The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS B

Tuesday, June 22, 2004 — 1:15 to 4:15 p.m., only

Print Your Name: Steve Sibol

Print Your School's Name: HSCR

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will *not* be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.

This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. The formulas that you may need to answer some questions in this examination are found on page 19.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice. . .

A graphing calculator, a straightedge (ruler), and a compass must be available for your use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [40]

2-2i -3+4i

1 What is the sum of $2 - \sqrt{-4}$ and $-3 + \sqrt{-16}$ expressed in $a + bi$ form?

(1) $-1 + 2i$
(2) $-1 + i\sqrt{20}$
(3) $-1 + 12i$
(4) $-14 + i$

Use this space for computations.

2 The Hiking Club plans to go camping in a State park where the probability of rain on any given day is 0.7. Which expression can be used to find the probability that it will rain on *exactly* three of the seven days they are there?

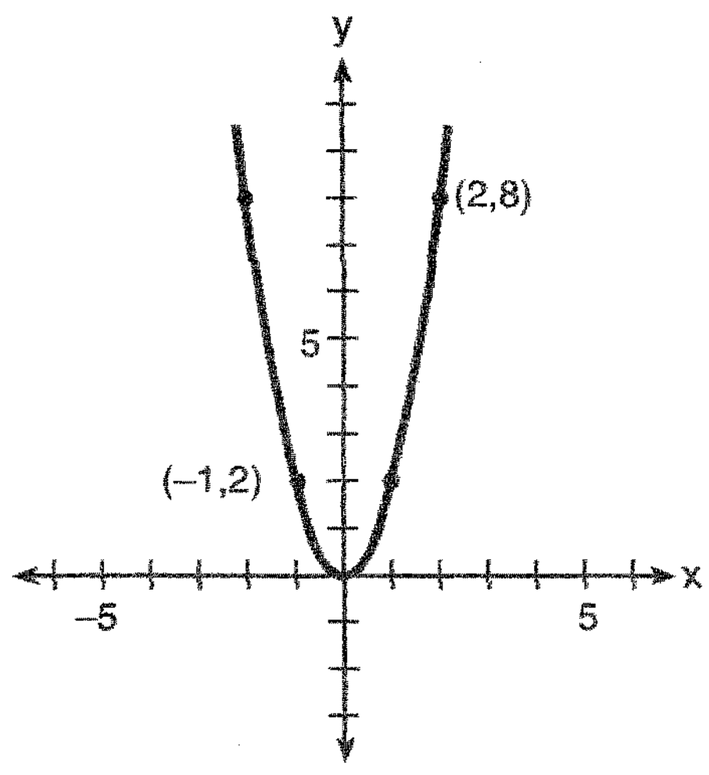
(1) ${}_7C_3(0.7)^3(0.3)^4$
(2) ${}_7C_3(0.3)^3(0.7)^4$
(3) ${}_4C_3(0.7)^3(0.7)^4$
(4) ${}_4C_3(0.4)^4(0.3)^3$

n=7
r=3
p=.7
q=.3

3 What is the amplitude of the function $y = \frac{2}{3}\sin 4x$?

(1) $\frac{\pi}{2}$
(2) $\frac{2}{3}$
(3) 3π
(4) 4

4 Which quadratic function is shown in the accompanying graph?

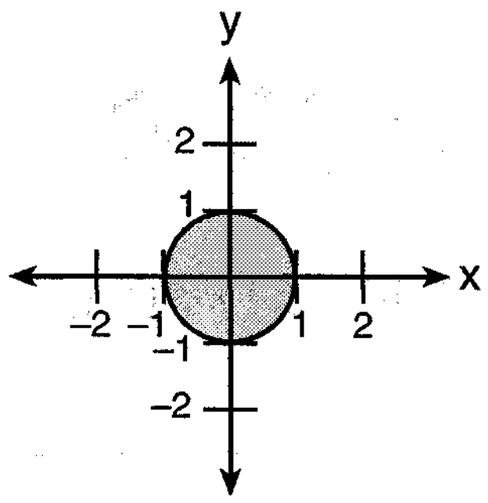


(1) $y = -2x^2$
(2) $y = 2x^2$
(3) $y = -\frac{1}{2}x^2$
(4) $y = \frac{1}{2}x^2$

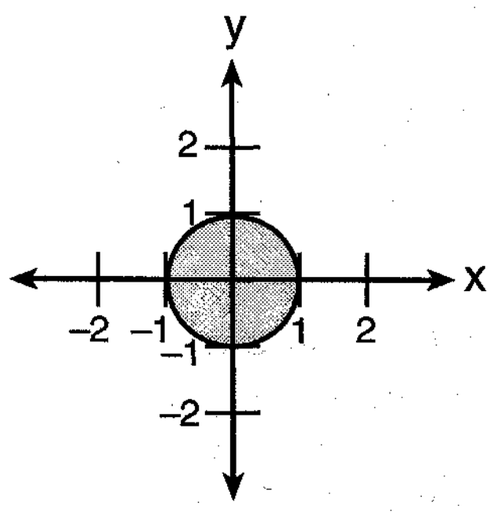
[2]

5 In the accompanying graph, the shaded region represents set A of all points (x,y) such that $x^2 + y^2 \leq 1$. The transformation T maps point (x,y) to point $(2x,4y)$.

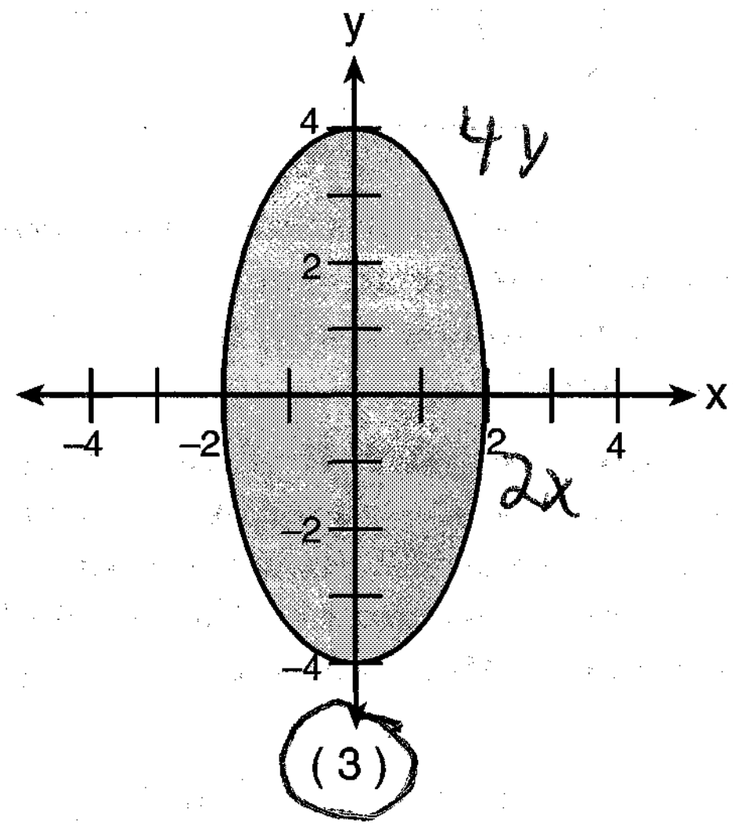
Use this space for computations.



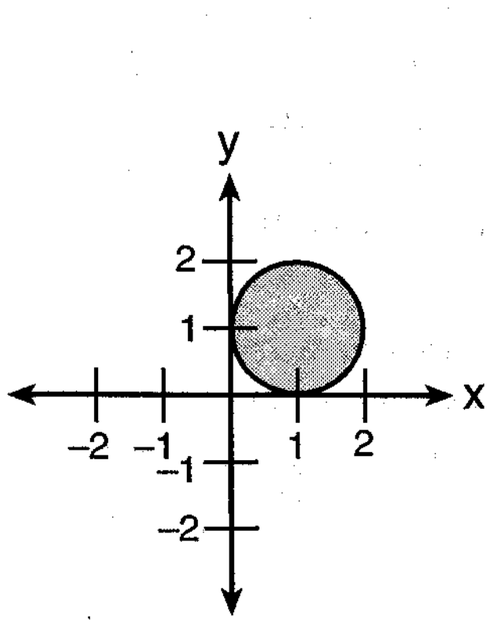
Which graph shows the mapping of set A by the transformation T?



(1)



(3)



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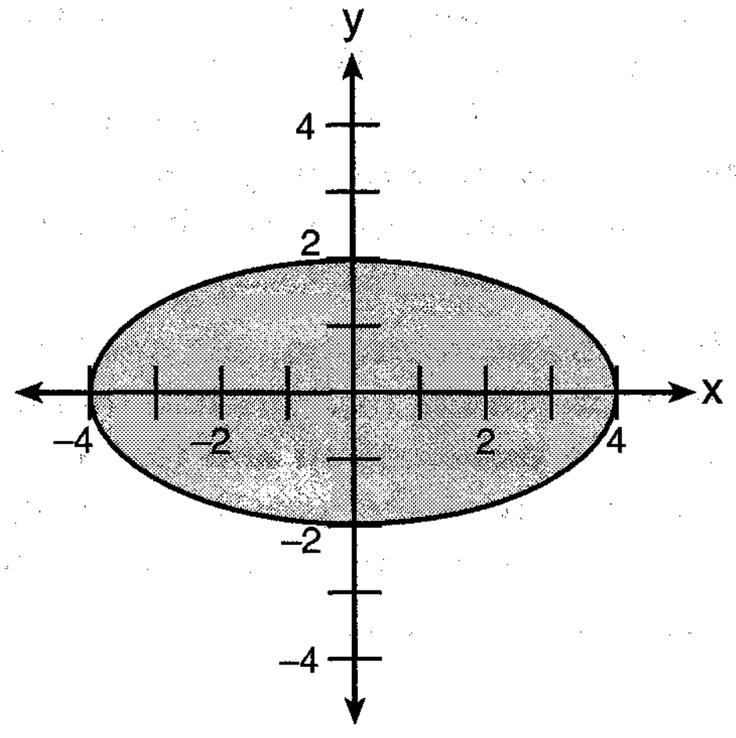


(4)

$4 \cdot 4^0 + (4 \cdot 4)^{-1}$

6 If $f(x) = 4x^0 + (4x)^{-1}$, what is the value of f(4)?

(1) –12
(2) 0
(3) $1\frac{1}{16}$
(4) $4\frac{1}{16}$

$4 \cdot 1 + 16^{-1}$

$4 + \frac{1}{16}$

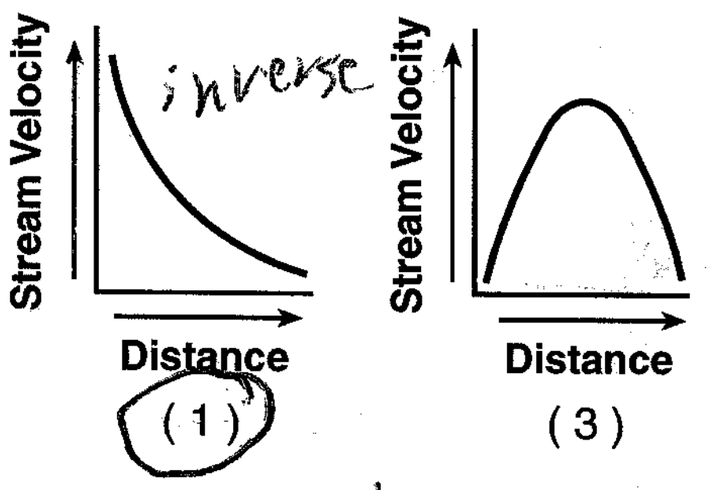
Use this space for computations.

7 What is the domain of the function $f(x) = \frac{2x^2}{x^2 - 9}$?

(1) all real numbers except 0
(2) all real numbers except 3
(3) all real numbers except 3 and –3
(4) all real numbers

$x^2 - 9 = 0$
$(x+3)(x-3) = 0$
$x = -3 \quad x = 3$

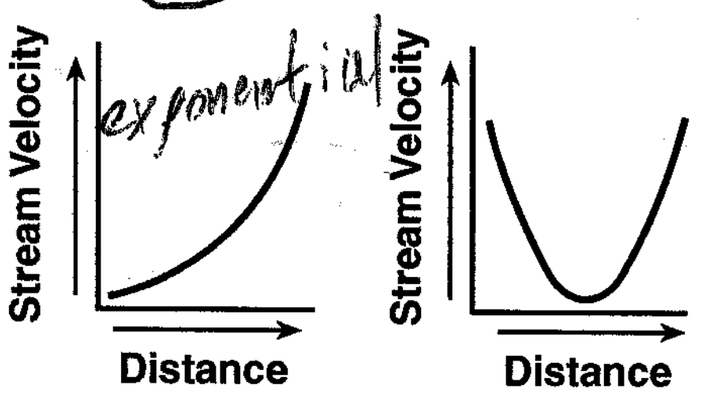
8 Which graph represents an inverse variation between stream velocity and the distance from the center of the stream?



(1)

(3)

quadratic



(2)

(4)

quadratic

9 If $\log_b x = y$, then x equals

(1) $y \cdot b$
(2) $\frac{y}{b}$
(3) y^b
(4) b^y

$\frac{10}{4} = 2 \text{ R}2$

10 The expression $i^0 \cdot i^1 \cdot i^2 \cdot i^3 \cdot i^4$ is equal to

(1) 1
(2) –1
(3) i
(4) $-i$

$= i^{0+1+2+3+4} = i^{10} = i^2 = -1$

11 Which equation models the data in the accompanying table?

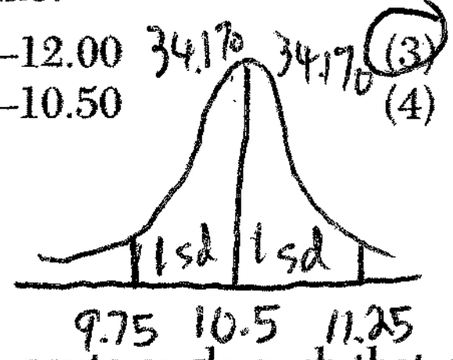
Use this space for computations.

Time in hours, x	0	1	2	3	4	5	6
Population, y	5	10	20	40	80	160	320

(1) $y = 2x + 5$
(2) $y = 2^x$
(3) $y = 2x$
(4) $y = 5(2^x)$

12 The amount of juice dispensed from a machine is normally distributed with a mean of 10.50 ounces and a standard deviation of 0.75 ounce. Which interval represents the amount of juice dispensed about 68.2% of the time?

(1) 9.00–12.00
(2) 9.75–10.50
(3) 9.75–11.25
(4) 10.50–11.25



68.2% is within one standard deviation of the mean.

13 If θ is an acute angle such that $\sin\theta = \frac{5}{13}$, what is the value of $\sin 2\theta$?

(1) $\frac{12}{13}$
(2) $\frac{10}{26}$
(3) $\frac{60}{169}$
(4) $\frac{120}{169}$

$\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(\frac{5}{13}\right)\left(\frac{12}{13}\right)$
$= \frac{120}{169}$

If $\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$ because $\sin^2\theta + \cos^2\theta = 1$

14 Which function is symmetrical with respect to the origin?

(1) $y = \sqrt{x+5}$
(2) $y = |5 - x|$
(3) $y = -\frac{5}{x}$
(4) $y = 5^x$

15 The expression $\dfrac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{x^2} - \frac{1}{y^2}}$ is equivalent to

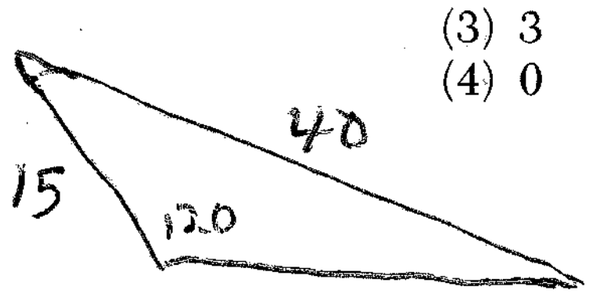
(1) $\frac{xy}{y-x}$
(2) $\frac{xy}{x-y}$
(3) $\frac{y-x}{xy}$
(4) $y - x$

$\frac{x+y}{xy} \div \frac{y^2 - x^2}{x^2y^2}$

$\frac{x+y}{xy} \times \frac{x^2y^2}{(y-x)(y+x)} = \frac{xy}{y-x}$

16 Sam is designing a triangular piece for a metal sculpture. He tells Martha that two of the sides of the piece are 40 inches and 15 inches, and the angle opposite the 40-inch side measures 120°. Martha decides to sketch the piece that Sam described. How many different triangles can she sketch that match Sam's description?

(1) 1
(2) 2
(3) 3
(4) 0



Use this space for computations.

$C \approx 161°$

$161 + 120 \not< 180$

NOT POSSIBLE

$\frac{40}{\sin 120} = \frac{15}{\sin C}$

$C \approx 19°$ $19 + 120 < 180$

17 If $f(x) = x + 1$ and $g(x) = x^2 - 1$, the expression $(g \circ f)(x)$ equals 0 when x is equal to

(1) 1 and –1
(2) 0, only
(3) –2, only
(4) 0 and –2

$g(f(x)) = (x+1)^2 - 1 = 0$

$x^2 + 2x + 1 - 1 = 0$

$x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0 \quad x = -2$

18 If θ is a positive acute angle and $\sin \theta = a$, which expression represents $\cos \theta$ in terms of a?

(1) $\sqrt{a}$
(2) $\sqrt{1 - a^2}$
(3) $\frac{1}{\sqrt{a}}$
(4) $\frac{1}{\sqrt{1 - a^2}}$

$\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos\theta = \sqrt{1 - \sin^2\theta}$

$\cos\theta = \sqrt{1 - a^2}$

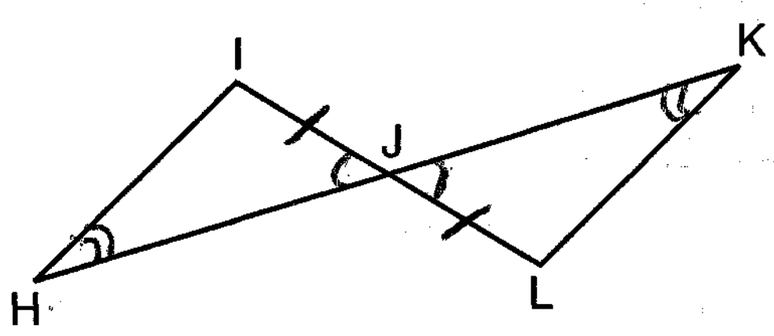
19 The expression $\sqrt[4]{16a^6b^4}$ is equivalent to

(1) $2a^2b$
(2) $2a^{\frac{3}{2}}b$
(3) $4a^2b$
(4) $4a^{\frac{3}{2}}b$

$\sqrt[4]{16a^6b^4} = (16a^6b^4)^{1/4} = 16^{1/4} a^{\frac{6}{4}} b^{\frac{4}{4}}$

$2a^{3/2}b$

20 In the accompanying diagram, $\overline{HK}$ bisects $\overline{IL}$ and $\angle H \cong \angle K$.



What is the most direct method of proof that could be used to prove $\triangle HIJ \cong \triangle KLJ$?

(1) HL ≅ HL
(2) SAS ≅ SAS
(3) AAS ≅ AAS
(4) ASA ≅ ASA

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

21 The projected total annual profits, in dollars, for the Nutyme Clothing Company from 2002 to 2004 can be approximated by the model $\sum_{n=0}^{2} (13{,}567n + 294)$, where n is the year and $n = 0$ represents 2002. Use this model to find the company's projected total annual profits, in dollars, for the period 2002 to 2004.

n	$13567n+294$
0	294
1	13861
2	27428
	$41,583

22 Solve algebraically for x: $27^{2x+1} = 9^{4x}$

$$27^{2x+1} = 9^{4x}$$

$$(3^3)^{2x+1} = (3^2)^{4x}$$

$$3^{6x+3} = 3^{8x}$$

$$8x = 6x+3$$
$$-6x \quad -6x$$
$$\frac{2x}{2} = \frac{3}{2}$$

23 Find all values of k such that the equation $3x^2 - 2x + k = 0$ has imaginary roots.

$a = 3$

$b = -2$

$c = k$

$$b^2 - 4ac < 0$$

$$(-2)^2 - 4(3)(k) < 0$$

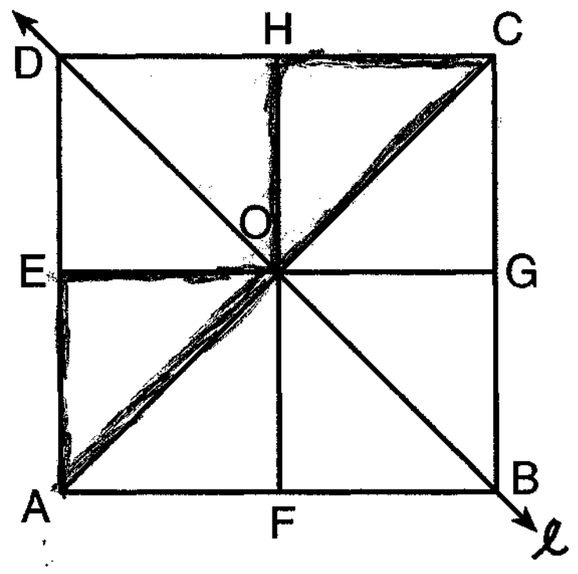
$$4 - 12k < 0$$

$$\frac{4}{12} < \frac{12k}{12}$$

$$\frac{1}{3} < k$$

$$k > \frac{1}{3}$$

24 In the accompanying diagram of square $ABCD$, F is the midpoint of $\overline{AB}$, G is the midpoint of $\overline{BC}$, H is the midpoint of $\overline{CD}$, and E is the midpoint of $\overline{DA}$.

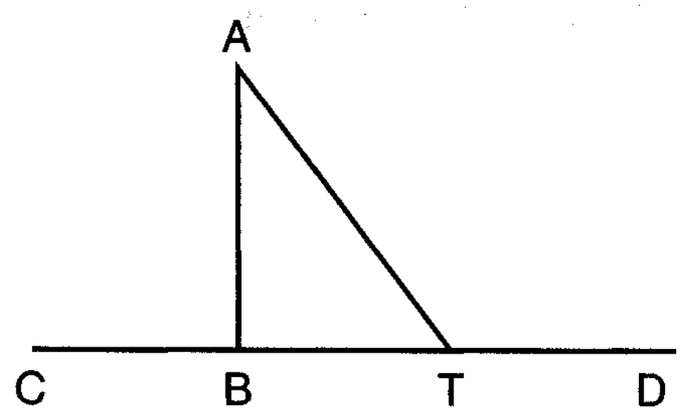


Find the image of $\triangle EOA$ after it is reflected in line ℓ. △HOC

Is this isometry direct or opposite? Explain your answer. opposite

Reflections are opposite isometries.

25 Given: $\triangle ABT$, $\overline{CBTD}$, and $\overline{AB} \perp \overline{CD}$



Write an indirect proof to show that $\overline{AT}$ is *not* perpendicular to $\overline{CD}$.

STATEMENT	REASON
(1) $\triangle ABT$, $\overline{CBTD}$, $\overline{AB} \perp \overline{CD}$	(1) Given
(2) Assume $AT \perp CD$	(2) Assumption
(3) $m\angle ATB = 90$	(3) Perpendicular lines Form 90° angles.
(4) Since $\overline{AB} \perp \overline{CD}$, $m\angle ABT = 90$	(4) Perpendicular lines Form 90° angles
(5) A triangle may not have two right angles	(5) The sum of the angles of a triangle is 180°
(6) Contradiction: Therefore $\overline{AT} \not\perp \overline{CD}$	(6) Assumption is wrong.

26 The equation $V = 20\sqrt{C + 273}$ relates speed of sound, V, in meters per second, to air temperature, C, in degrees Celsius. What is the temperature, in degrees Celsius, when the speed of sound is 320 meters per second? [The use of the accompanying grid is optional.]

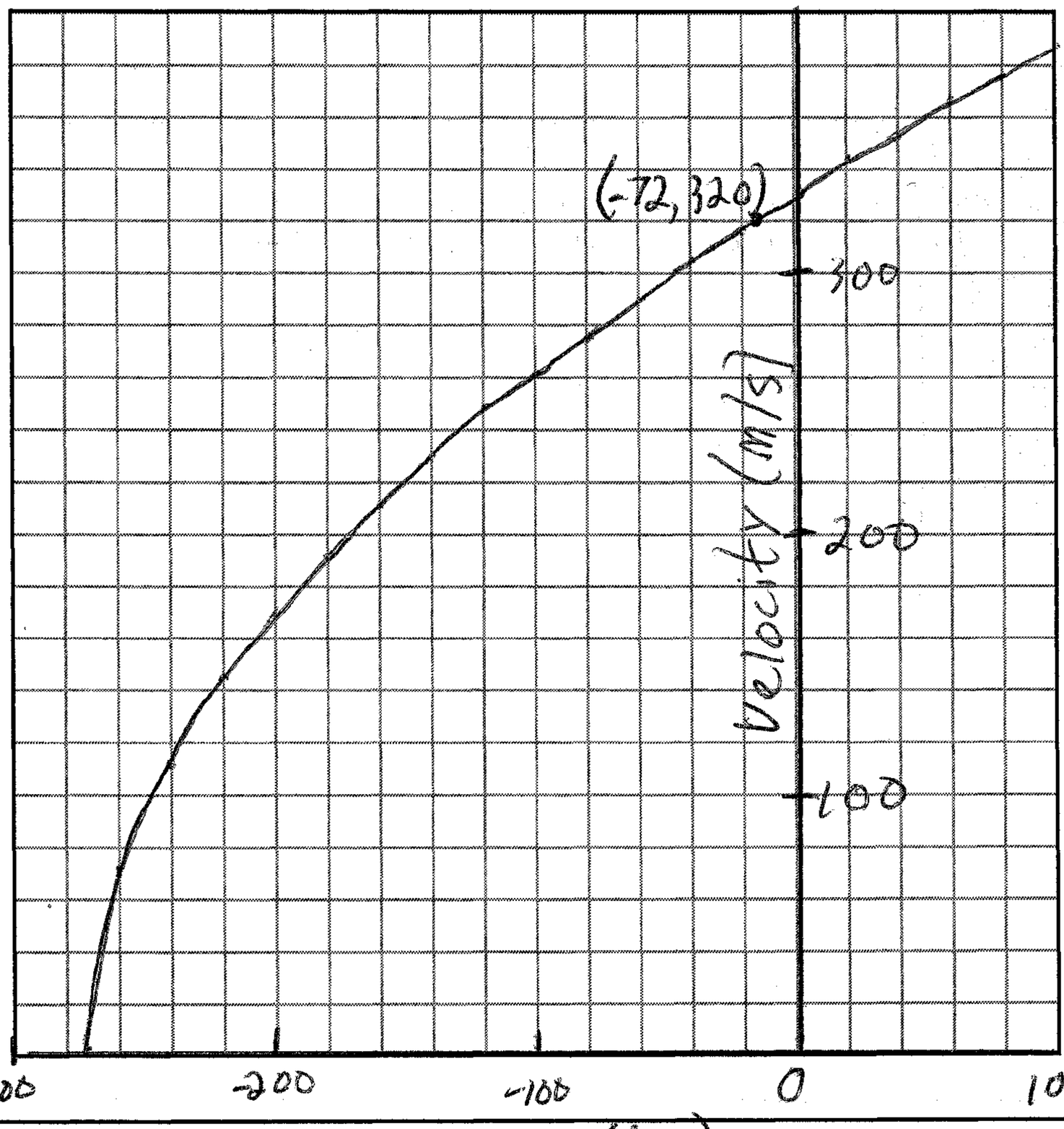
$V = 20\sqrt{C+273}$

$\frac{320}{20} = \frac{20\sqrt{C+273}}{20}$

$16 = \sqrt{C+273}$

$256 = C + 273$

$C = -17$



Part III

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [24]

27 Navigators aboard ships and airplanes use nautical miles to measure distance. The length of a nautical mile varies with latitude. The length of a nautical mile, L, in feet, on the latitude line θ is given by the formula $L = 6{,}077 - 31 \cos 2\theta$.

Find, to the *nearest degree*, the angle θ, $0° \leq \theta \leq 90°$, at which the length of a nautical mile is approximately 6,076 feet.

$6077 - 31\cos 2\theta = 6076$

$-6077 \qquad -6077$

$\frac{-31\cos 2\theta}{-31} = \frac{-1}{-31}$

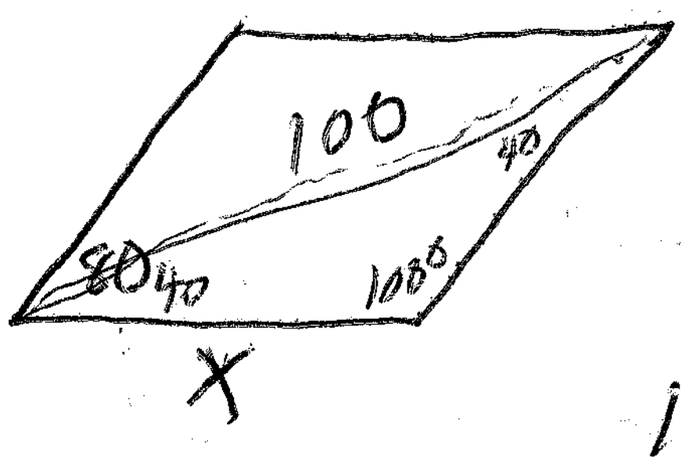
$\cos 2\theta = \frac{1}{31}$

$2\theta = \cos^{-1}\left(\frac{1}{31}\right)$

$\theta = \frac{\cos^{-1}\left(\frac{1}{31}\right)}{2}$

$\theta \approx 44$

28 Two equal forces act on a body at an angle of 80°. If the resultant force is 100 newtons, find the value of one of the two equal forces, to the *nearest hundredth of a newton*.



Because the forces are equal, the quadrilateral is a rhombus and the resultant bisects the forces.

$$\frac{100}{\sin 100} = \frac{X}{\sin 40}$$

$$X \approx 65.27$$

29 Solve for x and express your answer in simplest radical form:

$$\frac{4}{x} - \frac{3}{x+1} = 7$$

$$\frac{4(x+1)-3x}{x(x+1)} = 7$$

$$\frac{4x+4-3x}{x^2+x} = 7$$

$$x+4 = 7x^2+7x$$

$$7x^2+6x-4 = 0$$

$$a=7 \quad b=6 \quad c=-4$$

$$X = \frac{-6 \pm \sqrt{6^2-4(7)(-4)}}{2(7)}$$

$$\frac{-6 \pm \sqrt{148}}{14}$$

$$\frac{-6 \pm \sqrt{4}\sqrt{37}}{14}$$

$$\frac{-6 \pm 2\sqrt{37}}{14}$$

$$\frac{-3 \pm \sqrt{37}}{7}$$

30 A baseball player throws a ball from the outfield toward home plate. The ball's height above the ground is modeled by the equation $y = -16x^2 + 48x + 6$, where y represents height, in feet, and x represents time, in seconds. The ball is initially thrown from a height of 6 feet.

How many seconds after the ball is thrown will it again be 6 feet above the ground?

What is the maximum height, in feet, that the ball reaches? [The use of the accompanying grid is optional.]

$-16x^2+48x+6=6$

$-16x^2+48x=0$

$-16x(x-3)=0$

$-16x=0 \quad x-3=0$

$x=0 \quad x=3$

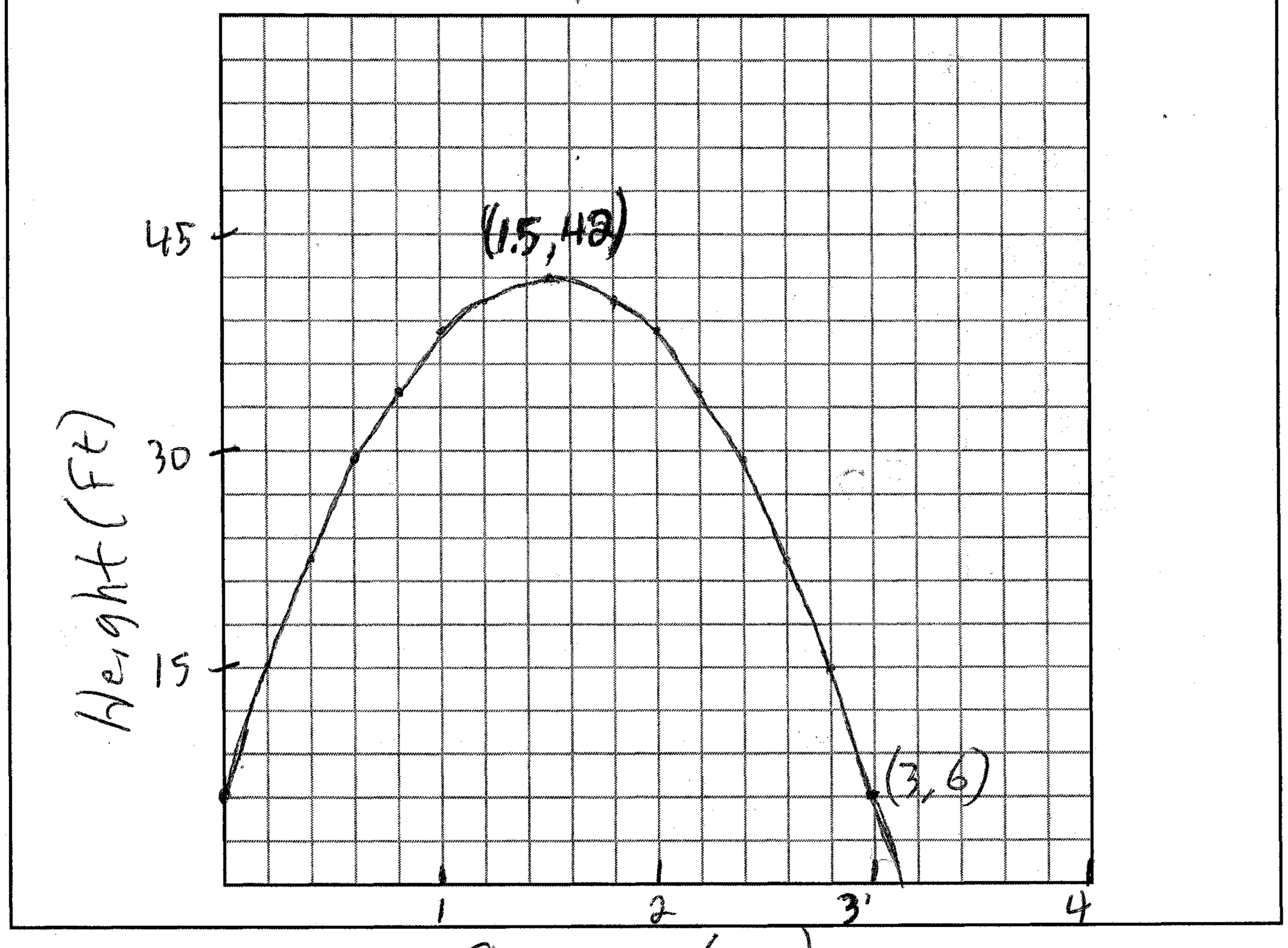
The maximum occurs at the axis of symmetry

$x=\frac{-b}{2a}=\frac{-48}{2(-16)}$

$=\frac{48}{32}=1.5$

$y=-16(1.5)^2+48(1.5)+6$

$=42$



31 An archaeologist can determine the approximate age of certain ancient specimens by measuring the amount of carbon-14, a radioactive substance, contained in the specimen. The formula used to determine the age of a specimen is $A = A_0 2^{\frac{-t}{5760}}$, where A is the amount of carbon-14 that a specimen contains, A_0 is the original amount of carbon-14, t is time, in years, and 5760 is the half-life of carbon-14.

A specimen that originally contained 120 milligrams of carbon-14 now contains 100 milligrams of this substance. What is the age of the specimen, to the *nearest hundred years*?

$A = A_0 2^{\frac{-t}{5760}}$

$\frac{100}{120} = \frac{120 \cdot 2^{\frac{-t}{5760}}}{120}$

$\log \frac{5}{6} = \log 2^{\frac{-t}{5760}}$

$\log \frac{5}{6} = \frac{-t}{5760} \log 2$

$\frac{-5760 \log \frac{5}{6}}{\log 2} = t$

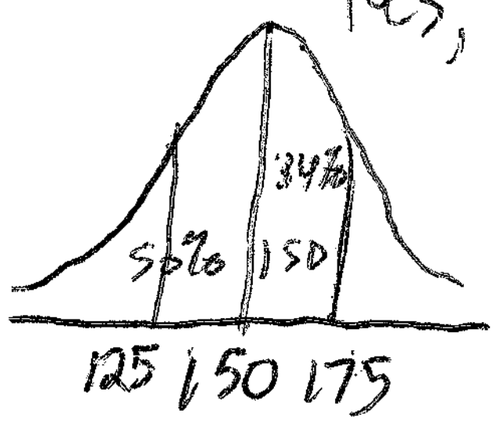
$1500 \approx t$

32 Mrs. Ramírez is a real estate broker. Last month, the sale prices of homes in her area approximated a normal distribution with a mean of $150,000 and a standard deviation of $25,000.

A house had a sale price of $175,000. What is the percentile rank of its sale price, to the *nearest whole number*? Explain what that percentile means.

84. 84% of the sales prices were below $175,000.

Mrs. Ramírez told a customer that most of the houses sold last month had selling prices between $125,000 and $175,000. Explain why she is correct.

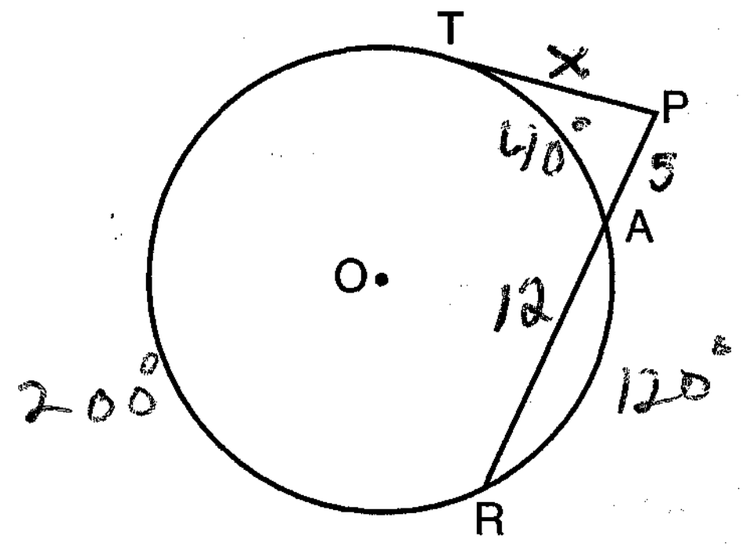


Yes, this range is within one standard deviation of the mean and is 68% of the data.

Part IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

33 The accompanying diagram shows a circular machine part that has rods $\overline{PT}$ and $\overline{PAR}$ attached at points T, A, and R, which are located on the circle; $m\widehat{TA}:m\widehat{AR}:m\widehat{RT} = 1:3:5$; $RA = 12$ centimeters; and $PA = 5$ centimeters.



Find the measure of $\angle P$, in degrees, and find the length of rod $\overline{PT}$, to the *nearest tenth of a centimeter.*

$m\widehat{TA}$ 1 $\frac{1}{9} \times 360 = 40$

$m\widehat{AR}$ 3 $\frac{3}{9} \times 360 = 120$

$m\widehat{RT}$ $\frac{5}{9}$ $\frac{5}{9} \times 360 = 200$

$\angle P = \frac{m\widehat{RT} - m\widehat{TA}}{2} = \frac{200 - 40}{2} = 80$

$x^2 = 5(12+5)$

$x^2 = 85$

$x \approx 9.2$

34 A surveyor is mapping a triangular plot of land. He measures two of the sides and the angle formed by these two sides and finds that the lengths are 400 yards and 200 yards and the included angle is 50°.

What is the measure of the third side of the plot of land, to the *nearest yard*?

What is the area of this plot of land, to the *nearest square yard*?

$K = \frac{1}{2}ab\sin C$

$= \frac{1}{2}(400)(200)\sin 50$

≈ 39642

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 400^2 + 200^2 - 2(400)(200)\cos 50$

$a^2 \approx 97154$

$a \approx 312$