The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS B

Friday, January 25, 2008 — 9:15 a.m. to 12:15 p.m., only

Print Your Name: Steve Sibol

Print Your School's Name: HSCR

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.

The formulas that you may need to answer some questions in this examination are found on page 23. This sheet is perforated so you may remove it from this booklet.

This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice. . .

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

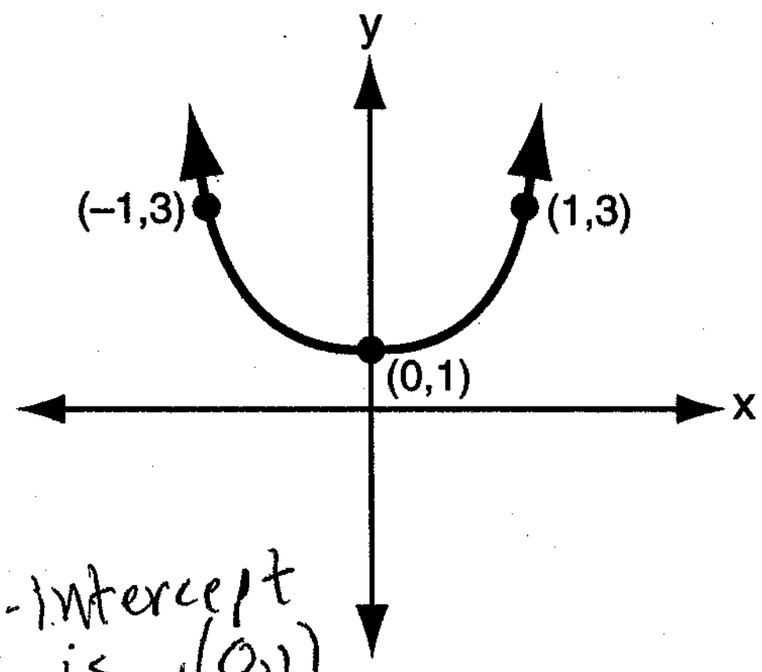
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [40]

Use this space for computations.

1 Which equation is represented by the accompanying graph?



y-intercept is (0,1)

(1) $y = 2x^2 + 1$
(2) $y = 2(x^2 + 1)$
(3) $y = x^2$
(4) $y = 2x^2$

2 What is the value of x in the equation $\sqrt{3+x} - 5 = -2$?

(1) 46
(2) 12
(3) 3
(4) 6

$+5 \quad +5$

$\sqrt{3+x} = 3$

$3 + x = 9$

$x = 6$

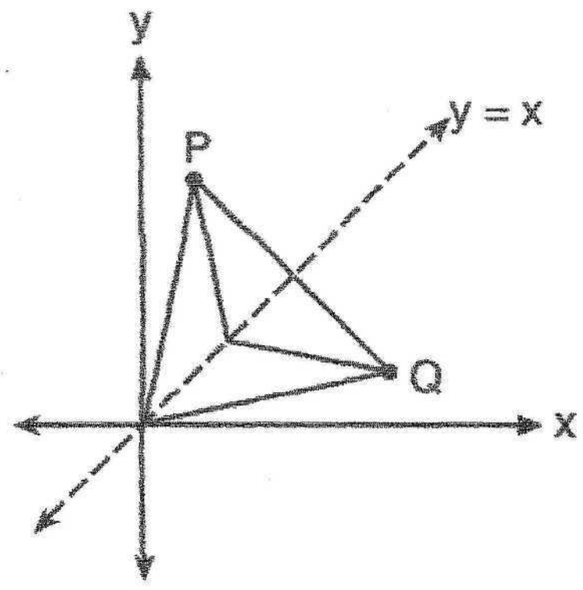
3 Under a dilation where the center of dilation is the origin, the image of $A(-2,-3)$ is $A'(-6,-9)$. What are the coordinates of B', the image of $B(4,0)$ under the same dilation?

×3

(1) (–12,0)
(2) (12,0)
(3) (–4,0)
(4) (4,0)

Use this space for computations.

4 Matthew is a fan of the Air Force's Thunderbirds flying team and is designing a jacket patch for the team, as shown in the accompanying diagram.



If P has the coordinates (a,b), what are the coordinates of Q, the reflection of P in the line $y = x$?

(1) (a,b)
(2) (b,a)
(3) $(-a,b)$
(4) (y,x)

5 Sean tells prospective clients that the probability of rain at the dive location is .2 each day. Which expression can be used to calculate the probability that it will rain on *exactly* 5 days of the 7 days at the dive location?

(1) ${}_7C_5(.2)^5(.8)^2$
(2) ${}_7C_5(.2)^2(.8)^5$
(3) ${}_7C_5(.5)(.7)$
(4) ${}_7C_2(.5)(.7)$

6 Jack wants to plant a border of flowers in the shape of an arc along the edge of a circular walkway. If the circle has a radius of 5 yards and the angle subtended by the arc measures $1\frac{1}{2}$ radians, what is the length, in yards, of the border?

(1) 0.5
(2) 2
(3) 5
(4) 7.5

$\theta = \frac{S}{R}$

$1\frac{1}{2} = \frac{S}{5}$

$7.5 = S$

Use this space for computations.

7 Mayken collected data about the size of the honors classes in her school building. This set of data is shown in the accompanying table.

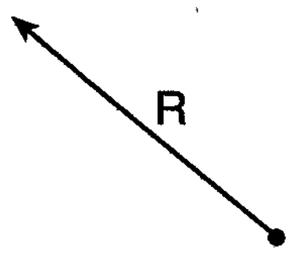
Class Size	Frequency
8	1
10	3
14	2

range = max - min
14 - 8 = 6
mean = $\frac{(8\times1)+(10\times3)+(14\times2)}{6}$ = 11
standard deviation ≈ 2.2

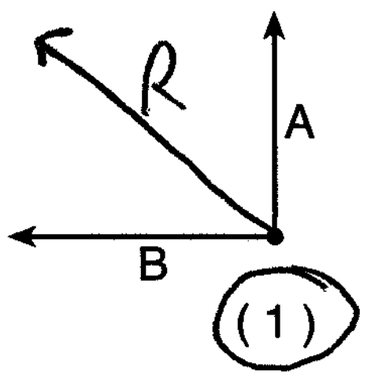
Which statement about the range of this sample is true?

(1) range = mean
(2) range > mean
(3) range < mean
(4) range < standard deviation

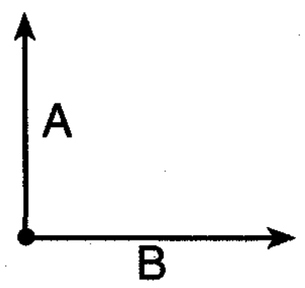
8 The accompanying diagram shows a resultant force vector, R.



Which diagram best represents the pair of component force vectors, A and B, that combined to produce the resultant force vector R?



(1)



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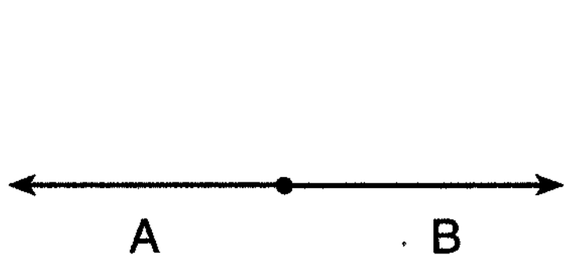

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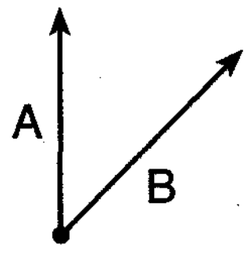
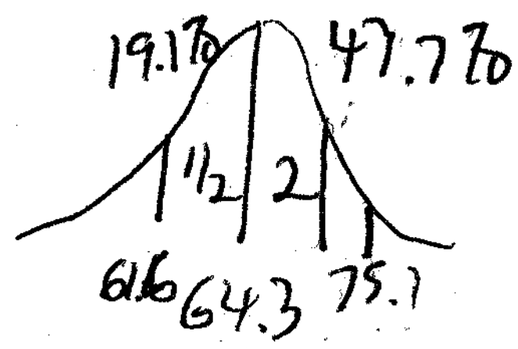


(4)

Use this space for computations.

9 On a standardized test with a normal distribution, the mean was 64.3 and the standard deviation was 5.4. What is the best approximation of the percent of scores that fell between 61.6 and 75.1?

(1) 38.2%
(2) 66.8%
(3) 68.2%
(4) 95%



10 A wave displayed by an oscilloscope is represented by the equation $y = 3 \sin x$. What is the period of this function?

(1) 2π
(2) 2
(3) 3
(4) 3π

$\frac{2\pi}{1} = 2\pi$

11 The expression $\frac{10}{3+i}$ is equivalent to

(1) $3 - i$
(2) $3 + i$
(3) $\frac{15+5i}{4}$
(4) $\frac{5}{4}$

$\frac{10}{3+i} \cdot \frac{3-i}{3-i} = \frac{10(3-i)}{9-i^2} = \frac{10(3-i)}{9-(-1)}$

$\frac{10(3-i)}{10}$

12 The accompanying tables define functions f and g.

x	1	2	3	4	5
f(x)	3	4	5	6	7

x	3	4	5	6	7
g(x)	4	6	8	10	12

What is $(g \circ f)(3)$?

(1) 6
(2) 2
(3) 8
(4) 4

$F(3) = 5$
$g(5) = 8$

Use this space for computations.

13 A radioactive substance has an initial mass of 100 grams and its mass halves every 4 years. Which expression shows the number of grams remaining after t years?

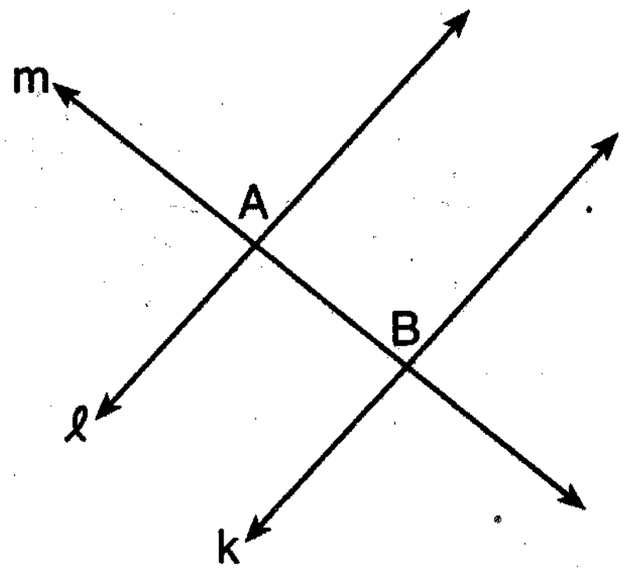
(1) $100(4)^{\frac{t}{4}}$

(2) $100(4)^{-2t}$

(3) $100\left(\frac{1}{2}\right)^{\frac{t}{4}}$

(4) $100\left(\frac{1}{2}\right)^{4t}$

14 In the accompanying diagram, line ℓ is perpendicular to line m at A, line k is perpendicular to line m at B, and lines ℓ, m, and k are in the same plane.



Which statement is the first step in an indirect proof to prove that ℓ is parallel to k?

(1) Assume that ℓ, m, and k are not in the same plane.
(2) Assume that ℓ is perpendicular to k.
(3) Assume that ℓ is not perpendicular to m.
(4) Assume that ℓ is not parallel to k.

15 Which method of collecting data would most likely result in an unbiased random sample?

(1) selecting every third teenager leaving a movie theater to answer a survey about entertainment

(2) placing a survey in a local newspaper to determine how people voted in the 2004 presidential election

(3) selecting students by the last digit of their school ID number to participate in a survey about cafeteria food

(4) surveying honor students taking Mathematics B to determine the average amount of time students in a school spend doing homework each night

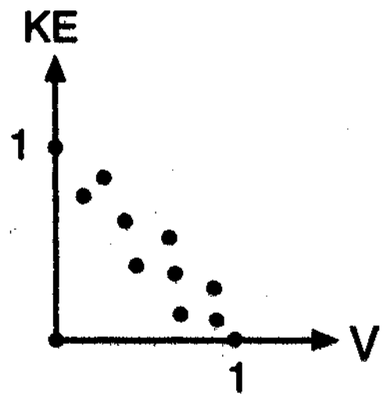
too many movie-goers

too many newspaper readers

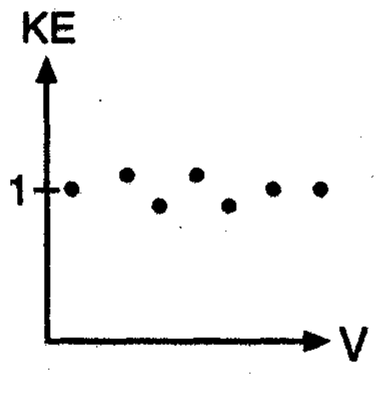
too many bright students

Use this space for computations.

16 In the physics lab, Thelma determined the kinetic energy, *KE*, of an object at various velocities, *V*, and found the linear correlation coefficient between *KE* and *V* to be +0.8. Which graph shows this relationship?



(1)



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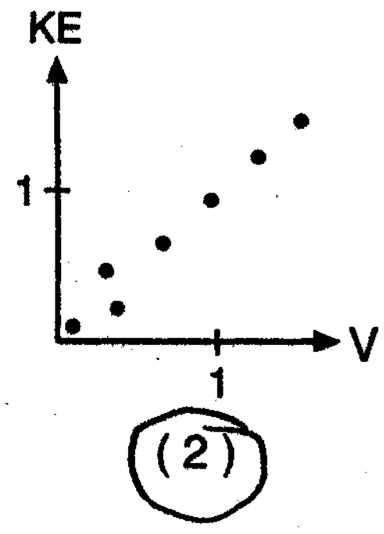

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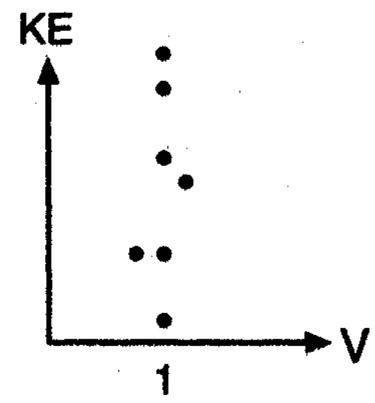


(4)

17 Which equation has roots that are real, rational, and unequal?

(1) $x^2 + x + 1 = 0$

(2) $x^2 - 4x + 4 = 0$

(3) $x^2 - 4 = 0$ $x = \pm 2$

(4) $x^2 - 2 = 0$ $x = \pm\sqrt{2}$ irrational

(1) $b^2 - 4ac = (-4)(1)(1) = -3$ imaginary

(2) $(x-2)^2 = 0$ $x = 2$ equal

18 The expression $\cos(\pi - x)$ is equivalent to

(1) $\sin x$

(2) $-\sin x$

(3) $\cos x$

(4) $-\cos x$

Use this space for computations.

19 If $\log_x 9 = -2$, what is the value of x?

(1) 81
(2) $\frac{1}{81}$
(3) 3
(4) $\frac{1}{3}$

$x^{-2} = 9$

$x^2 = \frac{1}{9}$

$x = \frac{1}{3}$

20 What is the coefficient of the fifth term in the expansion of $(x + 1)^8$?

(1) 8
(2) 28
(3) 56
(4) 70

$n C_{r-1}$

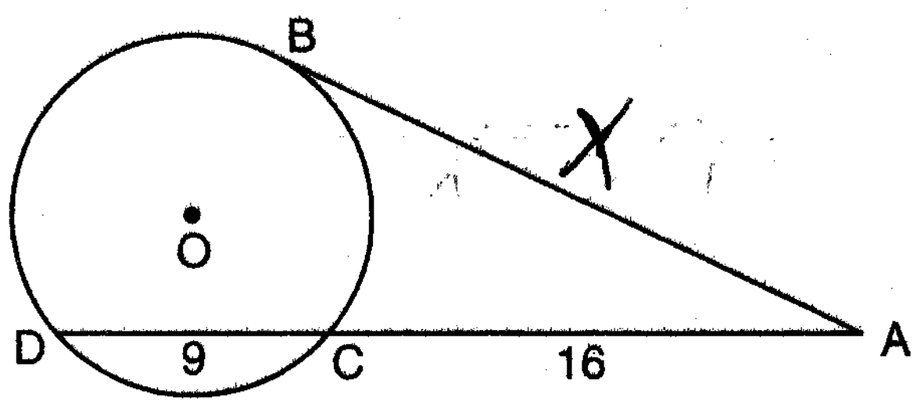
$8 C_{5-1}$

$8 C_4 = 70$

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

21 In the accompanying diagram, $\overline{AB}$ is tangent to circle O at B. If $AC = 16$ and $CD = 9$, what is the length of $\overline{AB}$?



$\overline{AD} = 9+16=25$

$x^2 = 25(16)$

$x^2 = 400$

$x = 20$

22 Solve for all values of x: $|2x-5|=3$

$2x-5=3$

$+5 \quad +5$

$\frac{2x}{2} = \frac{8}{2}$

$x=4$

$2x-5=-3$

$+5 \quad +5$

$\frac{2x}{2} = \frac{2}{2}$

$x=1$

23 The amount of money each member of a band earns playing at a graduation party varies inversely as the number of members in the band. If the band has five members, each member earns \$70. Write an equation that models the relationship between the number of members in a band, n, and the amount each member earns, d.

Use the equation to calculate the amount each member earns if there are four members in the band.

$d = \frac{350}{n}$

$d = \frac{350}{4} = \$87.50$

24 Simplify the expression $\left(m^6\right)^{-\frac{2}{3}}$ and write your answer using a positive exponent.

$m^{\frac{-12}{3}} = m^{-4} = \frac{1}{m^4}$

25 If $i = \sqrt{-1}$, what is the value of the expression $\sum_{n=1}^{20} i^{4n}$?

$i^{4n} = 1$

$20 \times 1 = 20$

26 Express in simplest form: $\dfrac{x-\dfrac{4}{x}}{\dfrac{2+x}{x}}$

$$\frac{\frac{x^2-4}{x}}{\frac{2+x}{x}} = \frac{x^2-4}{x} \div \frac{x+2}{x}$$

$$\frac{x^2-4}{x} \times \frac{x}{x+2}$$

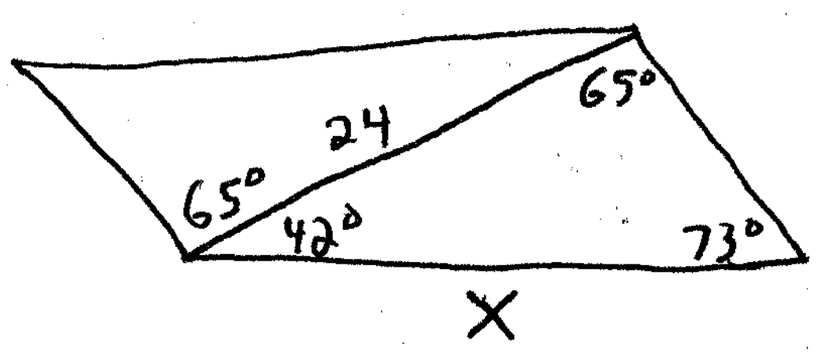
$$(x+2)(x-2) \times \frac{1}{x+2}$$

$$x-2$$

Part III

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [24]

27 The measures of the angles between the resultant and two applied forces are 65° and 42°, and the magnitude of the resultant is 24 pounds. Find, to the *nearest pound*, the magnitude of the larger force.



180
65+42= 107°
73°

$$\frac{24}{\sin 73} = \frac{x}{\sin 65}$$

$$x \approx 23$$

28 The number of houses in Central Village, New York, grows every year according to the function $H(t) = 540(1.039)^t$, where H represents the number of houses, and t represents the number of years since January 1995. A civil engineering firm has suggested that a new, larger well must be built by the village to supply its water when the number of houses exceeds 1,000. During which year will this first happen?

$$\frac{540(1.039)^t}{540} = \frac{1000}{540}$$

$$1.039^t = \frac{50}{27}$$

$$\log 1.039^t = \log \frac{50}{27}$$

$$\frac{t \log 1.039}{\log 1.039} = \frac{\log \frac{50}{27}}{\log 1.039}$$

$$t \approx 16.1$$

2011

29 Find all values of x in the interval $0° \leq x < 360°$ that satisfy the equation $3 \cos x + \sin 2x = 0$.

$3\cos x + 2\sin x \cos x = 0$

$\cos x(3 + 2\sin x) = 0$

$\cos x = 0$

$x = 90°, 270°$

$3 + 2\sin x = 0$

$2\sin x = -3$

$\sin x = \frac{-3}{2}$

No Solution

$-3x-2=6$

30 Write a quadratic equation such that the sum of its roots is –5 and the product of its roots is 6.

$-3+-2=-5$

What are the roots of this equation?

$(x+3)(x+2)=0$

$x^2+5x+6=0$

roots are -3 and -2

31 Water is draining from a tank maintained by the Yorkville Fire Department. Students measured the depth of the water in 15-second intervals and recorded the results in the accompanying table.

Time (x) (in seconds)	**Depth of Water (y)** (in feet)
15	11.8
30	9.9
45	8.2
60	6.3
75	5.9

Write the power regression equation for this set of data, rounding all values to the *nearest ten thousandth*.

Using this equation, predict the depth of the water at 2 minutes, to the *nearest tenth of a foot*.

120 seconds

$y = 42.2326x^{-.4494}$

$y = 42.2326(120)^{-.4494}$

≈ 4.9

32 The horizontal distance, in feet, that a golf ball travels when hit can be determined by the formula $d = \frac{v^2 \sin 2\theta}{g}$, where v equals initial velocity, in feet per second; g equals acceleration due to gravity; θ equals the initial angle, in degrees, that the path of the ball makes with the ground; and d equals the horizontal distance, in feet, that the ball will travel.

A golfer hits the ball with an initial velocity of 180 feet per second and it travels a distance of 840 feet. If g = 32 feet per second per second, what is the smallest initial angle the path of the ball makes with the ground, to the *nearest degree*?

$$d = \frac{v^2 \sin 2\theta}{g}$$

$$840 = \frac{180^2 \sin 2\theta}{32}$$

$$\sin 2\theta \approx .8296$$

$$2\theta \approx \sin^{-1}(.8296)$$

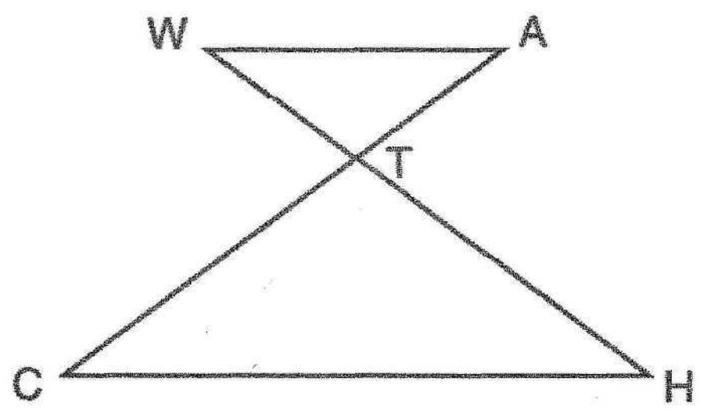
$$\frac{2\theta}{2} \approx \frac{56}{2}$$

$$\theta \approx 28°$$

Part IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

33 In the accompanying diagram, $\overline{WA} \parallel \overline{CH}$ and $\overline{WH}$ and $\overline{AC}$ intersect at point T. Prove that $(WT)(CT)=(HT)(AT)$.



STATEMENT	REASON
① $\overline{WA} \parallel \overline{CH}$, $\overline{WH}$ and $\overline{AC}$ intersect at T	① Given
② $\angle WTA \cong \angle HTC$	② Vertical angles
③ $\overline{ATC}$ and $\overline{WTH}$ are transversals	③ Definition of transversal
④ $\angle WHC \cong \angle AWH$ and $\angle ACH \cong \angle WAC$	④ Alternate interior angles are congruent
⑤ $\Delta TCH \sim \Delta TAW$	⑤ AA
⑥ $\frac{WH}{AT} = \frac{HT}{CT}$	⑥ Corresponding sides of similar triangles are in proportion
⑦ $(WT)(CT) = (HT)(AT)$	⑦ The product of the means equals the product of the extremes.

34 The members of the Lincoln High School Prom Committee are trying to raise money for their senior prom. They plan to sell teddy bears. The senior advisor told them that the profit equation for their project is $y = -0.1x^2 + 9x - 50$, where x is the price at which the teddy bears will be sold and y is the profit, in dollars.

On the grid on the next page, graph this relationship so that $0 \leq x \leq 90$ and $-50 \leq y \leq 160$.

How much profit can the committee expect to make if they sell the teddy bears for \$20 each?

What price should they charge for the teddy bears to make the maximum profit possible?

$y = -0.1x^2 + 9x - 50$

$= -0.1(20)^2 + 9(20) - 50$

$= \$90$

$X = \frac{-b}{2a} = \frac{-9}{2(-.1)} = \$45$ for maximum profit

Question 34 continued