The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

# MATHEMATICS B

**Friday,** January 27, 2006 — 9:15 a.m. to 12:15 p.m., only

**Print Your Name:** Steven Sibol

**Print Your School's Name:** HSER

**Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.**

**Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.**

**The formulas that you may need to answer some questions in this examination are found on page 23. This sheet is perforated so you may remove it from this booklet.**

**This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.**

**When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.**

**Notice. . .**

**A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.**

**The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.**

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

MATHEMATICS B

## Part I

**Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.** [40]

**1** What is the value of $\sum_{n=1}^{5}(-2n+100)$?

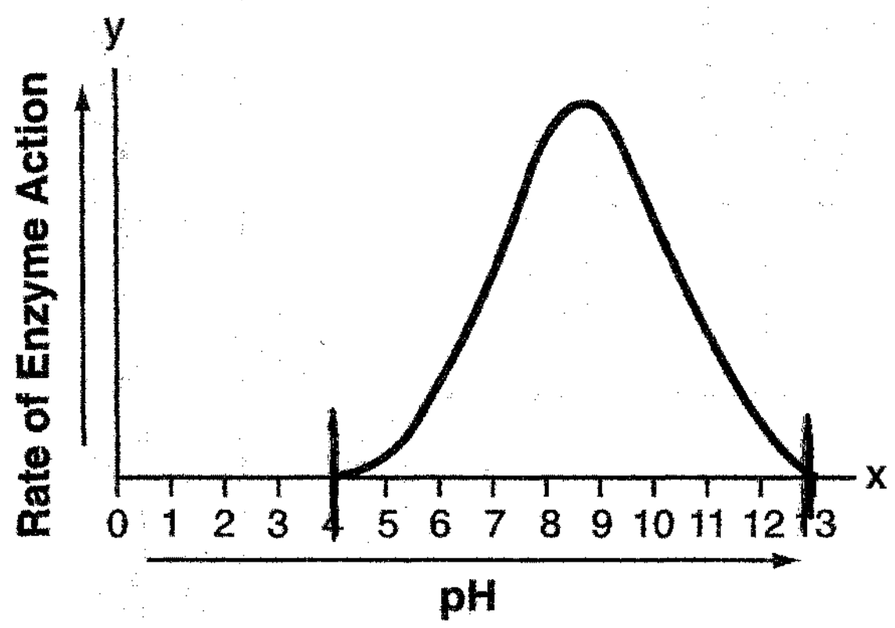
(1) 70
(2) 130
(3) 470
(4) 530

**Use this space for computations.**

| $n$ | $-2n+100$ |
|---|---|
| 1 | 98 |
| 2 | 96 |
| 3 | 94 |
| 4 | 92 |
| 5 | 90 |

470

**2** The effect of pH on the action of a certain enzyme is shown on the accompanying graph.

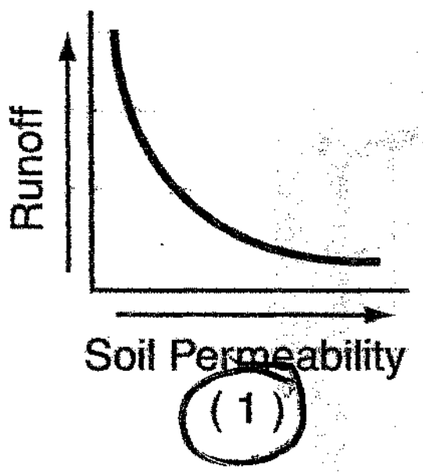



What is the domain of this function?

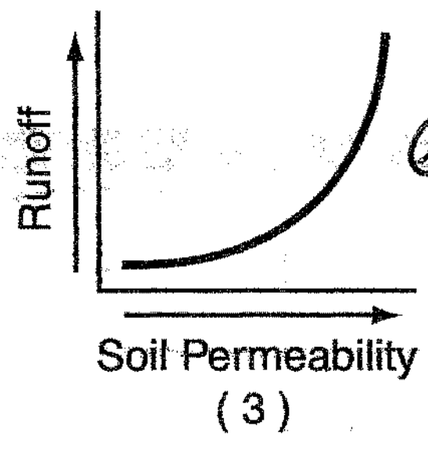
(1) $4 \le x \le 13$
(2) $4 \le y \le 13$
(3) $x \ge 0$
(4) $y \ge 0$

**Use this space for computations.**

3 Which graph shows that soil permeability varies inversely to runoff?

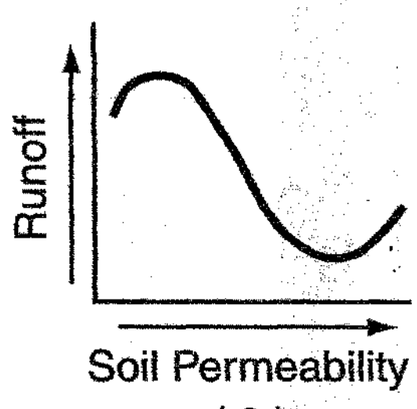



(1)

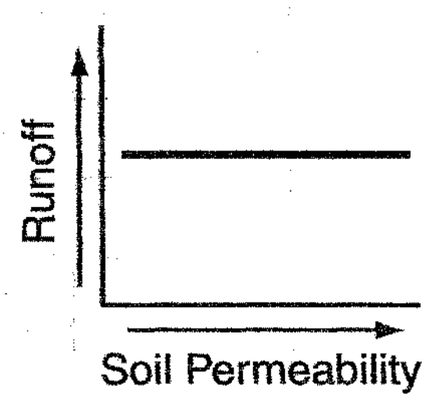



(3)

exponential




(2)




(4)

linear

trigonometric

4 On a standardized test, a score of 86 falls exactly 1.5 standard deviations below the mean. If the standard deviation for the test is 2, what is the mean score for this test?

(1) 84
(2) 84.5
(3) 87.5
(4) 89

$86 + 2(1.5) = 89$

5 Which transformation of the graph of $y = x^2$ would result in the graph of $y = x^2 + 2$?

(1) $D_2$
(2) $T_{0,2}$
(3) $r_{y=2}$
(4) $R_{0,90}$

6 A sound wave is modeled by the curve $y = 3 \sin 4x$. What is the period of this curve?

(1) $\pi$
(2) $\frac{\pi}{2}$
(3) 3
(4) 4

$\text{period} = \frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$

**7** If $\sqrt{2x-1}+2=5$, then $x$ is equal to

(1) 1
(2) 2
(3) 5
(4) 4

$\sqrt{2x-1} = 3$

$2x-1 = 9$

$\frac{2x}{2} = \frac{10}{2} = 5$

**Use this space for computations.**

**8** The expression $(1 + \cos x)(1 - \cos x)$ is equivalent to

(1) 1
(2) $\sec^2 x$
(3) $\sin^2 x$
(4) $\csc^2 x$

$(1+\cos x)(1-\cos x)$

$1-\cos x+\cos x-\cos^2 x$

$1-\cos^2 x$

$\sin^2 x$

**9** If $\theta$ is a positive acute angle and $\sin 2\theta = \frac{\sqrt{3}}{2}$, then $(\cos\theta + \sin\theta)^2$ equals

(1) 1
(2) $1+\frac{\sqrt{3}}{2}$
(3) 30°
(4) 60°

$(\cos\theta+\sin\theta)(\cos\theta+\sin\theta)$

$\cos^2\theta+\cos\theta\sin\theta+\cos\theta\sin\theta+\sin^2\theta$

$\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta$

$1 + \sin 2\theta$ [Function of Double Angle]

$1 + \sqrt{3}/2$

**10** What is the solution of the inequality $|y + 8| > 3$?

(1) $y > -5$ or $y < -11$
(2) $y > -5$
(3) $-11 < y < -5$
(4) $-5 < y < 11$

$y+8>3$ $\quad$ $y+8<-3$

$-8\ \ -8$ $\quad$ $-8\ \ -8$

$y>-5$ $\quad$ $y<-11$

**11** The speed of sound, $v$, at temperature $T$, in degrees Kelvin, is represented by the equation $v = 1087\sqrt{\frac{T}{273}}$. Which expression is equivalent to $\log v$?

(1) $1087 + \frac{1}{2}\log T - \log 273$
(2) $1087\left(\frac{1}{2}\log T - \frac{1}{2}\log 273\right)$
(3) $\log 1087 + \frac{1}{2}\log T - \frac{1}{2}\log 273$
(4) $\log 1087 + 2\log(T + 273)$

$\log v = \log 1087\sqrt{\frac{T}{273}}$

$= \log 1087 + \log\left(\frac{T}{273}\right)^{1/2}$

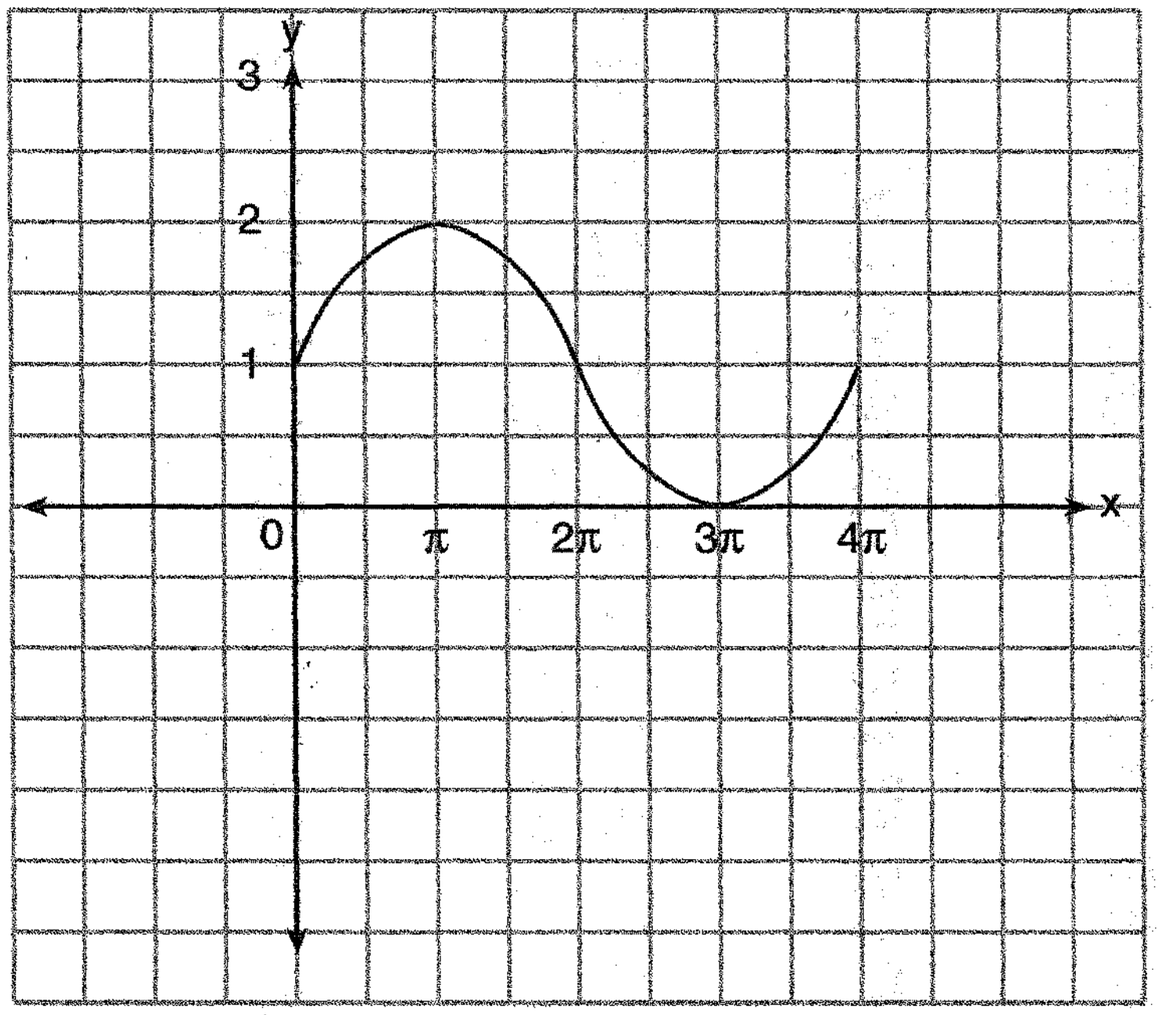
$= \log 1087 + \frac{1}{2}\log\frac{T}{273}$

$= \log 1087 + \frac{1}{2}\log T - \frac{1}{2}\log 273$

[4]

**12** In physics class, Eva noticed the pattern shown in the accompanying diagram on an oscilloscope.

**Use this space for computations.**




Which equation best represents the pattern shown on this oscilloscope?

(1) $y = \sin\left(\frac{1}{2}x\right) + 1$

(2) $y = \sin x + 1$

(3) $y = 2 \sin x + 1$

(4) $y = 2\sin\left(-\frac{1}{2}x\right) + 1$

period: $\frac{2\pi}{b}$

$4\pi = \frac{2\pi}{b}$

$b = \frac{1}{2}$

**13** The expression $\frac{5}{\sqrt{5}-1}$ is equivalent to

(1) $\frac{5}{4}$

(2) $\frac{5\sqrt{5}+5}{4}$

(3) $\frac{5\sqrt{5}-5}{4}$

(4) $\frac{5\sqrt{5}-5}{6}$

$\frac{5}{\sqrt{5}-1}\frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{5(\sqrt{5}+1)}{5-1} = \frac{5\sqrt{5}+5}{4}$

**14** The roots of the equation $2x^2 - 5 = 0$ are

(1) imaginary
(2) real, rational, and equal
(3) real, rational, and unequal
(4) real and irrational

$a = 2$
$c = -5$

$b^2 - 4ac$
$0^2 - 4(2)(-5)$
$40$

**15** What is the radian measure of the angle formed by the hands of a clock at 2:00 p.m.?

(1) $\frac{\pi}{2}$
(2) $\frac{\pi}{3}$
(3) $\frac{\pi}{4}$
(4) $\frac{\pi}{6}$

**Use this space for computations.**

$2\pi \times \frac{2}{12} = \frac{4\pi}{12} = \frac{\pi}{3}$

**16** If $\theta$ is an angle in standard position and $P(-3,4)$ is a point on the terminal side of $\theta$, what is the value of $\sin\theta$?

(1) $\frac{3}{5}$
(2) $-\frac{3}{5}$
(3) $\frac{4}{5}$
(4) $-\frac{4}{5}$

The terminal side of $\theta$ passes through $(\frac{-3}{5}, \frac{4}{5})$ on the unit circle.

**17** When simplified, the expression $\left(\sqrt[3]{m^4}\right)\left(m^{-\frac{1}{2}}\right)$ is equivalent to

(1) $\sqrt[3]{m^{-2}}$
(2) $\sqrt[4]{m^3}$
(3) $\sqrt[5]{m^{-4}}$
(4) $\sqrt[6]{m^5}$

$\left(m^{\frac{4}{3}}\right)\left(m^{-1/2}\right) = m^{\frac{8-3}{6}} = m^{5/6}$

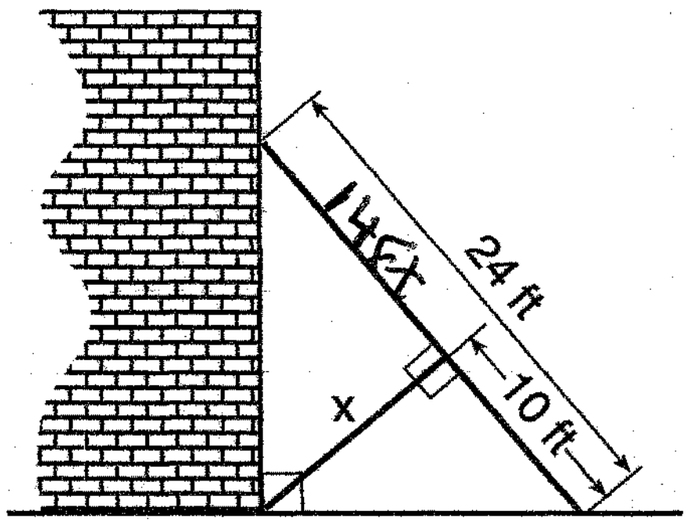
**18** What are the coordinates of point $A'$, the image of point $A(-4,1)$ after the composite transformation $R_{90°} \circ r_{y=x}$ where the origin is the center of rotation?

(1) (–1,–4)
(2) (–4,–1)
(3) (1,4)
(4) (4,1)

$(x,x)$ after the reflection $(1,-4)$
$(-y,x)$ after the rotation $(4,1)$

**Use this space for computations.**

**19** The accompanying diagram shows a 24-foot ladder leaning against a building. A steel brace extends from the ladder to the point where the building meets the ground. The brace forms a right angle with the ladder.




If the steel brace is connected to the ladder at a point that is 10 feet from the foot of the ladder, which equation can be used to find the length, $x$, of the steel brace?

(1) $\frac{10}{x} = \frac{x}{14}$

(2) $\frac{10}{x} = \frac{x}{24}$

(3) $10^2 + x^2 = 14^2$

(4) $10^2 + x^2 = 24^2$

**20** The center of a circle represented by the equation $(x-2)^2 + (y+3)^2 = 100$ is located in Quadrant

(1) I

(2) II

(3) III

(4) IV

(2, -3)

## Part II

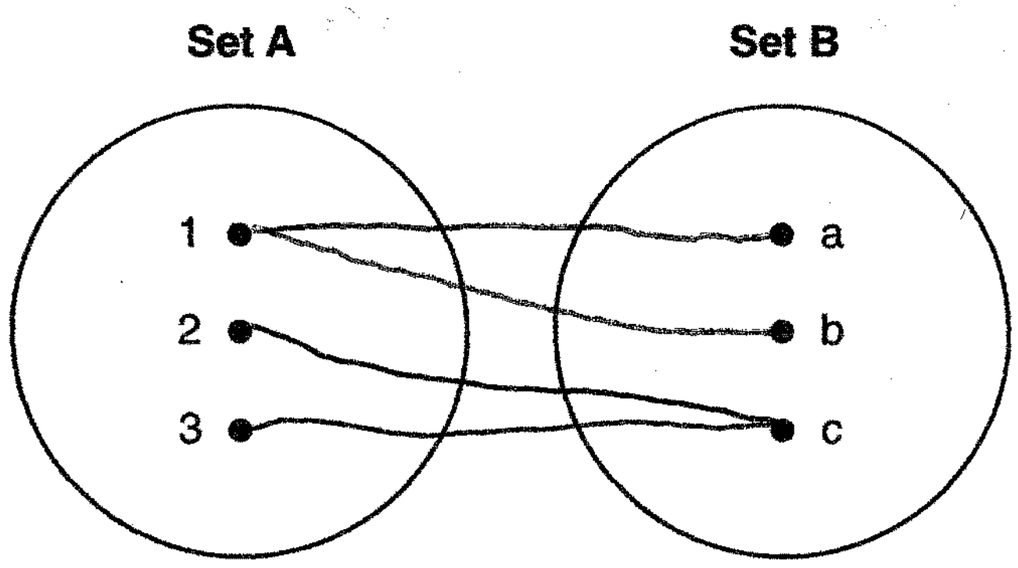
**Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.** [12]

**21** If $f(x) = 5x^2 - 1$ and $g(x) = 3x - 1$, find $g(f(1))$.

$F(1) = 5(1)^2 - 1 = 4$

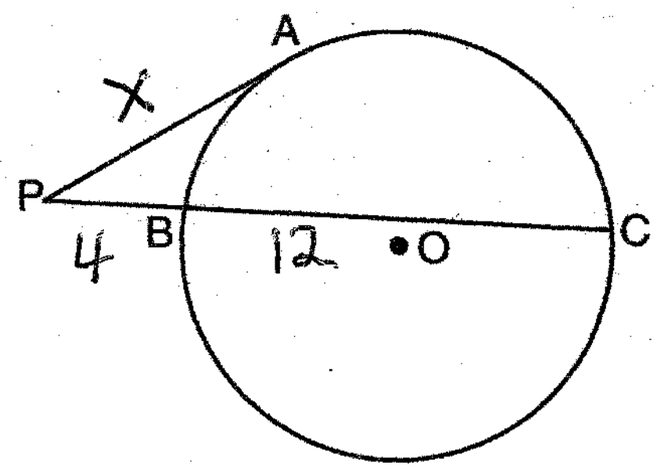
$g(4) = 3(4) - 1 = 11$

**22** On the accompanying diagram, draw a mapping of a relation from set *A* to set *B* that is *not* a function. Explain why the relationship you drew is *not* a function.




This relation is not a function because 1 maps to both a and b.

**23** In the accompanying diagram, $\overline{PA}$ is tangent to circle $O$ at $A$, secant $\overline{PBC}$ is drawn, $PB = 4$, and $BC = 12$. Find $PA$.




$X^2 = 4(12+4)$

$X^2 = 64$

$X = 8$

**24** The time it takes to travel to a location varies inversely to the speed ~~traveled. It takes 4 hours driving at an average speed of 55 miles per~~ hour to reach a location. To the *nearest tenth of an hour*, how long will it take to reach the same location driving at an average speed of 50 miles ~~per hour~~?

$$\frac{4 \times 55}{50} = \frac{h \cdot 50}{50}$$

$$4.4 = h$$

**25** During a recent survey, students at Franconia College were asked if they drink coffee in the morning. The results showed that two-thirds of the students drink coffee in the morning and the remainder do not. What is the probability that of six students selected at random, *exactly* two of them drink coffee in the morning? Express your answer as a fraction or as a decimal rounded to *four decimal places*.

$n = 6$

$r = 2$

$p = \frac{2}{3}$

$q = \frac{1}{3}$

$${}_6C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^4$$

$$15 \left(\frac{4}{9}\right)\left(\frac{1}{81}\right) = \frac{60}{729} = \frac{20}{243}$$

**26** Solve algebraically for $x$: $8^{2x} = 4^6$

$$8^{2x} = 4^6$$

$$(2^3)^{2x} = (2^2)^6$$

$$2^{6x} = 2^{12}$$

$$\frac{6x}{6} = \frac{12}{6}$$

$$x = 2$$

## Part III

**Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.** [24]

**27** In physics class, Taras discovers that the behavior of electrical power, $x$, in a particular circuit can be represented by the function $f(x) = x^2 + 2x + 7$. If $f(x) = 0$, solve the equation and express your answer in simplest $a + bi$ form.

$x^2+2x+7=0$ complete the square

$x^2+2x+1 = -7+1$

$(x+1)^2 = -6$

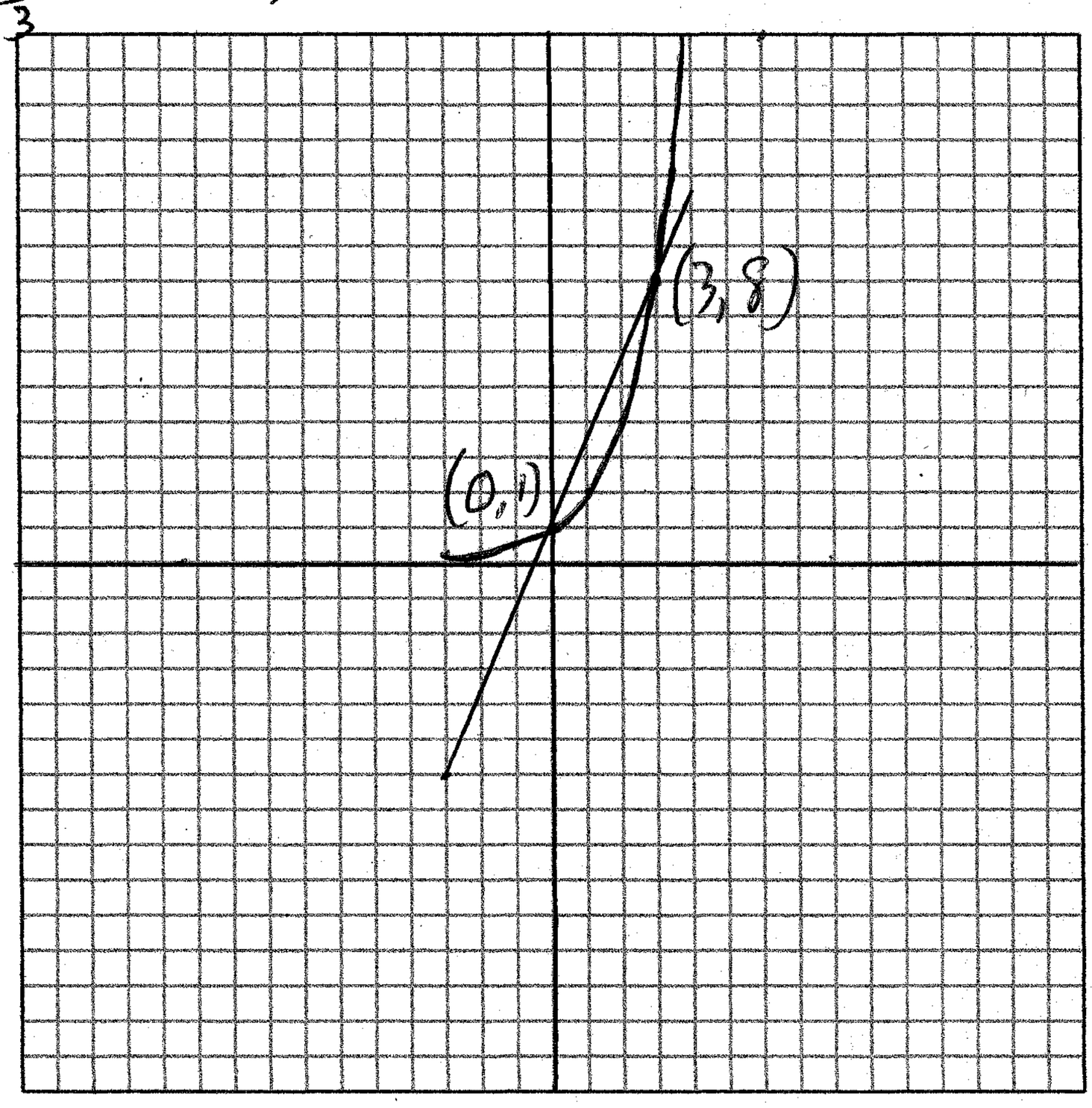
$x+1 = \pm\sqrt{-6}$

$x = -1 \pm \sqrt{-6}$

$x = -1 \pm i\sqrt{6}$

**28** On the accompanying grid, sketch the graphs of $y = 2^x$ and $3y = 7x + 3$ over the interval $-3 \leq x \leq 4$. Identify and state the coordinates of all points of intersection.

$\frac{3y}{3} = \frac{7x+3}{3}$ $\frac{3}{3}$ $y = \frac{7}{3}x + 1$



**29** Simplify completely: $\dfrac{\dfrac{1-m}{m}}{m-\dfrac{1}{m}}$

$$\frac{1-m}{m} \div \left(m - \frac{1}{m}\right)$$

$$\frac{1-m}{m} \div \frac{m^2-1}{m}$$

$$\frac{\cancel{1-m}}{\cancel{m}} \times \frac{\cancel{m}-1}{(m+1)\cancel{(m-1)}}$$

$$\frac{-1}{m+1}$$

**30** An architect is using a computer program to design the entrance of a ~~railroad tunnel. The outline of the opening is modeled by the function~~ $f(x) = 8 \sin x + 2$, in the interval $0 \leq x \leq \pi$, where $x$ is expressed in radians.

Solve algebraically for all values of $x$ in the interval $0 \leq x \leq \pi$, where the height of the opening, $f(x)$, is 6. Express your answer in terms of $\pi$.

If the $x$-axis represents the base of the tunnel, what is the maximum height of the entrance of the tunnel?

$8 \sin x + 2 = 6$

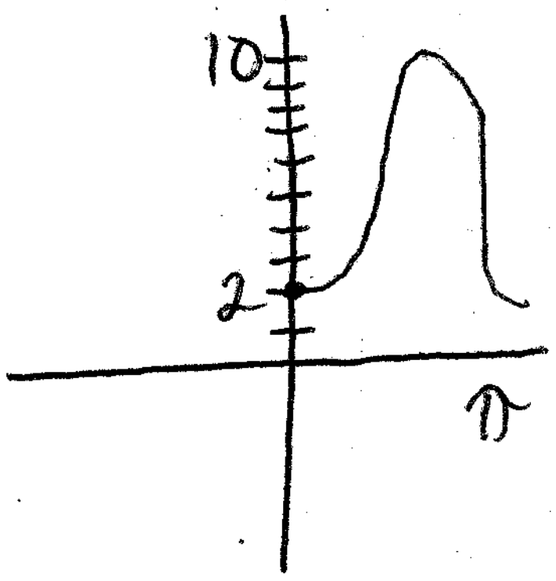
$-2 \quad -2$

$\frac{8 \sin x}{8} = \frac{4}{8}$

$\sin x = \frac{1}{2}$

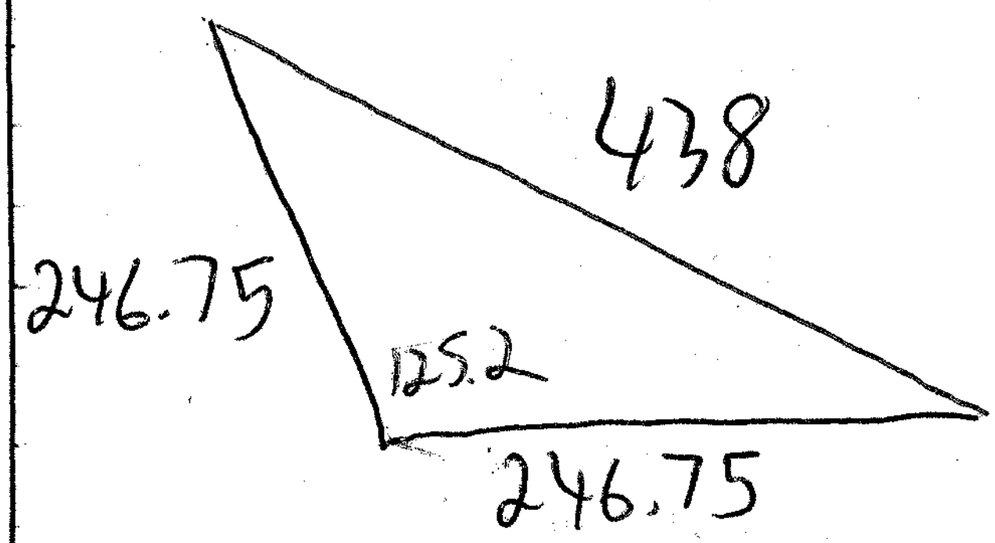
$x = \sin^{-1}\left(\frac{1}{2}\right)$

$x = \frac{\pi}{6}$ and $\frac{5\pi}{6}$




Maximum height is 10.

**31** The Vietnam Veterans Memorial in Washington, D.C., is made up of ~~two walls, each 246.75 feet long, that meet at an angle of 125.2°~~. Find, to the *nearest foot*, the distance between the ends of the walls that do not meet.




$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 246.75^2 + 246.75^2 - 2(246.75)(246.75)\cos 125.2$

$a^2 \approx 191964$

$a \approx 438$

**32** The current population of Little Pond, New York, is 20,000. The population is *decreasing*, as represented by the formula $P = A(1.3)^{-0.234t}$, where $P$ = final population, $t$ = time, in years, and $A$ = initial population.

What will the population be 3 years from now? Round your answer to the *nearest hundred people*.

To the *nearest tenth of a year*, how many years will it take for the population to reach half the present population? [The use of the grid on the next page is optional.]

$P = A(1.3)^{-.234t}$

$= 20,000\,(1.3)^{(-.234)(3)}$

$\approx 16,600$

$P = A(1.3)^{-.234t}$

$\frac{10,000}{20,000} = \frac{20,000\,(1.3)^{-.234t}}{20,000}$

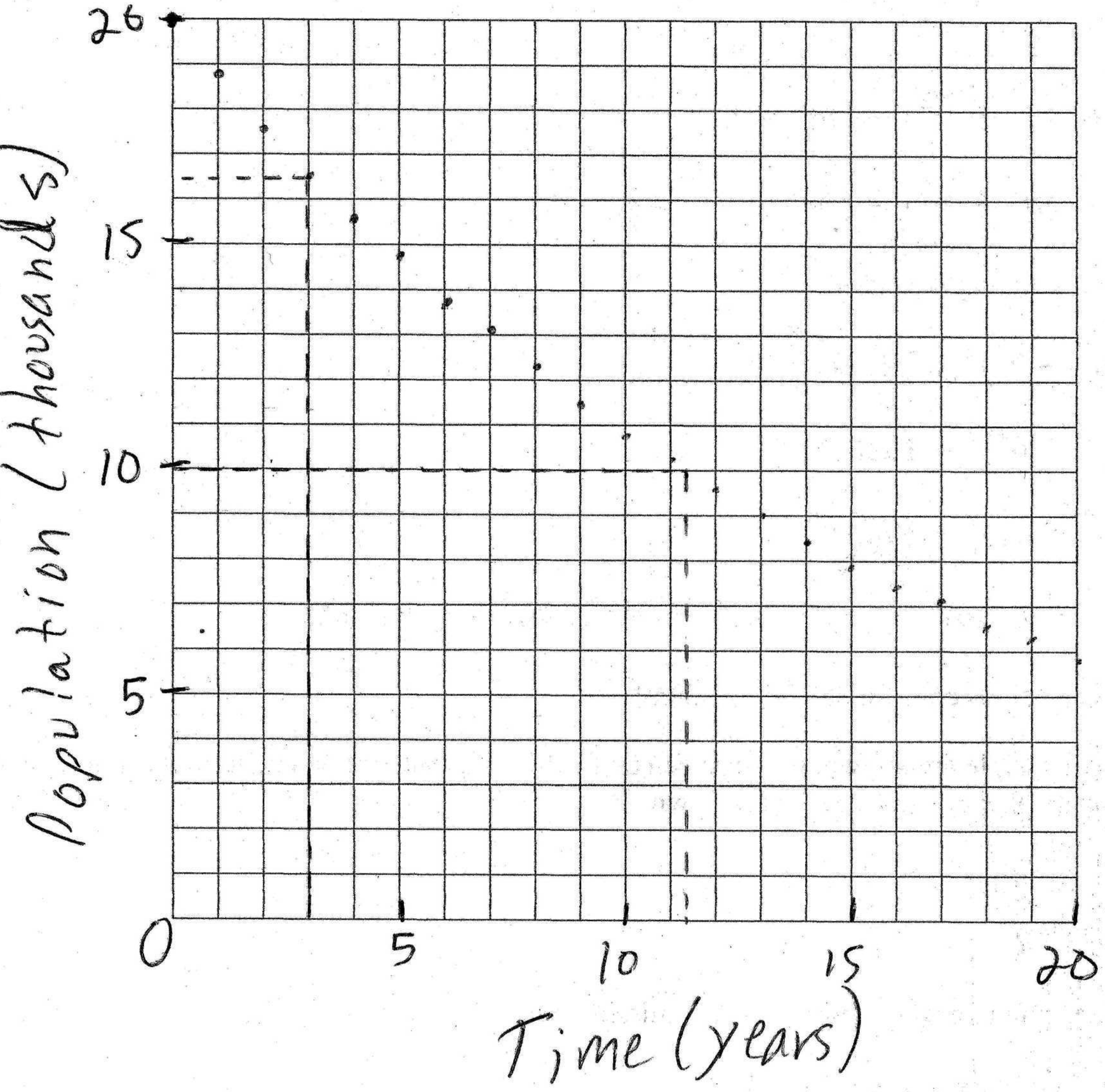
$\log .5 = \log 1.3^{-.234t}$

$\log .5 = -.234t \cdot \log 1.3$

$t = \frac{\log .5}{(-.234)(\log 1.3)}$

$t \approx 11.3$

**Question 32 continued**



## Part IV

**Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.** [12]

**33** Since 1990, fireworks usage nationwide has grown, as shown in the accompanying table, where $t$ represents the number of years since 1990, and $p$ represents the fireworks usage per year, in millions of pounds.

| **Number of Years Since 1990** ($t$) | 0 | 2 | 4 | 6 | 7 | 8 | 9 | 11 |
|---|---|---|---|---|---|---|---|---|
| **Fireworks Usage per Year, in Millions of Pounds** ($p$) | 67.6 | 88.8 | 119.0 | 120.1 | 132.5 | 118.3 | 159.2 | 161.6 |

Find the equation of the linear regression model for this set of data, where $t$ is the independent variable. Round values to *four decimal places*. $y = 8.1875t + 72.7860$

Using this equation, determine in what year fireworks usage would have reached 99 million pounds.

Based on this linear model, how many millions of pounds of fireworks would be used in the year 2008? Round your answer to the *nearest tenth*.

$99 = 8.1875t + 72.7860$

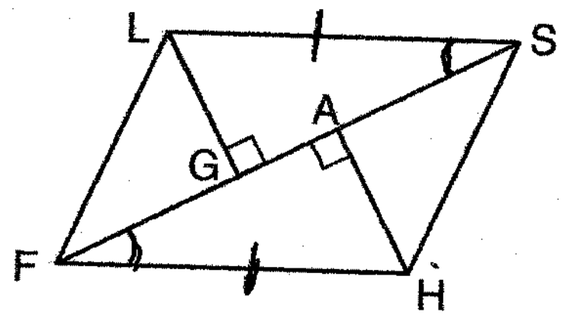
$3 \approx t$ or 1993

2008 is 18 years after 1990

$y = 8.1875(18) + 72.7860$

$y \approx 220.2$

**34** Given: parallelogram $FLSH$, diagonal $\overline{FGAS}$, $\overline{LG} \perp \overline{FS}$, $\overline{HA} \perp \overline{FS}$




Prove: $\triangle LGS \cong \triangle HAF$

| | STATEMENT | REASON |
|---|---|---|
| (1) | Parallelogram FLSH, diagonal $\overline{FGAS}$, $\overline{LG} \perp \overline{FS}$, $\overline{HA} \perp \overline{FS}$ | (1) Given |
| (2) | $\overline{FH} \cong \overline{SL}$ | (2) Opposite sides of a parallelogram are congruent |
| (3) | $\overline{FH} \parallel \overline{SL}$ | (3) Opposite sides of a parallelogram are parallel |
| (4) | Because $\overline{FGAS}$ is a transversal, $\angle AFH$ and $\angle LSG$ are congruent | (4) Alternate interior angles formed by parallel lines and a transversal are congruent |
| (5) | $\triangle LGS \cong \triangle HAF$ | (5) AAS |