined: $\dfrac{1-\frac{2}{a}}{\frac{4}{a^2}-1}$

$$\frac{\frac{a-2}{a}}{\frac{4-a^2}{a^2}} = \frac{a-2}{a} \times \frac{a^2}{4-a^2} = \frac{a-2}{4-a^2} = \frac{-1(a-2)a}{(2-a)(2+a)}$$

$$= \frac{-a}{a+2}$$

29 Solve algebraically for x: $\sqrt{3x+1}+1=x$

$$\sqrt{3x+1} = x-1$$

$$3x+1 = (x-1)^2$$

$$3x+1 = x^2-2x+1$$

$$0 = x^2-5x$$

$$0 = x(x-5)$$

$x = 0$ or $x = 5$

$\sqrt{3(0)+1} = 0-1$

$\sqrt{1} = -1$

FALSE

EXTRANEOUS SOLUTION

$\sqrt{3(5)+1}+1 = 5$

$\sqrt{16}+1 = 5$

$4+1 = 5$

30 The number of children of each of the first 41 United States presidents is given in the accompanying table. For this population, determine the mean and the standard deviation to the *nearest tenth*.

How many of these presidents fall within one standard deviation of the mean?

Number of Children (x_i)	Number of Presidents (f_i)
0	6
1	2
2	8
3	6
4	7
5	3
6	5
7	1
8	1
10	1
15	1

mean = 3.6
standard deviation = 2.9

$3.6 - 2.9 < x_i < 3.6 + 2.9$

$.7 < x_i < 6.5$

$f_i = 31$

31 A factory is producing and stockpiling metal sheets to be shipped to an automobile manufacturing plant. The factory ships only when there is a minimum of 2,050 sheets in stock. The accompanying table shows the day, x, and the number of sheets in stock, $f(x)$.

Day (x)	Sheets in Stock ($f(x)$)
1	860
2	930
3	1000
4	1150
5	1200
6	1360

Write the linear regression equation for this set of data, rounding the coefficients to *four decimal places*.

Use this equation to determine the day the sheets will be shipped.

$y = 98.8571x + 737.3333$

$2050 = 98.8571x + 737.3333$

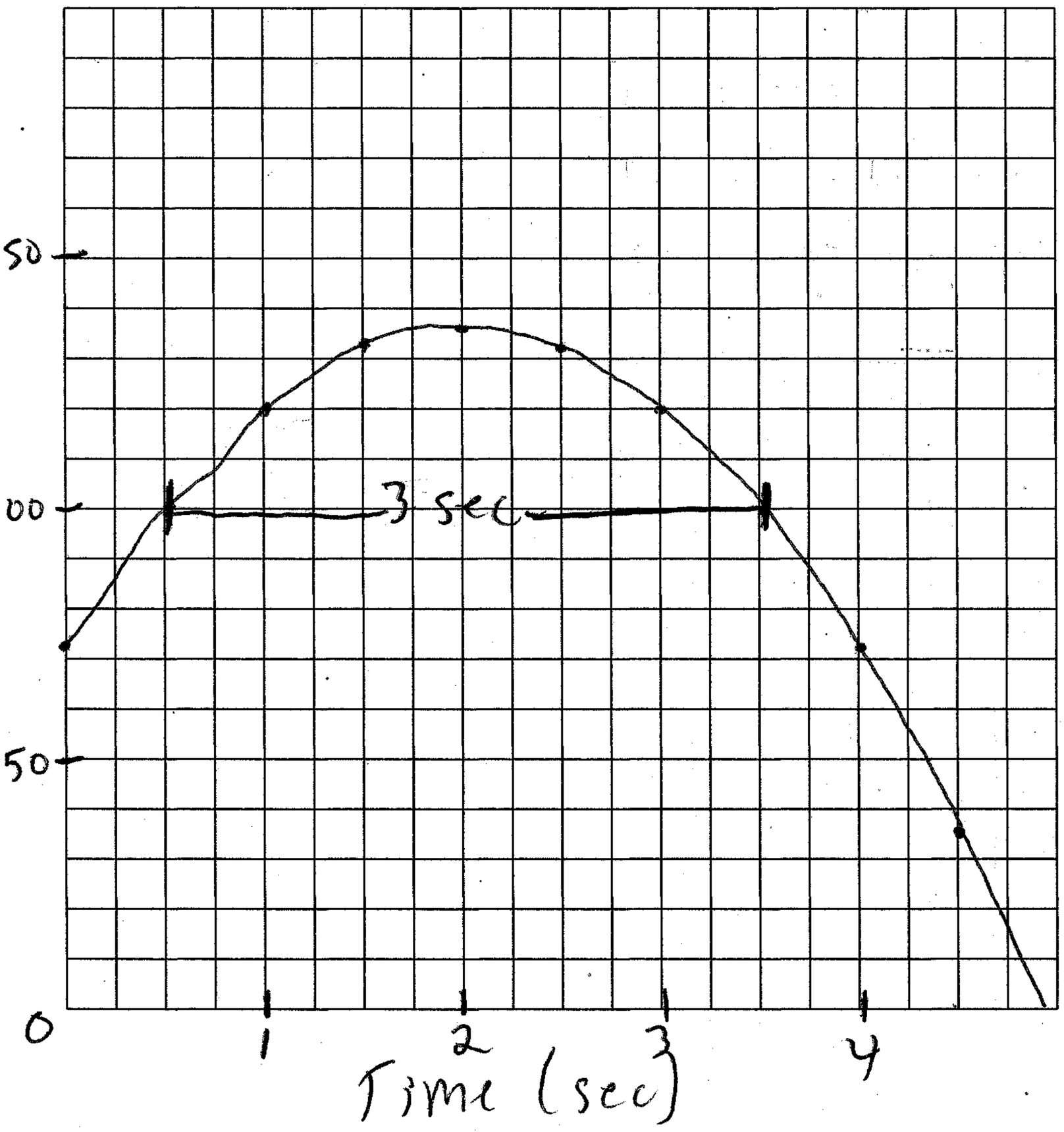
$1312.6667 = 98.8571x$

$13.2 = x$

day 14

32 A small rocket is launched from a height of 72 feet. The height of the rocket in feet, h, is represented by the equation $h(t) = -16t^2 + 64t + 72$, where t = time, in seconds. Graph this equation on the accompanying grid.

Use your graph to determine the number of seconds that the rocket will remain at or above 100 feet from the ground. [Only a graphic solution can receive full credit.]



Part IV

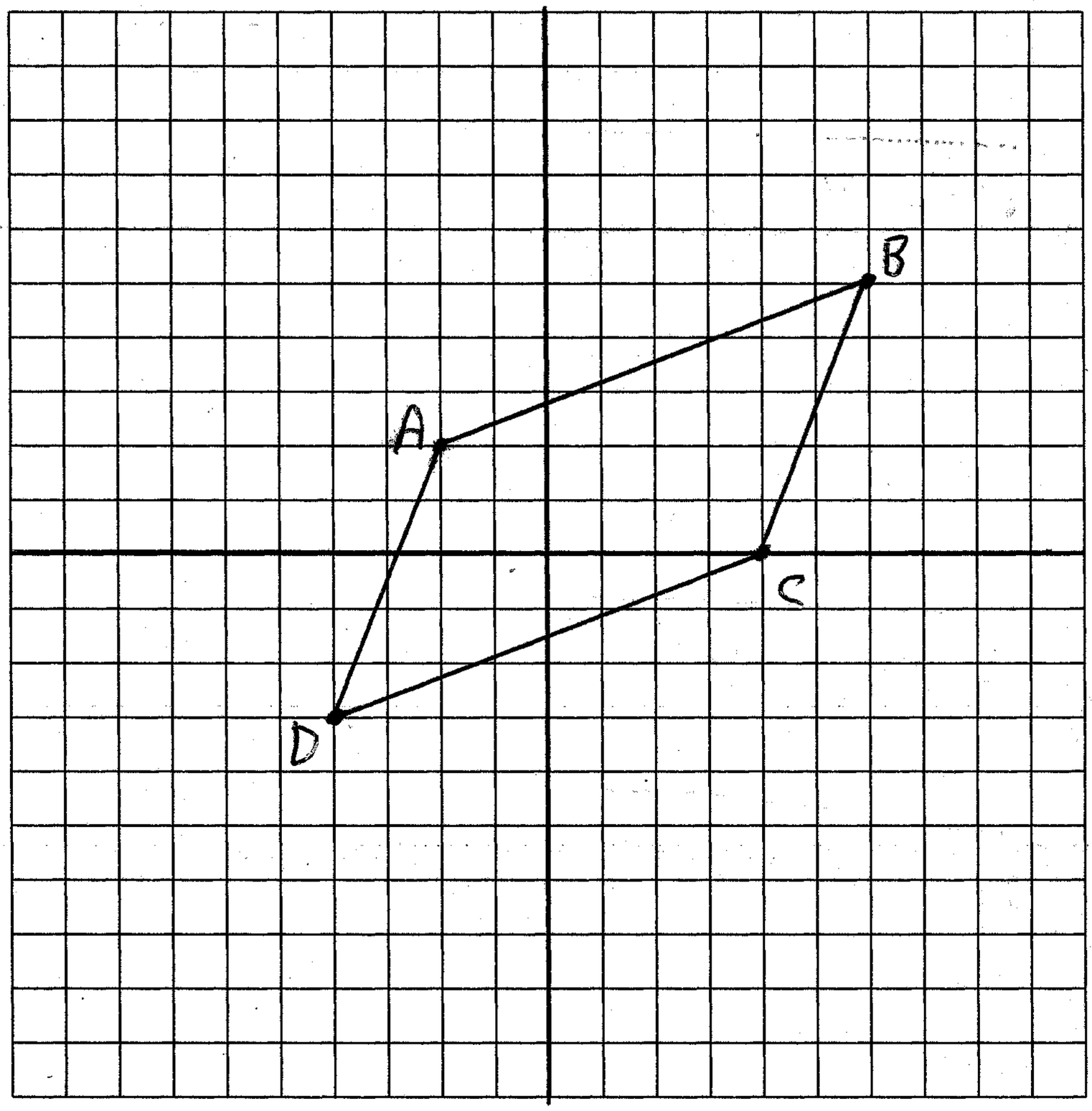
Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

33 Given: $A(-2,2)$, $B(6,5)$, $C(4,0)$, $D(-4,-3)$

Prove: $ABCD$ is a parallelogram but not a rectangle. [The use of the grid on the next page is optional.]

Statement	Reason
① Quadrilateral ABCD with A(-2,2), B(6,5), C(4,0), D(-4,-3)	① Given
② Slope of $\overline{AB}$ $\frac{5-2}{6--2}=\frac{3}{8}$ Slope of $\overline{BC}$ $\frac{5-0}{6-4}=\frac{5}{2}$ slope of $\overline{CD}$ $\frac{0--3}{4--4}=\frac{3}{8}$ slope of $\overline{AD}$ $\frac{2--3}{-2--4}=\frac{5}{2}$	② Definition of slope
③ $\overline{AB}$ is parallel to $\overline{CD}$ $\overline{AD}$ is parallel to $\overline{BC}$	③ Any two lines with equal slope are parallel
④ ABCD is a parallelogram	④ If both pairs of opposite sides of a quadrilateral are congruent, the quadrilateral is a parallelogram
⑤ $\overline{AB} \not\perp \overline{BC}$ $\overline{BC} \not\perp \overline{CD}$ $\overline{CD} \not\perp \overline{AD}$ $\overline{AD} \not\perp \overline{AB}$	⑤ Lines whose slopes are not negative reciprocals are not perpendicular.
⑥ $\angle ABC$, $\angle BCD$, $\angle CDA$ & $\angle DAC$ are not right angles	⑥ Definition of perpendicular
⑦ ABCD is not a rectangle	⑦ Definition of rectangle

Question 33 continued



34 A triangular plot of land has sides that measure 5 meters, 7 meters, and 10 meters. What is the area of this plot of land, to the *nearest tenth of a square meter*?

Use Heron's Formula for area of a $\triangle$ with sides a, b, c

$\sqrt{s(s-a)(s-b)(s-c)}$ where s is semi-perimeter

Perimeter = 5+7+10 = 22

Semi-perimeter = 11

$A = \sqrt{11(11-5)(11-7)(11-10)}$

$= \sqrt{11 \cdot 6 \cdot 4}$

$= 16.2$

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS B

Thursday, June 15, 2006 — 1:15 to 4:15 p.m., only

ANSWER SHEET

Student Steve Sibol Sex: ☐ Male ☐ Female Grade .....

Teacher ..... School HSCR

Your answers to Part I should be recorded on this answer sheet.

Part I

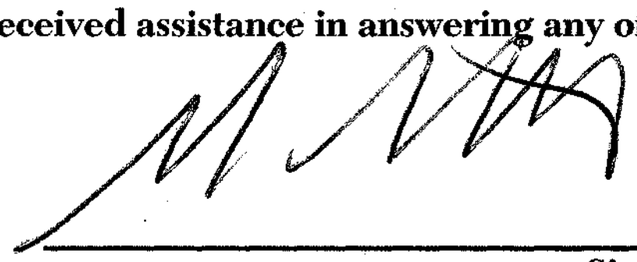
Answer all 20 questions in this part.

1 3	6 1	11 2	16 4
2 1	7 1	12 2	17 1
3 2	8 2	13 4	18 2
4 4	9 3	14 4	19 3
5 3	10 1	15 2	20 1

Your answers for Parts II, III, and IV should be written in the test booklet.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.



Signature