The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS B

Friday, January 28, 2005 — 9:15 a.m. to 12:15 p.m., only

Print Your Name: Steve Sigol

Print Your School's Name: HSCR

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will *not* be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.

This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. The formulas that you may need to answer some questions in this examination are found on page 23.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice. . .

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [40]

1 If $f(x) = -2x + 7$ and $g(x) = x^2 - 2$, then $f(g(3))$ is equal to

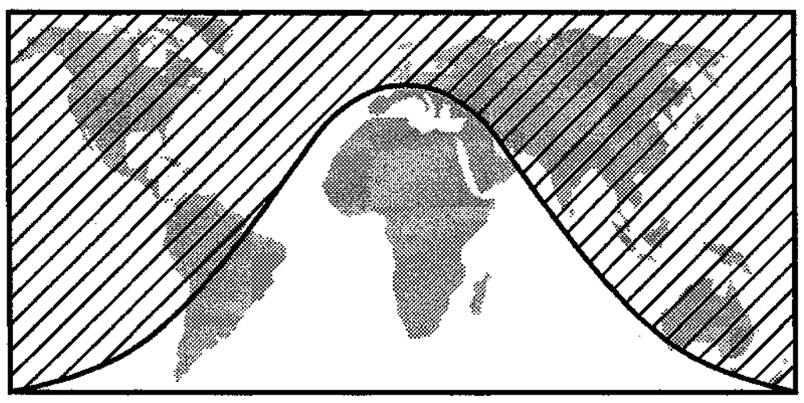
(1) -7
(2) -3
(3) -1
(4) 7

$g(3) = 3^2 - 2 = 7$

$f(7) = -2(7) + 7 = -7$

Use this space for computations.

2 The shaded portion of the accompanying map indicates areas of night, and the unshaded portion indicates areas of daylight at a particular moment in time.



Which type of function best represents the curve that divides the area of night from the area of daylight?

(1) quadratic
(2) cosine
(3) tangent
(4) logarithmic

3 If R varies inversely as S, when S is doubled, R is multiplied by

(1) $\frac{1}{2}$
(2) 2
(3) $\frac{1}{4}$
(4) 4

$25 \cdot \frac{R}{2} = 5R$

4 What is the domain of the function $f(x) = \frac{3x^2}{x^2 - 49}$?

(1) $\{x | x \in \text{real numbers}, x \neq 7\}$
(2) $\{x | x \in \text{real numbers}, x \neq \pm 7\}$
(3) $\{x | x \in \text{real numbers}\}$
(4) $\{x | x \in \text{real numbers}, x \neq 0\}$

$x^2 - 49 = 0$

$(x+7)(x-7) = 0$

$x = -7 \quad x = 7$

5 The value of $\sum_{r=2}^{4} {}_5C_r$ is

(1) 5
(2) 10
(3) 25
(4) 45

r	${}_5C_r$
2	${}_5C_2 = 10$
3	${}_5C_3 = 10$
4	${}_5C_4 = 5$
	24

Use this space for computations.

6 The product of $(5ab)$ and $(-2a^2b)^3$ is

(1) $-30a^6b^4$
(2) $-30a^7b^4$
(3) $-40a^6b^4$
(4) $-40a^7b^4$

$5a^1b^1(-8a^6b^3) = -40a^7b^4$

7 Which transformation is an example of an opposite isometry?

translation (1) $(x,y) \rightarrow (x + 3, y - 6)$
dilation (2) $(x,y) \rightarrow (3x,3y)$
(3) $(x,y) \rightarrow (y,x)$ reflection over $y = x$
(4) $(x,y) \rightarrow (y,-x)$ 270° rotation

8 The expression $\frac{\tan \theta}{\sec \theta}$ is equivalent to

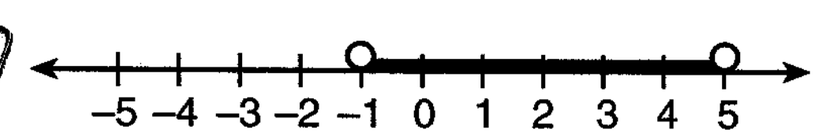
(1) $\frac{\cos^2 \theta}{\sin \theta}$
(2) $\frac{\sin \theta}{\cos^2 \theta}$
(3) $\cos \theta$
(4) $\sin \theta$

$\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}} = \frac{\sin\theta}{\cos\theta} \div \frac{1}{\cos\theta}$

$\frac{\sin\theta}{\cos\theta} \times \cos\theta$

$\sin\theta$

9 Which graph represents the solution set of the inequality $x^2 - 4x - 5 < 0$?

(1)


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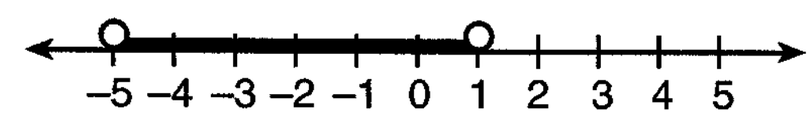

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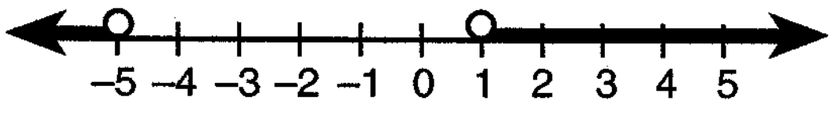
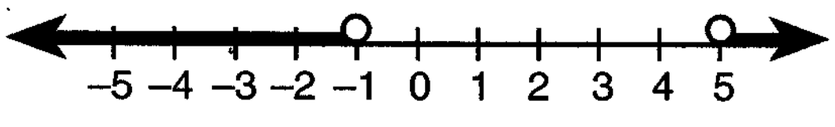


(4)


$x^2 - 4x - 5 < 0$

$(x-5)(x+1) < 0$

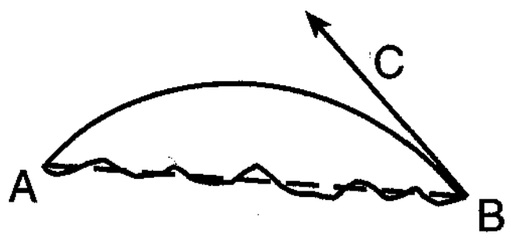
$x - 5 < 0$ and $x + 1 > 0$ OR $x - 5 > 0$ and $x + 1 < 0$

$x < 5$ and $x > -1$ $x > 5$ and $x < -1$

NOT POSSIBLE

10 A small fragment of something brittle, such as pottery, is called a shard. The accompanying diagram represents the outline of a shard from a small round plate that was found at an archaeological dig.

Use this space for computations.

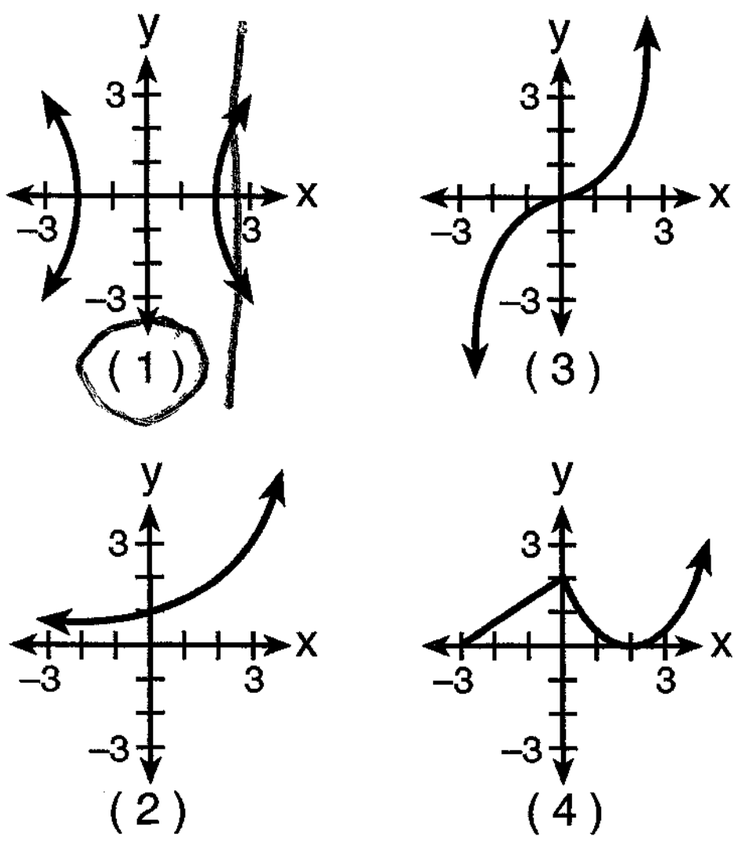


The arc intercepted by a tangent and chord is double the angle created by the tangent and chord

If $\overrightarrow{BC}$ is a tangent to $\overset{\frown}{AB}$ at B and m$\angle ABC$ = 45, what is the measure of $\overset{\frown}{AB}$, the outside edge of the shard?

(1) 45°
(2) 90°
(3) 135°
(4) 225°

11 Which graph is *not* a function?



(1) Fails the vertical line test

12 If A is a positive acute angle and $\sin A = \frac{\sqrt{5}}{3}$, what is $\cos 2A$?

(1) $\frac{1}{9}$
(2) $-\frac{1}{9}$
(3) $\frac{1}{3}$
(4) $-\frac{1}{3}$

$\cos 2A = 1 - 2\sin^2 A$

$= 1 - 2\left(\frac{\sqrt{5}}{3}\right)^2$

$= 1 - 2\left(\frac{5}{9}\right)$

$= 1 - \frac{10}{9} = -\frac{1}{9}$

a=2 b=-8 c=-4

13 The roots of the equation $2x^2 - 8x - 4 = 0$ are

(1) imaginary
(2) real, rational, and equal
(3) real, irrational, and unequal
(4) real, rational, and unequal

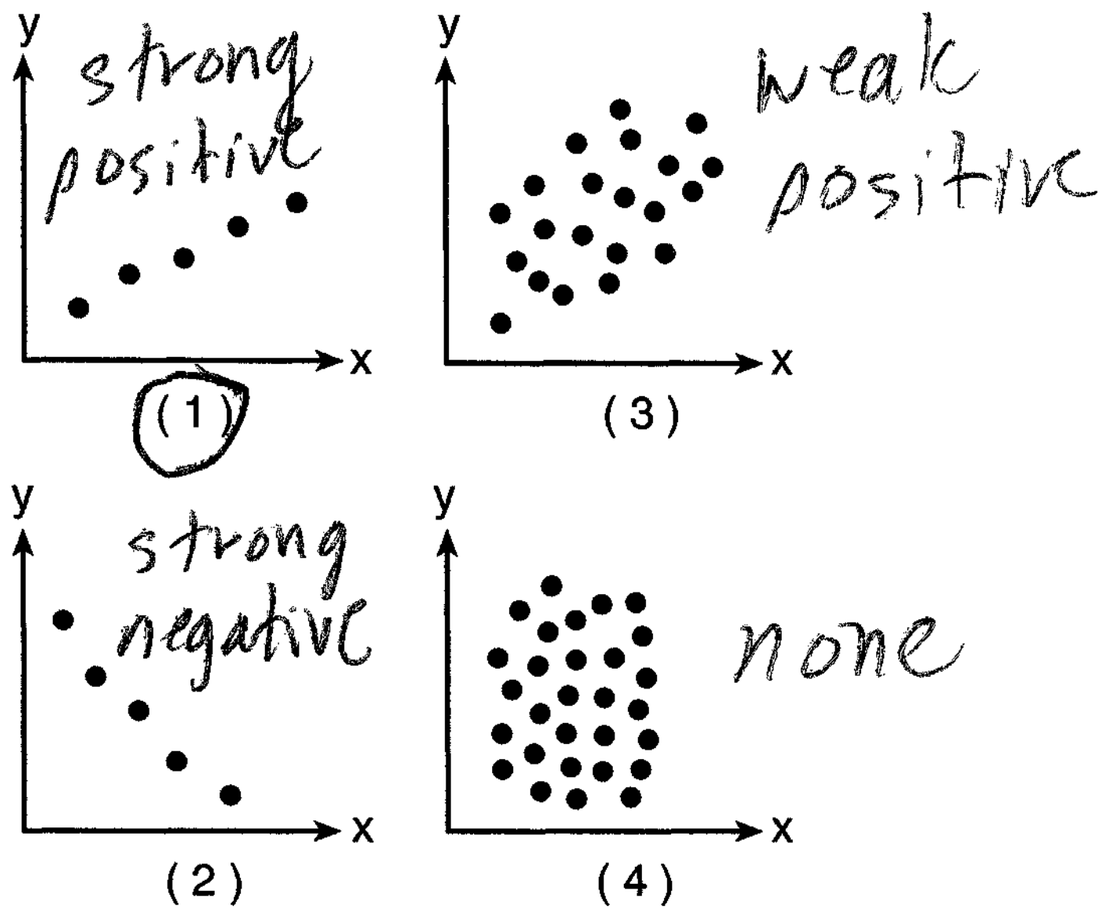
$b^2 - 4ac$
$(-8)^2 - 4(2)(-4)$
$64 + 32 = 96$

Use this space for computations.

14 What is the equation of a circle with center (–3,1) and radius 7?

(1) $(x - 3)^2 + (y + 1)^2 = 7$
(2) $(x - 3)^2 + (y + 1)^2 = 49$
(3) $(x + 3)^2 + (y - 1)^2 = 7$
(4) $(x + 3)^2 + (y - 1)^2 = 49$

15 Which scatter diagram shows the strongest positive correlation?



16 The expression $\frac{7}{3 - \sqrt{2}}$ is equivalent to

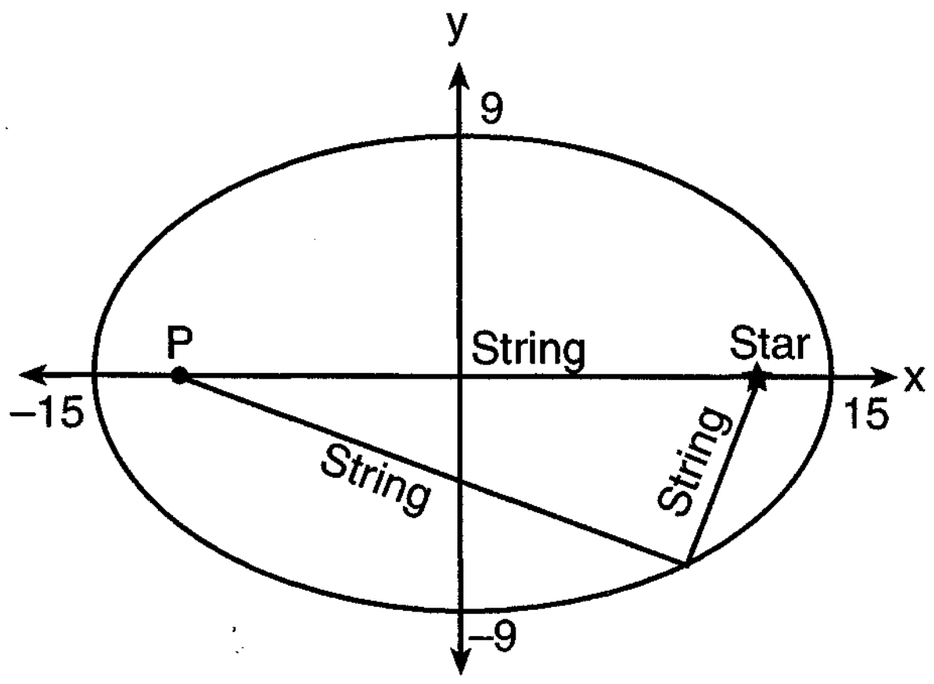
(1) $\frac{3 + \sqrt{2}}{7}$
(2) $\frac{21 + \sqrt{2}}{7}$
(3) $3 + \sqrt{2}$
(4) $3 - \sqrt{2}$

$\frac{7}{3-\sqrt{2}}\left(\frac{3+\sqrt{2}}{3+\sqrt{2}}\right) = \frac{7(3+\sqrt{2})}{9-2}$

$\frac{7(3+\sqrt{2})}{7}$

17 The accompanying diagram shows the construction of a model of an elliptical orbit of a planet traveling around a star. Point P and the center of the star represent the foci of the orbit.

Use this space for computations.



Which equation could represent the relation shown?

(1) $\frac{x^2}{81} + \frac{y^2}{225} = 1$

(3) $\frac{x^2}{15} + \frac{y^2}{9} = 1$

(2) $\frac{x^2}{225} + \frac{y^2}{81} = 1$

(4) $\frac{x^2}{15} - \frac{y^2}{9} = 1$

$\pm 15 \quad \pm 9$

18 The expression $\frac{i^{16}}{i^3}$ is equivalent to

(1) 1

(2) –1

(3) i

(4) $-i$

$\frac{i^{16}}{i^3} = i^{13} = i$ $\quad \frac{13}{4} = 3$ rem 1

19 If $\log_5 x = 2$, what is a value of $\sqrt{x}$?

(1) $2^{\frac{2}{5}}$

(2) $\sqrt{5}$

(3) 5

(4) 25

$\log_5 x = 2$

$5^2 = x$

$25 = x$

$5 = \sqrt{x}$

20 If the coordinates of point A are (–2,3), what is the image of A under $r_{y\text{–axis}} \circ D_3$?

(1) (–6,–9)

(2) (9,–6)

(3) (5,6)

(4) (6,9)

After the dilation → (-6, 9)

After the reflection → (6, 9)

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

21 The accompanying diagram shows the graph of the line whose equation is $y = -\frac{1}{3}x + 2$.

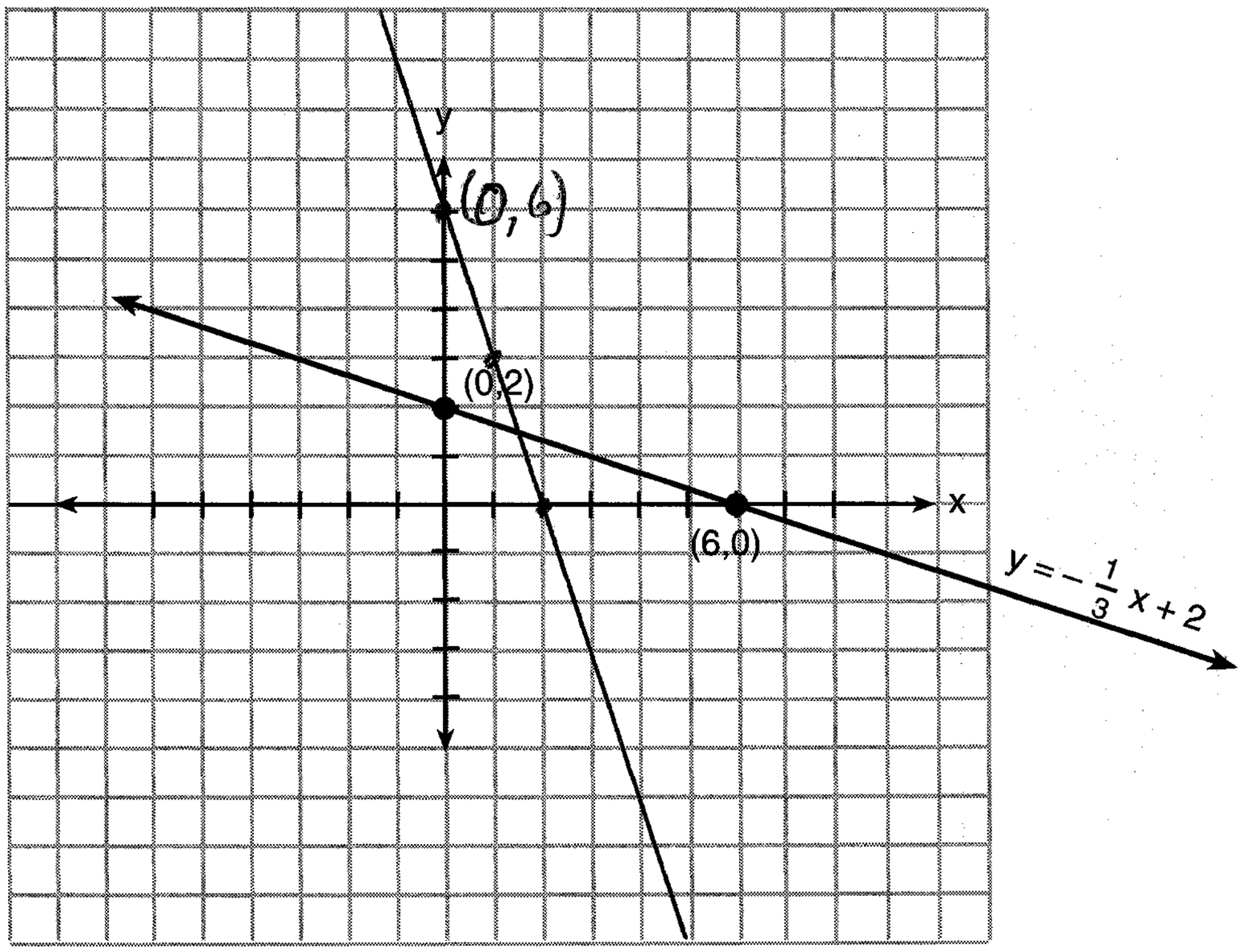
On the same set of axes, sketch the graph of the inverse of this function.

State the coordinates of a point on the inverse function.

$x = -\frac{1}{3}y + 2$

$-3(x - 2 = -\frac{1}{3}y)$

$-3x + 6 = y$



22 If $2 + 3i$ is one root of a quadratic equation with real coefficients, what is the sum of the roots of the equation?

If $2+3i$ is one root,
$2-3i$ is the other root

$2+3i+2-3i = 4$

23 Solve the following equation algebraically for all values of θ in the interval $0° \leq \theta \leq 180°$.

$$2 \sin \theta - 1 = 0$$

$+1 \quad +1$

$\frac{2\sin\theta}{2} = \frac{1}{2}$

$\sin\theta = \frac{1}{2}$

$\theta = 30°$ and $150°$

24 If the probability that it will rain on any given day this week is 60%, find the probability it will rain *exactly* 3 out of 7 days this week.

$n=7$

$r=3$

$p=.6$

$q=.4$

$${}_7C_3\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^4 = \frac{15120}{78125}$$

25 On January 1, 1999, the price of gasoline was \$1.39 per gallon. If the price of gasoline increased by 0.5% per month, what was the cost of one gallon of gasoline, to the *nearest cent*, on January 1 one year later?

$$1.39(1.005)^{12} \approx \$1.48$$

26 An arc of a circle that is 6 centimeters in length intercepts a central angle of 1.5 radians. Find the number of centimeters in the radius of the circle.

$$\theta = \frac{s}{r}$$

$$1.5 = \frac{6}{r}$$

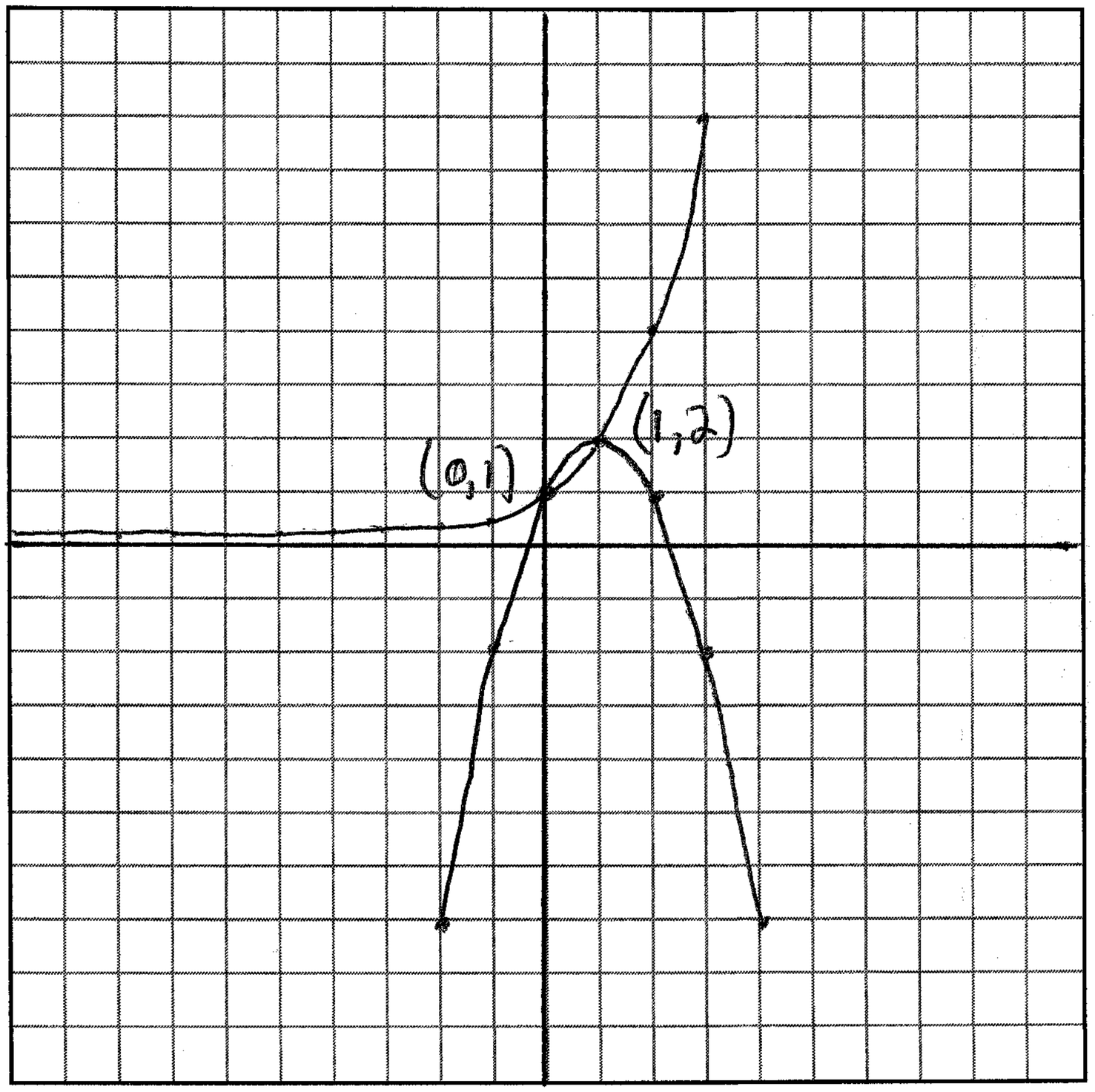
$$r = 4$$

Part III

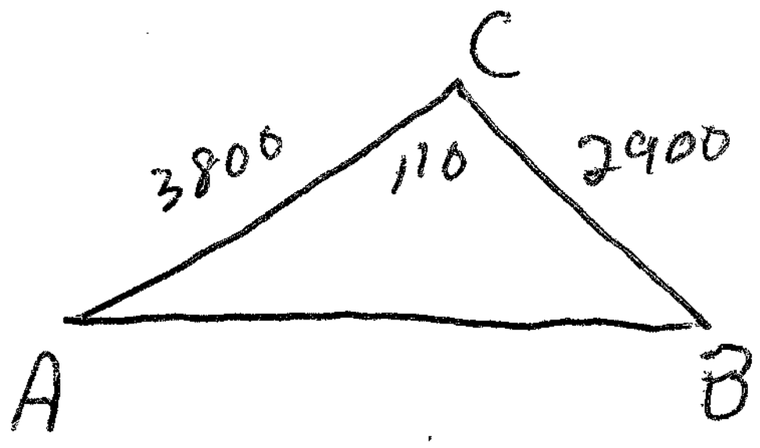
Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [24]

27 On the accompanying grid, solve the following system of equations graphically:

$$y = -x^2 + 2x + 1$$
$$y = 2^x$$



28 To measure the distance through a mountain for a proposed tunnel, surveyors chose points A and B at each end of the proposed tunnel and a point C near the mountain. They determined that $AC = 3{,}800$ meters, $BC = 2{,}900$ meters, and $m\angle ACB = 110$. Draw a diagram to illustrate this situation and find the length of the tunnel, to the *nearest meter*.



$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 2900^2 + 3800^2 - 2(2900)(3800)\cos 110$

$c \approx 5{,}513$

29 From 1984 to 1995, the winning scores for a golf tournament were 276, 279, 279, 277, 278, 278, 280, 282, 285, 272, 279, and 278. Using the standard deviation for the sample, S_x, find the percent of these winning scores that fall within one standard deviation of the mean.

$S_x \approx 3.1 \quad \bar{x} \approx 278.6$

$275.6 < x < 281.7$

$9/12 = 75\%$

30 A real estate agent plans to compare the price of a cottage, y, in a town on the seashore to the number of blocks, x, the cottage is from the beach. The accompanying table shows a random sample of sales and location data.

Write a linear regression equation that relates the price of a cottage to its distance from the beach. $y = -34739.71292x + 313309.0909$

Use the equation to predict the price of a cottage, to the *nearest dollar*, located three blocks from the beach.

Number of Blocks from the Beach (x)	**Price of a Cottage** (y)
5	$132,000
0	$310,000
4	$204,000
2	$238,000
1	$275,000
7	$60,800

$y = -34739.71292(3) + 313309.0909$

$\approx 209,090$

31 The heights, h, of the students in the chorus at Central Middle School satisfy the inequality $\left|\frac{h-57.5}{2}\right| \leq 3.25$, when h is measured *in inches*. Determine the interval in which these heights lie and express your answer to the *nearest tenth of a foot*. [Only an algebraic solution can receive full credit.]

$\frac{h-57.5}{2} \leq 3.25$

$h-57.5 \leq 6.5$

$h \leq 64$

$64 \text{ in.}\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) \approx 5.3 \text{ ft}$

$\frac{h-57.5}{2} \geq -3.25$

$h-57.5 \geq -6.5$

$h \geq 51$

$51 \text{ in}\left(\frac{1 \text{ ft}}{12 \text{ in}}\right) \approx 4.3 \text{ ft}$

$4.3 \leq h \leq 5.3$

32 The number of people, y, involved in recycling in a community is modeled by the function $y = 90\sqrt{3x} + 400$, where x is the number of months the recycling plant has been open.

Construct a table of values, sketch the function on the grid on the next page, and find the number of people involved in recycling exactly 3 months after the plant opened.

After how many months will 940 people be involved in recycling?

$90\sqrt{3x} + 400 = 940$

$-400 \quad -400$

$\frac{90\sqrt{3x}}{90} = \frac{540}{90}$

$\sqrt{3x} = 6$

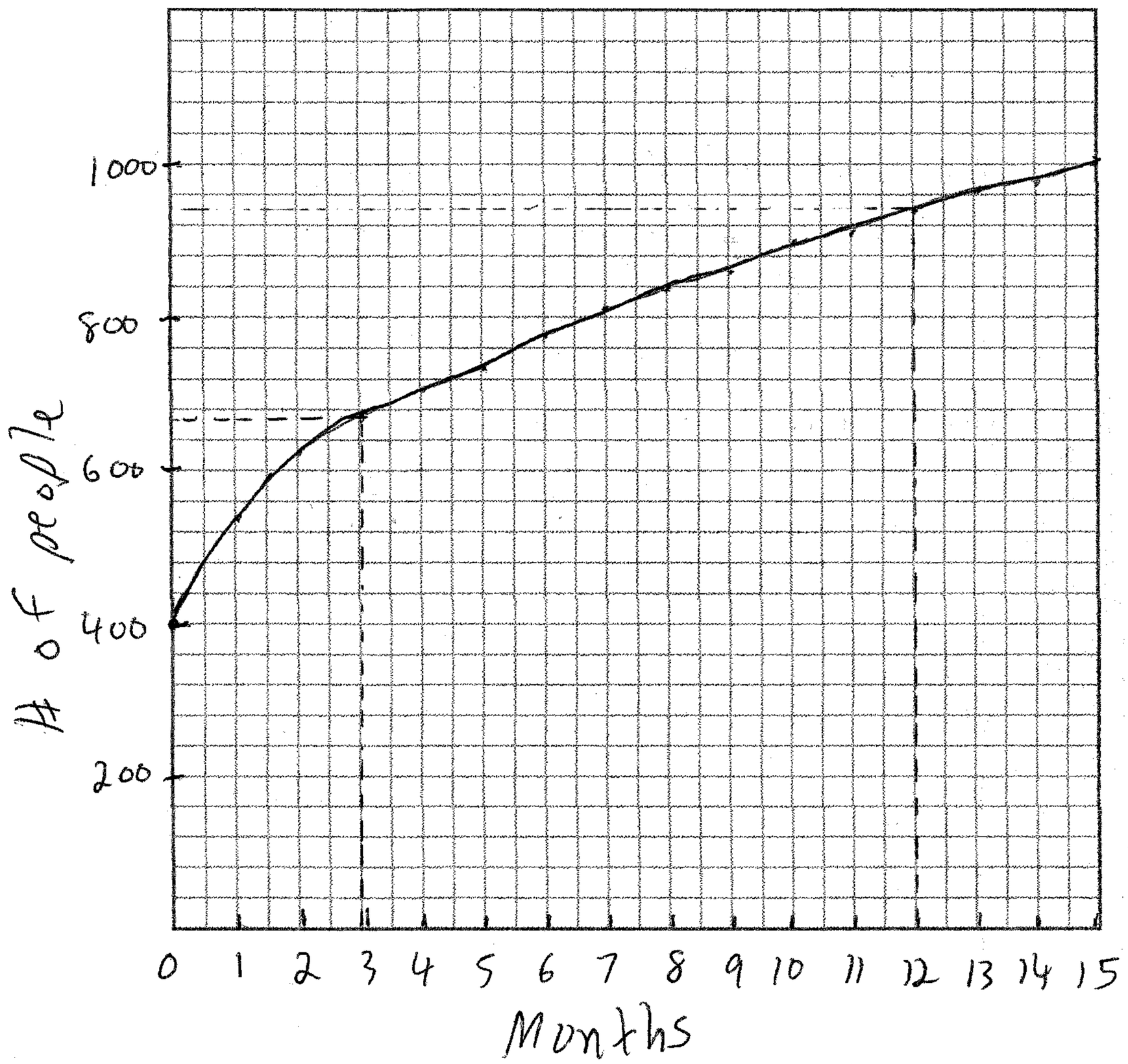
$\frac{3x}{3} = \frac{36}{3}$

$x = 12$ months

Question 32 continued

x	y
0	460
3	670 ←
12	940
27	1210

Recycling



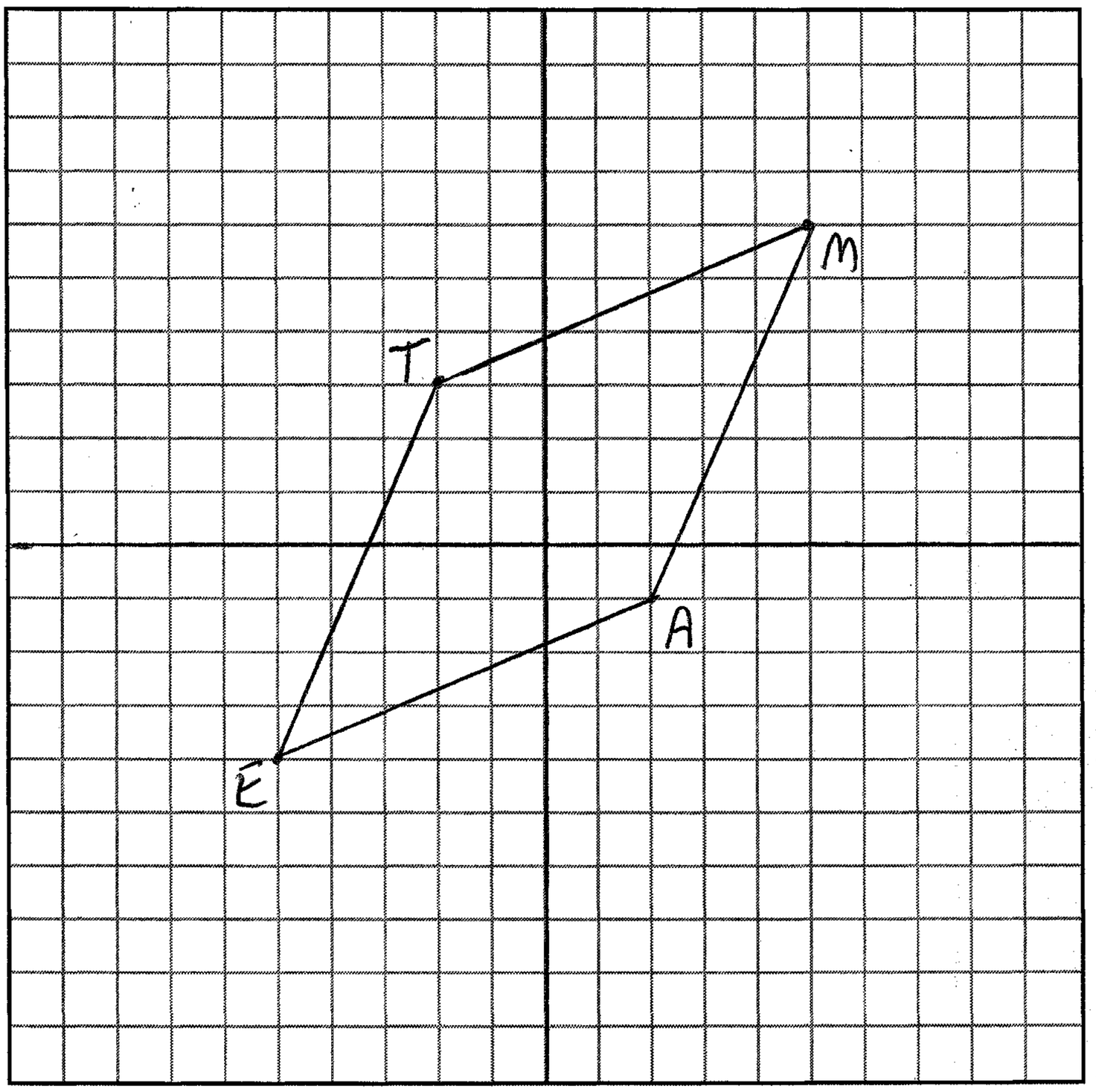
Part IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

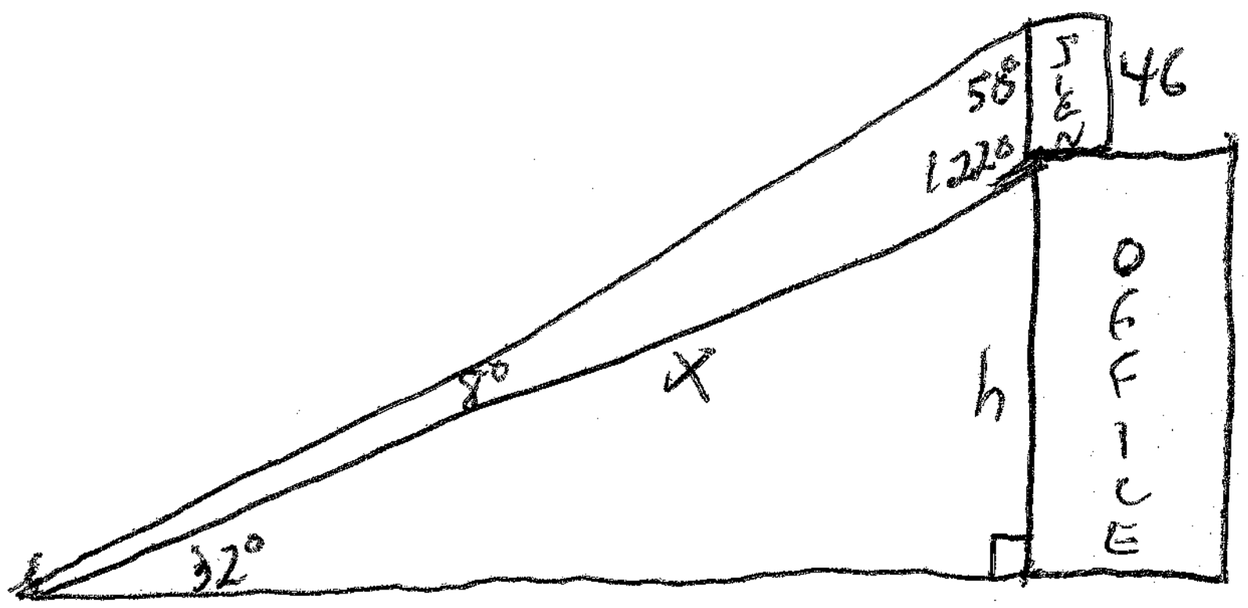
33 Jim is experimenting with a new drawing program on his computer. He created quadrilateral *TEAM* with coordinates $T(-2,3)$, $E(-5,-4)$, $A(2,-1)$, and $M(5,6)$. Jim believes that he has created a rhombus but not a square. Prove that Jim is correct. [The use of the grid on the next page is optional.]

STATEMENT	REASON
(1) Quadrilateral TEAM with coordinates $T(-2,3)$, $E(-5,-4)$, $A(2,-1)$ and $M(5,6)$	(1) Given
(2) $d_{\overline{ET}} = \sqrt{(-2-(-5))^2+(3-(-4))^2} = \sqrt{58}$ $d_{\overline{AM}} = \sqrt{(2-5)^2+(-1-6)^2} = \sqrt{58}$ $d_{\overline{AE}} = \sqrt{(-5-2)^2+(-4-(-1))^2} = \sqrt{58}$ $d_{\overline{MT}} = \sqrt{(-2-5)^2+(3-6)^2} = \sqrt{58}$	(2) Distance Formula
(3) Quadrilateral TEAM is a rhombus.	(3) A rhombus has four equal sides
(4) $m_{\overline{ET}} = \frac{-4-3}{-5-(-2)} = \frac{7}{3}$ $m_{\overline{AM}} = \frac{6-(-1)}{5-2} = \frac{7}{3}$ $m_{\overline{AE}} = \frac{-4-(-1)}{-5-2} = \frac{3}{7}$ $m_{\overline{MT}} = \frac{3-6}{-2-5} = \frac{3}{7}$	(4) Slope Formula
(5) $\overline{ET} \not\perp \overline{TM}$, $\overline{AE} \not\perp \overline{ET}$, $\overline{AM} \not\perp \overline{MT}$ and $\overline{AE} \perp \overline{AM}$	(5) If the slopes of 2 lines are not negative reciprocals, the lines are not perpendicular
(6) Quadrilateral TEAM is not a square.	(6) Consecutive sides of a square are perpendicular.

Question 33 continued



34 A sign 46 feet high is placed on top of an office building. From a point on the sidewalk level with the base of the building, the angle of elevation to the top of the sign and the angle of elevation to the bottom of the sign are 40° and 32°, respectively. Sketch a diagram to represent the building, the sign, and the two angles, and find the height of the building to the *nearest foot*.



$\frac{x}{\sin 50} = \frac{46}{\sin 8}$

$x = \frac{46 \sin 50}{\sin 8}$

$x \approx 253.2$

$\sin 32 \approx \frac{h}{253.2}$

$h \approx 134$