The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS B

Thursday, June 23, 2005 — 9:15 a.m. to 12:15 p.m., only

Print Your Name: Steve Sibol

Print Your School's Name: HSCR

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will *not* be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.

This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. The formulas that you may need to answer some questions in this examination are found on page 19.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice. . .

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

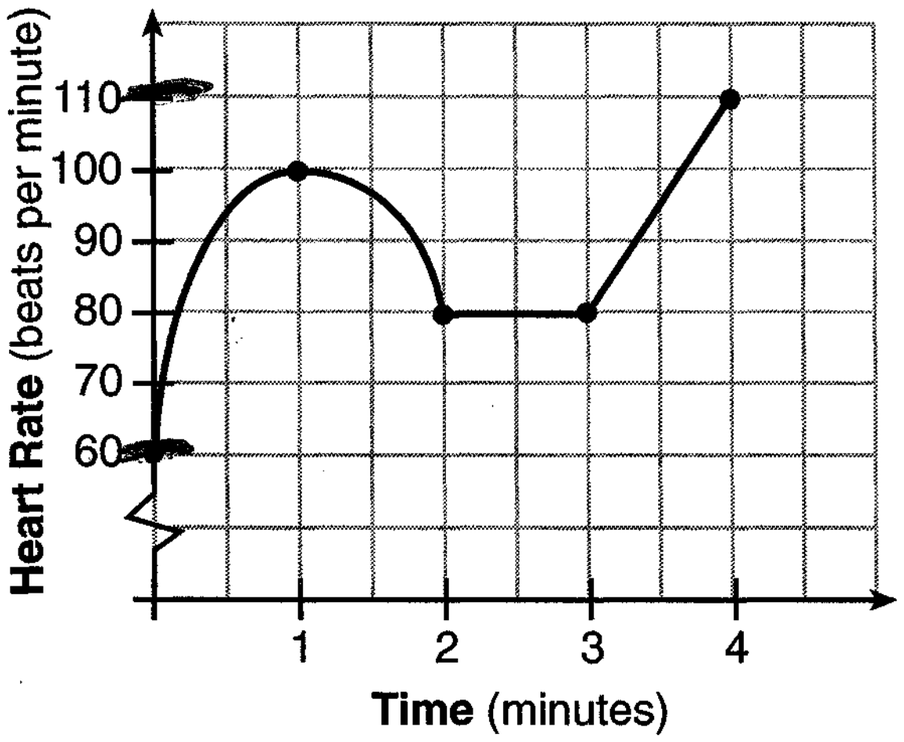
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [40]

1 The accompanying graph shows the heart rate, in beats per minute, of a jogger during a 4-minute interval.

Use this space for computations.



What is the range of the jogger's heart rate during this interval?

(1) 0–4
(2) 1–4
(3) 0–110
(4) 60–110

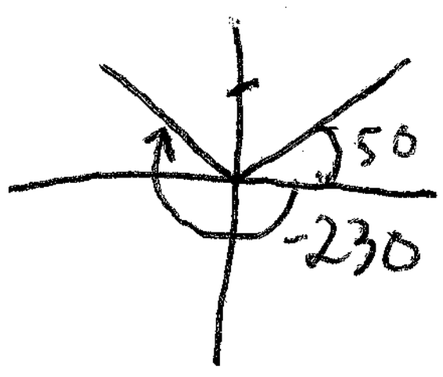
2 If sin θ is negative and cos θ is negative, in which quadrant does the terminal side of θ lie?

(1) I
(2) II
(3) III
(4) IV

(cos θ, sin θ)
(−, −)

3 Expressed as a function of a positive acute angle, sin (–230°) is equal to

(1) sin 50°
(2) –sin 50°
(3) cos 50°
(4) –cos 50°



[2]

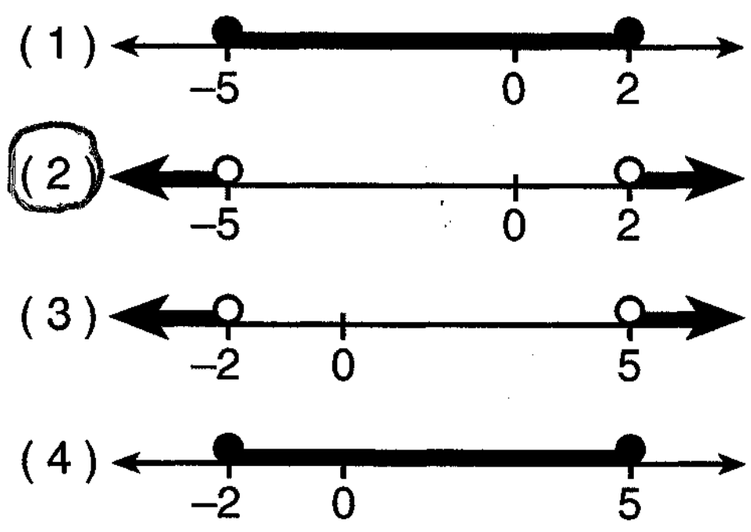
Use this space for computations.

4 Written in simplest form, the expression $\frac{x^2 - 9x}{45x - 5x^2}$ is equivalent to

(1) $\frac{1}{5}$
(2) $-\frac{1}{5}$
(3) 5
(4) –5

$\frac{-x(x-9)}{5x(9-x)} = -\frac{1}{5}$

5 Which graph represents the solution set for the expression $|2x + 3| > 7$?



$2x+3>7$ $2x+3<-7$

$2x>4$ $\frac{2x}{2} < \frac{-10}{2}$

$x>2$ $x<-5$

6 What are the coordinates of the center of the circle represented by the equation $(x + 3)^2 + (y - 4)^2 = 25$?

(1) (3,4)
(2) (3,–4)
(3) (–3,4)
(4) (–3,–4)

7 What is the mean of the data in the accompanying table?

Scores (x_i)	Frequency (f_i)
25	3
20	2
11	5
10	4

$\frac{(25\times3)+(20\times2)+(11\times5)+(16\times4)}{14}$

(1) 11
(2) 14.5
(3) 15
(4) 16

8 In a given rectangle, the length varies inversely as the width. If the length is doubled, the width will

(1) be divided by 2
(2) remain the same
(3) be multiplied by 2
(4) increase by 2

Use this space for computations.

$2l \cdot \frac{w}{2} = lw$

9 Impedance measures the opposition of an electrical circuit to the flow of electricity. The total impedance in a particular circuit is given by the formula $Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2}$. What is the total impedance of a circuit, Z_T, if $Z_1 = 1 + 2i$ and $Z_2 = 1 - 2i$?

(1) 1
(2) 0
(3) $\frac{5}{2}$
(4) $-\frac{3}{2}$

$\frac{(1+2i)(1-2i)}{1+2i+1-2i} = \frac{1-4i^2}{2} = \frac{1-4(-1)}{2}$

$\frac{5}{2}$

10 If $\log a = x$ and $\log b = y$, what is $\log a\sqrt{b}$?

(1) $x + 2y$
(2) $2x + 2y$
(3) $\frac{x + y}{2}$
(4) $x + \frac{y}{2}$

$\log a + \log \sqrt{b}$
$\log a + \log b^{1/2}$
$\log a + \frac{1}{2}\log b$
$x + \frac{y}{2}$

11 Which relation is a function?

(1) $xy = 7$
(2) $x = 7$
(3) $x^2 - y^2 = 7$ hyperbola
(4) $x^2 + y^2 = 7$ circle

12 Which equation, when graphed on a Cartesian coordinate plane, would best represent an elliptical racetrack?

(1) $3x^2 + 10y^2 = 288{,}000$
(2) $3x^2 - 10y^2 = 288{,}000$
(3) $3x + 10y = 288{,}000$ linear
(4) $30xy = 288{,}000$

13 The expression $\frac{2 + i}{3 + i}$ is equivalent to

(1) $\frac{6 + 5i}{8}$

(2) $\frac{6 + i}{8}$

(3) $\frac{7 - 5i}{10}$

(4) $\frac{7 + i}{10}$

Use this space for computations.

$\frac{2+i}{3+i}\left(\frac{3-i}{3-i}\right) = \frac{6-2i+3i-i^2}{9-i^2}$

$\frac{6+i-(-1)}{9-(-1)} = \frac{7+i}{10}$

14 For which quadratic equation is the axis of symmetry $x = 3$?

(1) $y = -x^2 + 3x + 5$

(2) $y = -x^2 + 6x + 2$

(3) $y = x^2 + 6x + 3$

(4) $y = x^2 + x + 3$

$x = \frac{-b}{2a} = \frac{-6}{2(-1)} = 3$

15 A crate weighing w pounds sits on a ramp positioned at an angle of θ with the horizontal. The forces acting on this crate are modeled by the equation $Mw \cos \theta = w \sin \theta$, where M is the coefficient of friction. What is an expression for M in terms of θ?

(1) $M = \tan \theta$

(2) $M = \cot \theta$

(3) $M = \sec \theta$

(4) $M = \csc \theta$

$\frac{Mw\cos\theta}{\cos\theta} = \frac{w\sin\theta}{\cos\theta}$

$M = \frac{\sin\theta}{\cos\theta} = \tan\theta$

16 If $(a^x)^{\frac{2}{3}} = \frac{1}{a^2}$, what is the value of x?

(1) 1

(2) 2

(3) –3

(4) –1

$a^{\frac{2x}{3}} = a^{-2}$

$\frac{2x}{3} = -2$

$\frac{2x}{2} = \frac{-6}{2} = -3$

17 What is the third term in the expansion of $(\cos x + 3)^5$?

(1) $90 \cos^2 x$

(2) $270 \cos^2 x$

(3) $60 \cos^3 x$

(4) $90 \cos^3 x$

$n = 5$

$r - 1 = 2$

${}_5C_2 (\cos x)^{5-2} (3)^2 = 90 \cos^3 x$

18 Which equation has imaginary roots?

(1) $x(5 + x) = 8$
(2) $x(5 - x) = -3$
(3) $x(x + 6) = -10$
(4) $(2x + 1)(x - 3) = 7$

$x^2+6x+10=0$

$a=1 \quad b=6 \quad c=10$

Use this space for computations.

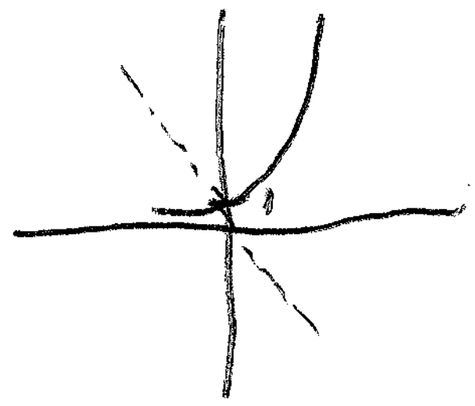
b^2-4ac

$6^2-4(1)(10)$

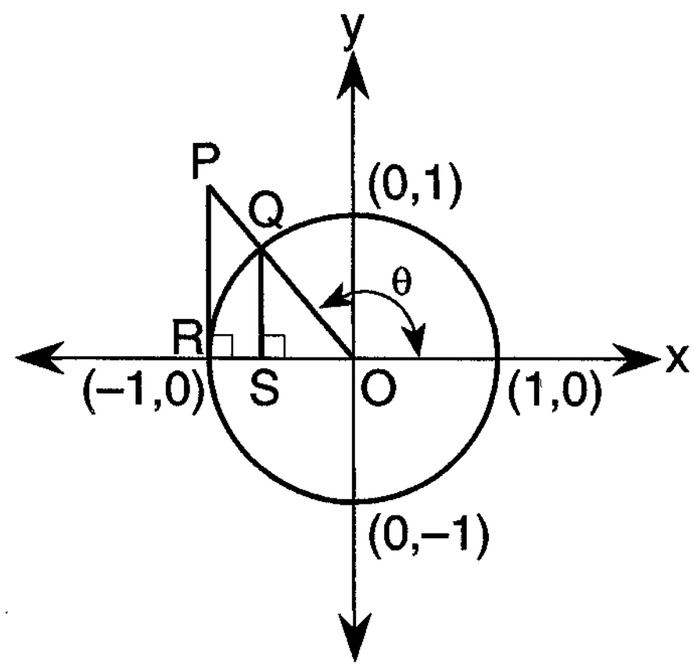
$36-40=-4$

19 The graphs of the equations $y = 2^x$ and $y = -2x + a$ intersect in Quadrant I for which values of a?

(1) $0 < a < 1$
(2) $a < 1$
(3) $a \geq 1$
(4) $a > 1$



20 In the accompanying diagram, $\overline{PR}$ is tangent to circle O at R, $\overline{QS} \perp \overline{OR}$, and $\overline{PR} \perp \overline{OR}$.



Which measure represents sin θ?

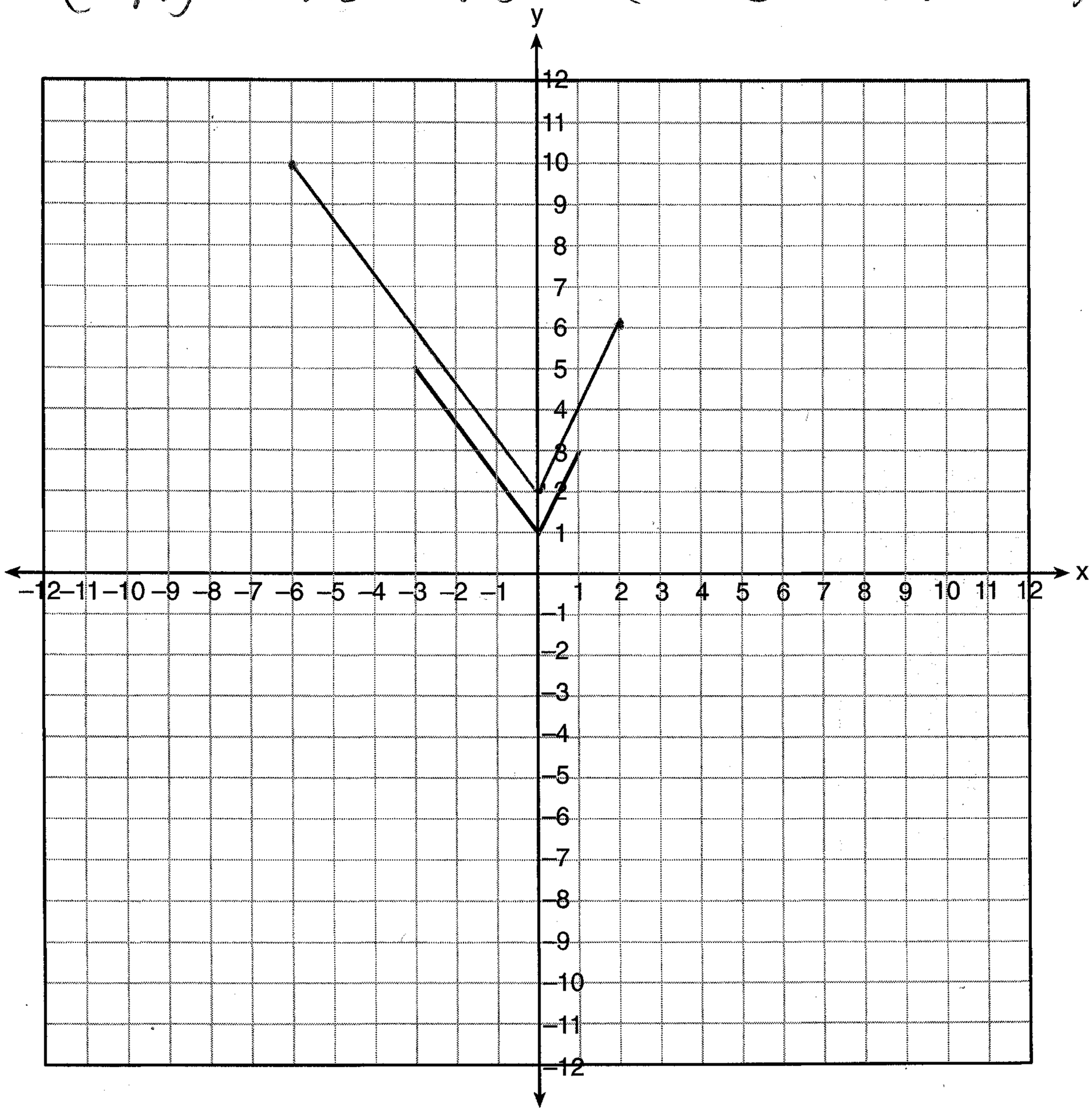
(1) SO
(2) RO
(3) PR
(4) QS

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

21 The graph of the function g(x) is shown on the accompanying set of axes. On the same set of axes, sketch the image of g(x) under the transformation D_2.

(−3, 5) (0, 1) (1, 3) (−6, 10) (0, 2) (2, 6)



22 Solve for m: $3^{m+1} - 5 = 22$

$+5 \quad +5$

$3^{m+1} = 27$

$3^{m+1} = 3^3$

$m = 2$

23 Evaluate: $\sum_{k=0}^{3}(3\cos k\pi + 1)$

$\sum_{k=0}^{3}(3\cos k\pi + 1)$

k	$3\cos k\pi + 1$
0	$3\cos 0 + 1 = 4$
1	$3\cos \pi + 1 = -2$
2	$3\cos 2\pi + 1 = 4$
3	$3\cos 3\pi + 1 = -2$
	4

24 Express in simplest form: $\frac{1}{x} + \frac{1}{x+3}$

$$\frac{x+3+x}{x(x+3)} = \frac{2x+3}{x^2+3x}$$

25 A landscape architect is designing a triangular garden to fit in the corner of a lot. The corner of the lot forms an angle of 70°, and the sides of the garden including this angle are to be 11 feet and 13 feet, respectively. Find, to the *nearest integer*, the number of square feet in the area of the garden.

$$\text{Area} = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2}(11)(13)\sin 70$$

$$\approx 67$$

26 A certain drug raises a patient's heart rate, $h(x)$, in beats per minute, according to the function $h(x) = 70 + 0.2x$, where x is the bloodstream drug level, in milligrams. The level of the drug in the patient's bloodstream is a function of time, t, in hours, according to the formula $g(t) = 300(0.8)^t$. Find the value of $h(g(4))$, the patient's heart rate in beats per minute, to the *nearest whole number*.

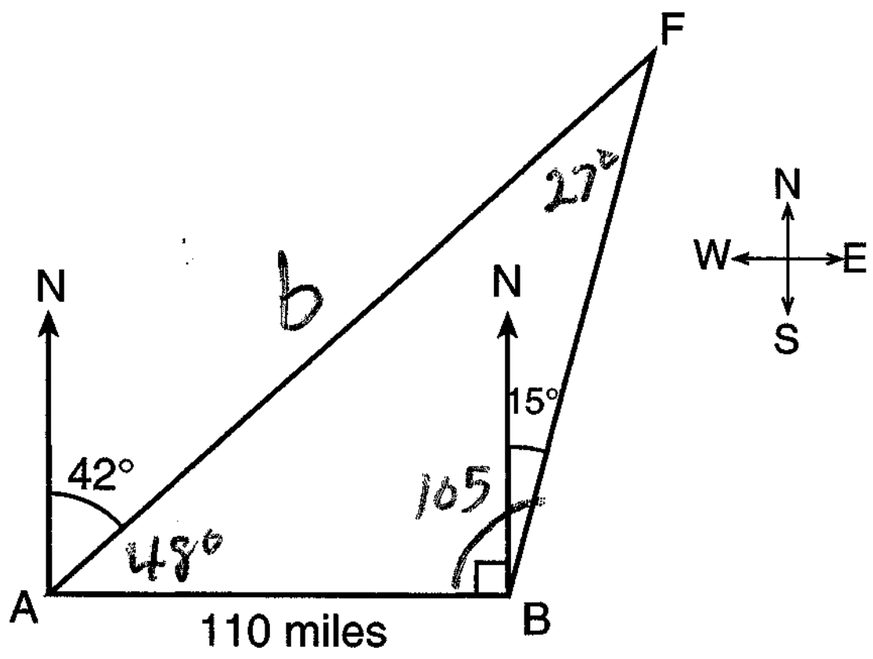
$g(4) = 300(0.8)^4 = 122.88$

$h(122.88) = 70 + 0.2(122.88) \approx 95$

Part III

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [24]

27 As shown in the accompanying diagram, two tracking stations, A and B, are on an east-west line 110 miles apart. A forest fire is located at F, on a bearing 42° northeast of station A and 15° northeast of station B. How far, to the *nearest mile*, is the fire from station A?



$$\frac{b}{\sin 105} = \frac{110}{\sin 27}$$

$$b = \frac{110 \sin 105}{\sin 27}$$

$$\approx 234$$

28 Solve for all values of q that satisfy the equation $\sqrt{3q + 7} = q + 3$.

$3q+7 = (q+3)^2$

$3q+7 = q^2+6q+9$

$q^2+3q+2 = 0$

$(q+2)(q+1) = 0$

$q = -2 \quad q = -1$

29 The probability that a planted watermelon seed will sprout is $\frac{3}{4}$. If Peyton plants seven seeds from a slice of watermelon, find, to the *nearest ten thousandth,* the probability that *at least* five will sprout.

$n = 7$

$r = 5, 6, 7$

$p = \frac{3}{4}$

$q = \frac{1}{4}$

$P(5) = {}_7C_5\left(\frac{3}{4}\right)^5\left(\frac{1}{4}\right)^2 = \frac{5103}{16384}$

$P(6) = {}_7C_6\left(\frac{3}{4}\right)^6\left(\frac{1}{4}\right)^1 = \frac{5103}{16384}$

$P(7) = {}_7C_7\left(\frac{3}{4}\right)^7\left(\frac{1}{4}\right)^0 = \frac{2187}{16384}$

$\frac{12,393}{16,384} \approx .7564$

30 Find, to the *nearest degree*, all values of θ in the interval $0° \le \theta < 360°$ that satisfy the equation $3\cos 2\theta + \sin\theta - 1 = 0$.

$3(1-2\sin^2\theta)+\sin\theta-1=0$

$3-6\sin^2\theta+\sin\theta-1=0$

$-6\sin^2\theta+\sin\theta+2=0$

$6\sin^2\theta-\sin\theta-2=0$

$(3\sin\theta-2)(2\sin\theta+1)=0$

$3\sin\theta-2=0$

$\frac{3\sin\theta}{3}=\frac{2}{3}$

$\sin\theta=\frac{2}{3}$

$\theta=\sin^{-1}\left(\frac{2}{3}\right)$

$\theta\approx 42°, 138°$

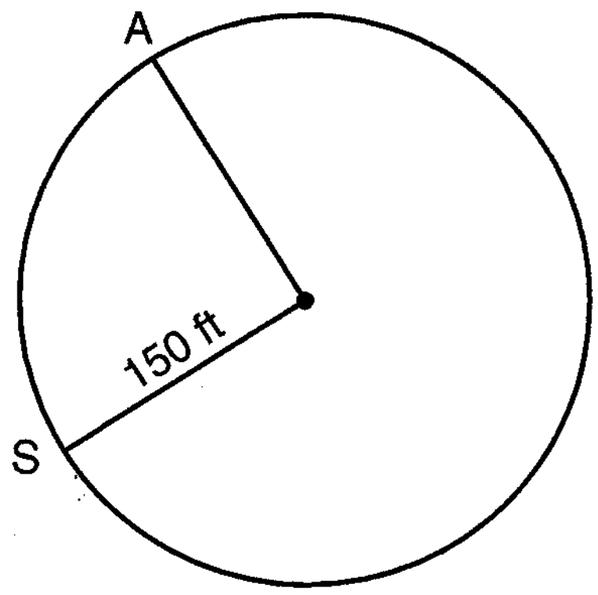
$2\sin\theta+1=0$

$\frac{2\sin\theta}{2}=\frac{-1}{2}$

$\sin\theta=-\frac{1}{2}$

$\theta=210°, 330°$

31 Kathy and Tami are at point A on a circular track that has a radius of 150 feet, as shown in the accompanying diagram. They run counterclockwise along the track from A to S, a distance of 247 feet. Find, to the *nearest degree,* the measure of minor arc AS.



$\theta = \frac{s}{r}$

$\theta = \frac{247}{150}$

$\approx 1.64\overline{6}$ radians

$1.64\overline{6} \cdot \frac{180}{\pi} \approx 94^\circ$

32 The height of a projectile is modeled by the equation $y = -2x^2 + 38x + 10$, where x is time, in seconds, and y is height, in feet. During what interval of time, to the *nearest tenth of a second,* is the projectile *at least* 125 feet above ground? [The use of the accompanying grid is optional.]

$-2x^2+38x+10 \geq 125$

$-2x^2+38x-115 \geq 0$

$a=-2$

$b=38$

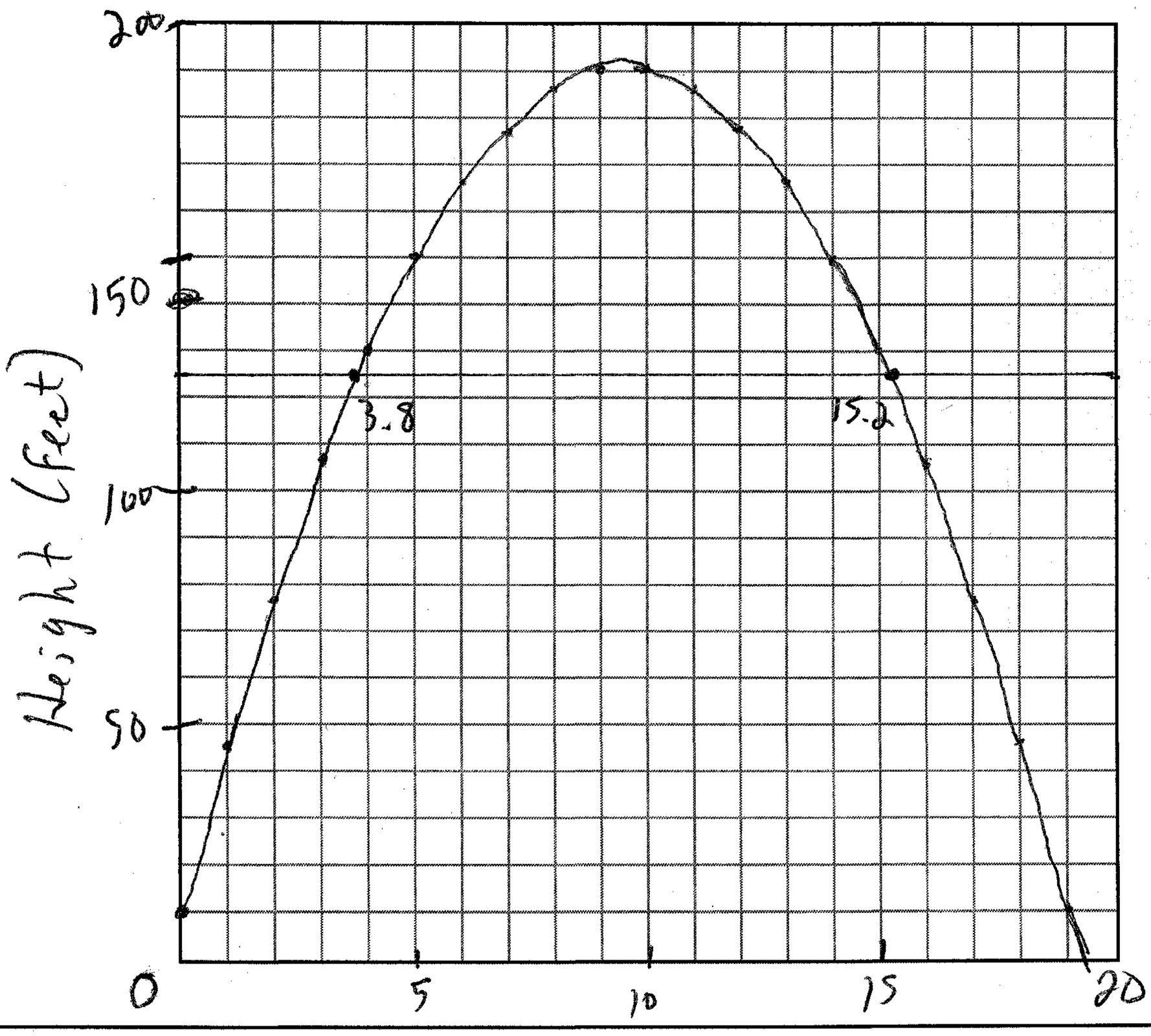
$c=-115$

$$\frac{-38 \pm \sqrt{38^2-4(-2)(-115)}}{2(-2)}$$

$$\frac{-38 \pm 22.89}{-4}$$

$$\frac{-38+22.89}{-4} \qquad \frac{-38-22.89}{-4}$$

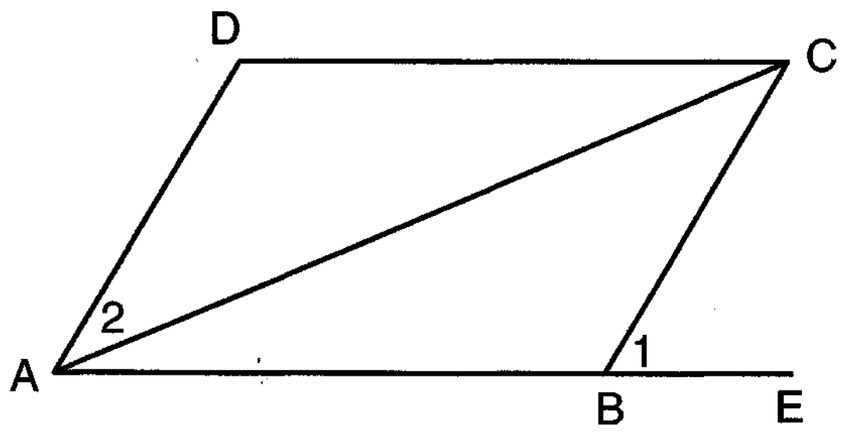
$3.8 \leq x \leq 15.2$



Part IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

33 Given: parallelogram $ABCD$, diagonal $\overline{AC}$, and $\overline{ABE}$



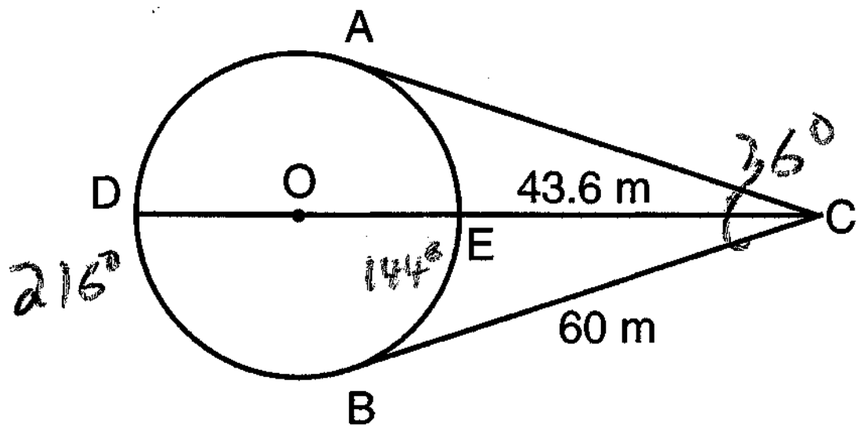
Prove: $m\angle 1 > m\angle 2$

	STATEMENT	REASON
①	Parallelogram ABCD, diagonal $\overline{AC}$, and $\overline{ABE}$	① Given
②	$\overline{AD} \parallel \overline{BC}$	② Opposite sides of a parallelogram are parallel.
③	$\angle BAD \cong \angle 1$	③ Corresponding angles formed by parallel lines and a transversal are congruent.
④	$m\angle BAD > m\angle 2$	④ Inspection
⑤	If $m\angle BAD > m\angle 2$ then $m\angle 1 > m\angle 2$	⑤ Substitution

34 An architect is designing a park with an entrance represented by point C and a circular garden with center O, as shown in the accompanying diagram. The architect plans to connect three points on the circumference of the garden, A, B, and D, to the park entrance, C, with walkways so that walkways $\overline{CA}$ and $\overline{CB}$ are tangent to the garden, walkway $\overline{DOEC}$ is a path through the center of the garden, $m\overset{\frown}{ADB}:m\overset{\frown}{AEB} = 3:2$, $BC = 60$ meters, and $EC = 43.6$ meters.

Find the measure of the angle between walkways $\overline{CA}$ and $\overline{CB}$.

Find the diameter of the circular garden, to the *nearest meter*.



$\frac{3}{5} \times 360 = 216°$ $\overset{\frown}{ADB}$

$\frac{2}{5} \times 360 = 144°$ $\overset{\frown}{AEB}$

$\frac{216-144}{2} = 36°$ $\angle C$

$m\overline{DOE} = d$

$60^2 = 43.6(43.6+d)$

$3600 = 1900.96 + 43.6d$

$d \approx 39$