The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

# MATHEMATICS B

**Thursday,** January 29, 2004 — 9:15 a.m. to 12:15 p.m., only

**Print Your Name:** Steven Sibol

**Print Your School's Name:** HSCR

**Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.**

**Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will *not* be scored. Write all your work in pen, except graphs and drawings, which should be done in pencil.**

**This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. The formulas that you may need to answer some questions in this examination are found on page 19.**

***When you have completed the examination, you must sign the statement printed* at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.**

**Notice. . .**

**A graphing calculator, a straightedge (ruler), and a compass must be available for your use while taking this examination.**

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

## Part I

**Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question.** [40]

**1** The expression $\cos 40° \cos 10° + \sin 40° \sin 10°$ is equivalent to

(1) $\cos 30°$
(2) $\cos 50°$
(3) $\sin 30°$
(4) $\sin 50°$

**Use this space for computations.**

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\cos(40-10) = \cos 40 \cos 10 + \sin 40 \sin 10$

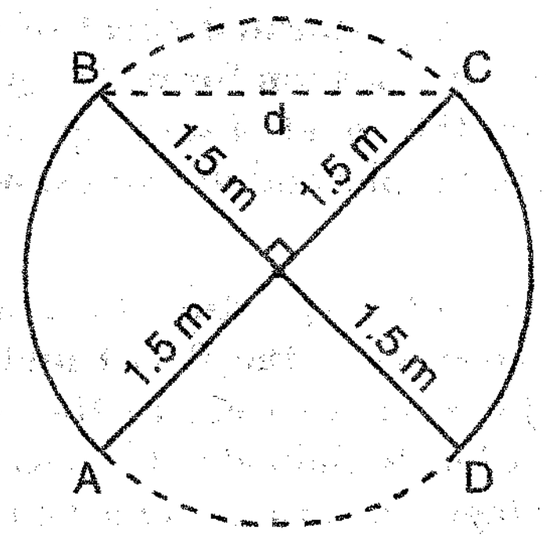
$\cos 30$

**2** The expression $\frac{\sec \theta}{\csc \theta}$ is equivalent to

(1) $\sin \theta$
(2) $\cos \theta$
(3) $\frac{\sin \theta}{\cos \theta}$
(4) $\frac{\cos \theta}{\sin \theta}$

$\frac{\sec \theta}{\csc \theta} = \frac{1/\cos \theta}{1/\sin \theta} = \frac{\sin \theta}{\cos \theta}$

**3** An overhead view of a revolving door is shown in the accompanying diagram. Each panel is 1.5 meters wide.




Use Pythagoras

$a^2 + b^2 = c^2$

$1.5^2 + 1.5^2 = c^2$

$4.5 = c^2$

$2.12 \approx c$

What is the approximate width of $d$, the opening from $B$ to $C$?

(1) 1.50 m
(2) 1.73 m
(3) 3.00 m
(4) 2.12 m

**4** What is a positive value of $x$ for which $9^{-\cos x} = \frac{1}{3}$?

(1) 30°
(2) 45°
(3) 60°
(4) 90°

$9^{-\cos x} = \frac{1}{3}$

$9^{-\cos x} = 9^{-\frac{1}{2}}$

$-\cos x = -\frac{1}{2}$

$\cos x = \frac{1}{2}$

$x = 60°$

**5** The expression $\frac{7}{2-\sqrt{3}}$ is equivalent to

(1) $14-7\sqrt{3}$

(2) $14+7\sqrt{3}$

(3) $\frac{2+\sqrt{3}}{7}$

(4) $\frac{14+\sqrt{3}}{7}$

$\frac{7}{2-\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{14+7\sqrt{3}}{4+2\sqrt{3}-2\sqrt{3}-1}$

$= 14+7\sqrt{3}$

**Use this space for computations.**

**6** Jean's scores on five mathematics tests were 98, 97, 99, 98, and 96. Her scores on five English tests were 78, 84, 95, 72, and 79. Which statement is true about the standard deviations for the scores?

(1) The standard deviation for the English scores is greater than the standard deviation for the math scores.

(2) The standard deviation for the math scores is greater than the standard deviation for the English scores.

(3) The standard deviations for both sets of scores are equal.

(4) More information is needed to determine the relationship between the standard deviations.

**7** In $\triangle ABC$, $a = 19$, $c = 10$, and $m\angle A = 111$. Which statement can be used to find the value of $\angle C$?

(1) $\sin C = \frac{10}{19}$

(2) $\sin C = \frac{19 \sin 69°}{10}$

(3) $\sin C = \frac{10 \sin 21°}{19}$

(4) $\sin C = \frac{10 \sin 69°}{19}$

$\sin 111 = \sin 69$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{19}{\sin 111} = \frac{10}{\sin C}$

$\sin C = \frac{10 \sin 111}{19}$

**8** If $f(x) = \frac{2}{x+3}$ and $g(x) = \frac{1}{x}$, then $(g \circ f)(x)$ is equal to

(1) $\frac{1+3x}{2x}$

(2) $\frac{2x}{1+3x}$

(3) $\frac{x+3}{2}$

(4) $\frac{x+3}{2x}$

$\frac{1}{\frac{2}{x+3}} = \frac{x+3}{2}$

**9** If $\log x = a$, $\log y = b$, and $\log z = c$, then $\log \frac{x^2y}{\sqrt{z}}$ is equivalent to

(1) $42a + b + \frac{1}{2}c$

(2) $2ab - \frac{1}{2}c$

(3) $a^2 + b - \frac{1}{2}c$

(4) $2a + b - \frac{1}{2}c$

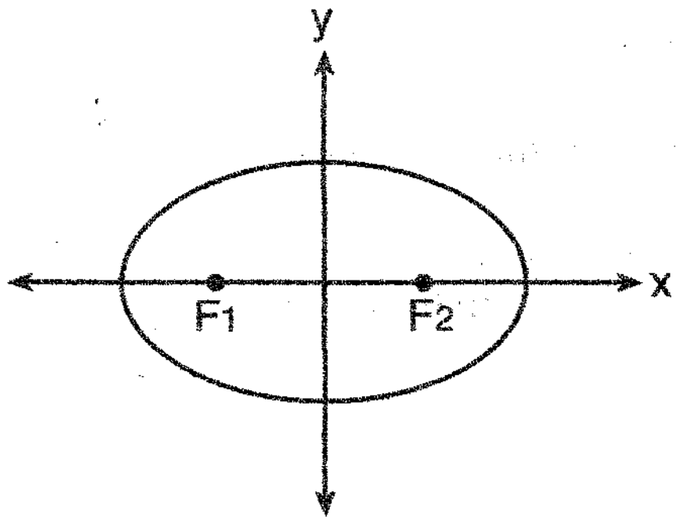
**Use this space for computations.**

$\log \frac{x^2y}{\sqrt{z}} = \log x^2 + \log y - \log z^{1/2}$

$= 2\log x + \log y - \frac{1}{2}\log z$

$= 2a + b - \frac{1}{2}c$

**10** The accompanying diagram shows the elliptical orbit of a planet. The foci of the elliptical orbit are $F_1$ and $F_2$.




If $a$, $b$, and $c$ are all positive and $a \neq b \neq c$, which equation could represent the path of the planet?

(1) $ax^2 - by^2 = c^2$ hyperbola

(2) $ax^2 + by^2 = c^2$

(3) $y = ax^2 + c^2$ parabola

(4) $x^2 + y^2 = c^2$ circle

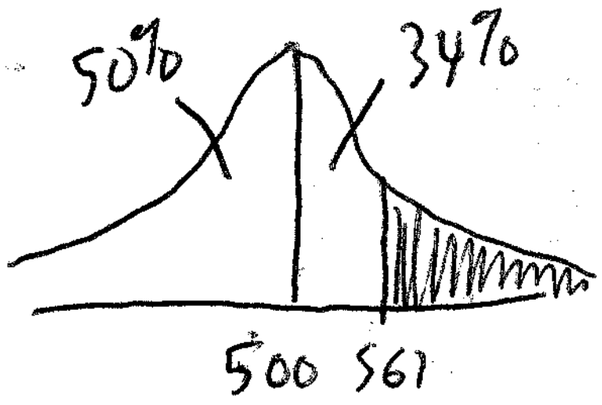
**11** Battery lifetime is normally distributed for large samples. The mean lifetime is 500 days and the standard deviation is 61 days. Approximately what percent of batteries have lifetimes *longer than* 561 days?

(1) 16%

(2) 34%

(3) 68%

(4) 84%




$100 - (50 + 34) = 16$

**12** The expression $\log_3 (8 - x)$ is defined for all values of $x$ such that

(1) $x > 8$

(2) $x \geq 8$

(3) $x < 8$

(4) $x \leq 8$

If $x \geq 8$, then the expression would be the log of zero or a negative number

**13** The expression $b^{-\frac{3}{2}}$, $b > 0$, is equivalent to

(1) $\frac{1}{(\sqrt[3]{b})^2}$

(2) $\frac{1}{(\sqrt{b})^3}$

(3) $-(\sqrt{b})^3$

(4) $(\sqrt[3]{b})^2$

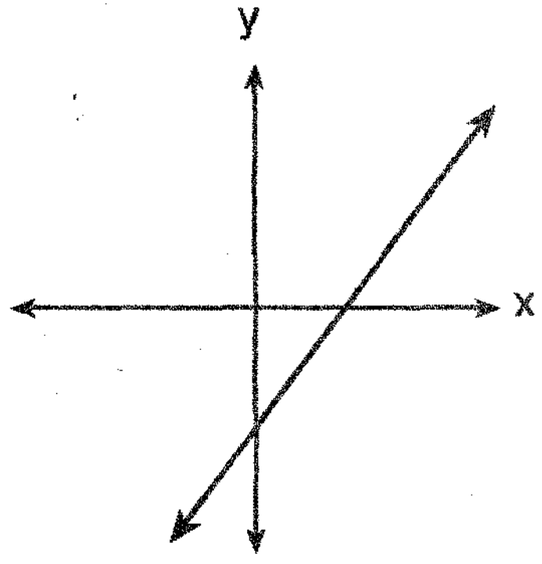
**Use this space for computations.**

$b^{-3/2}$

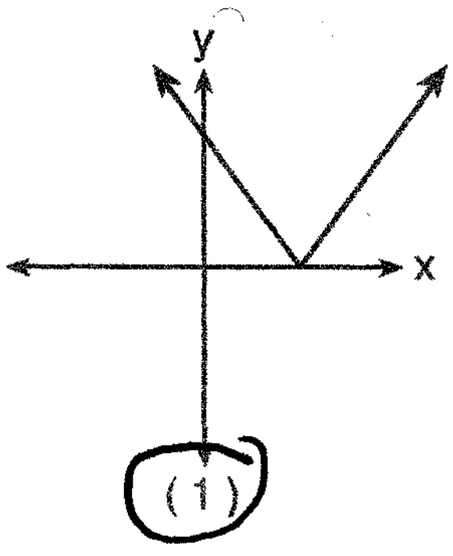
eliminate negative exponent $\Rightarrow \frac{1}{b^{3/2}}$

eliminate Fractional exponent $\Rightarrow \frac{1}{(\sqrt{b})^3}$

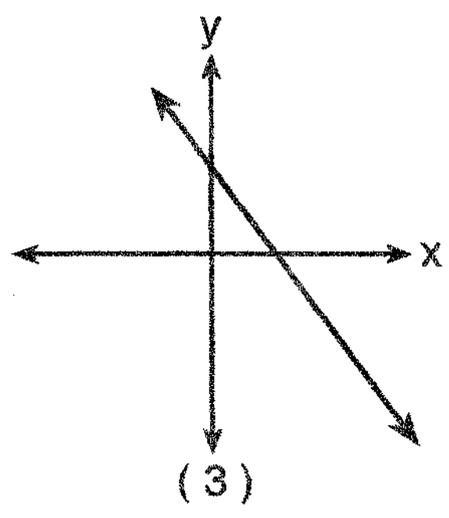
**14** The graph below represents f($x$).



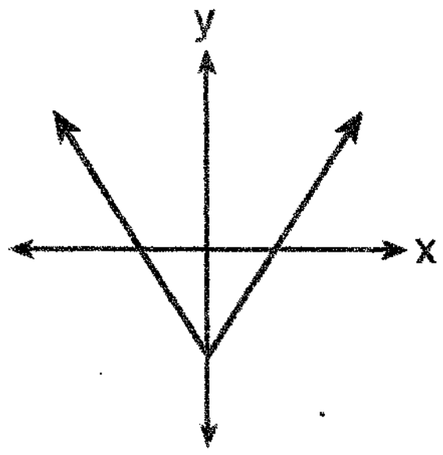

Which graph best represents $|f(x)|$?



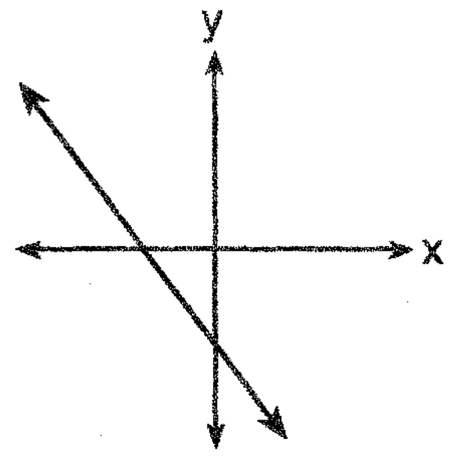

(1)




(3)




(2)




(4)

**15** If $f(x) = x^3 - 2x^2$, then $f(i)$ is equivalent to

(1) $-2 + i$
(2) $-2 - i$
(3) $2 + i$
(4) $2 - i$

$F(i) = i^3 - 2i^2$
$= -i - 2(-1)$
$= 2 - i$

**Use this space for computations.**

**16** Which statement must be true if a parabola represented by the equation $y = ax^2 + bx + c$ does *not* intersect the $x$-axis?

(1) $b^2 - 4ac = 0$
(2) $b^2 - 4ac < 0$
(3) $b^2 - 4ac > 0$, and $b^2 - 4ac$ is a perfect square.
(4) $b^2 - 4ac > 0$, and $b^2 - 4ac$ is not a perfect square.

**17** A garden in the shape of an equilateral triangle has sides whose lengths are 10 meters. What is the area of the garden?

(1) $25\ m^2$
(2) $25\sqrt{3}\ m^2$
(3) $50\ m^2$
(4) $50\sqrt{3}\ m^2$

$A = \frac{1}{2}ab\sin C$
$= \frac{1}{2}(10)(10)\sin 60$
$= 50\frac{\sqrt{3}}{2}$
$= 25\sqrt{3}$

**18** If $x$ is an acute angle and $\sin x = \frac{12}{13}$, then $\cos 2x$ equals

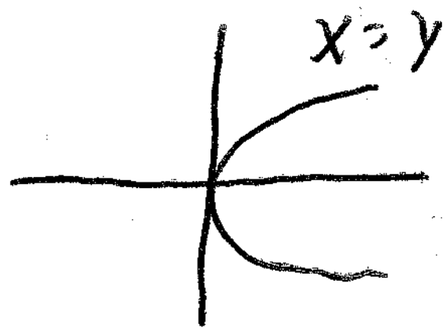
(1) $\frac{25}{169}$
(2) $\frac{119}{169}$
(3) $-\frac{25}{169}$
(4) $-\frac{119}{169}$

$\cos 2x = 1 - 2\sin^2 x$
$= 1 - 2\left(\frac{12}{13}\right)^2$
$= 1 - 2\left(\frac{144}{169}\right)$
$= 1 - \frac{288}{169}$
$= \frac{169}{169} - \frac{288}{169}$
$= \frac{-119}{169}$

**19** What is the axis of symmetry of the graph of the equation $x = y^2$?

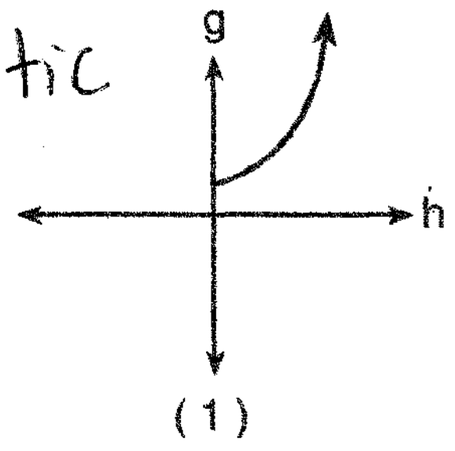
(1) $x$-axis
(2) $y$-axis
(3) line $y = x$
(4) line $y = -x$

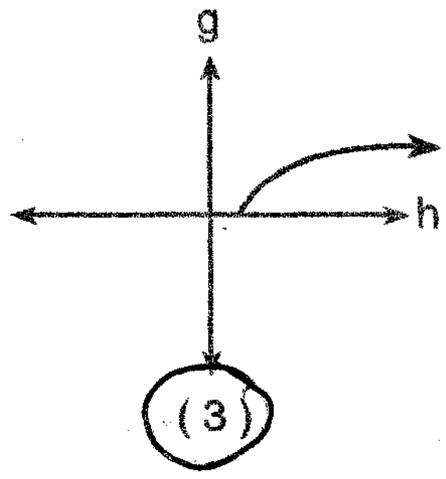
**Use this space for computations.**



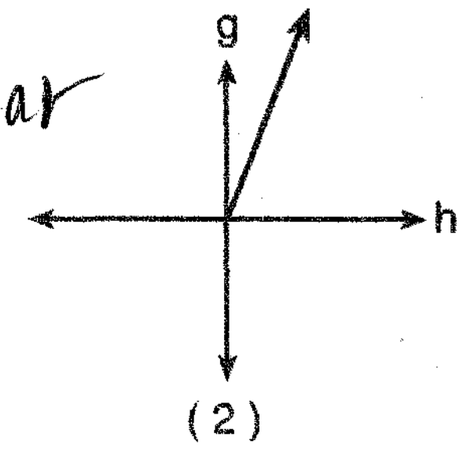

**20** The cells of a particular organism increase logarithmically. If $g$ represents cell growth and $h$ represents time, in hours, which graph best represents the growth pattern of the cells of this organism?

quadratic

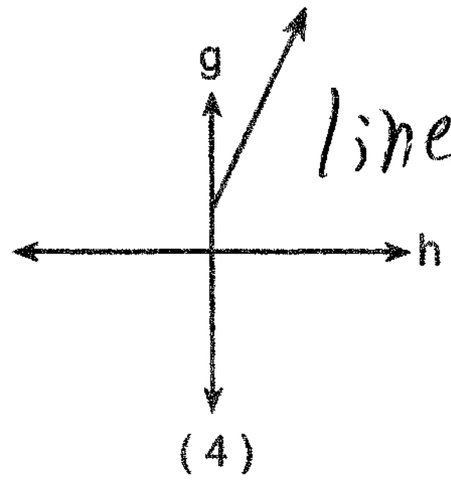



linear



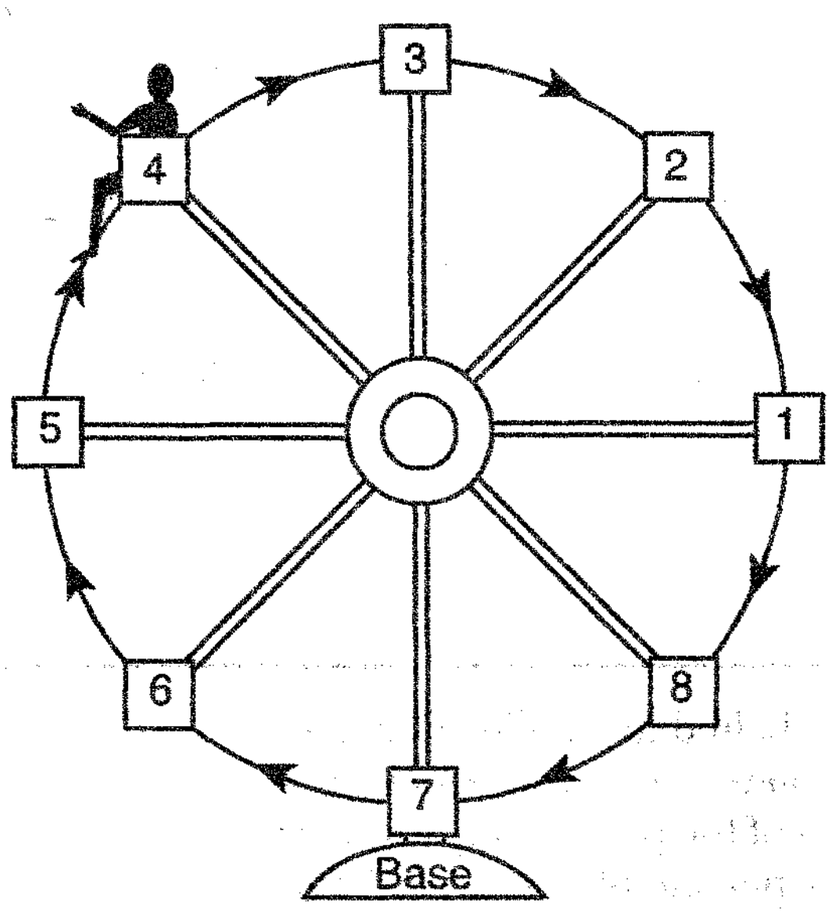

linear



## Part II

**Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.** [12]

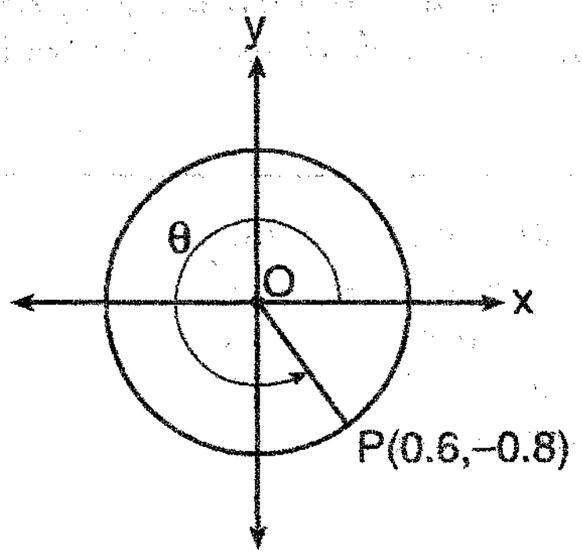
**21** Kristine is riding in car 4 of the Ferris wheel represented in the accompanying diagram. The Ferris wheel is rotating in the direction indicated by the arrows. The eight cars are equally spaced around the circular wheel. Express, in radians, the measure of the *smallest* angle through which she will travel to reach the bottom of the Ferris wheel.




An entire rotation would be $2\pi$ radians.

$2\pi \cdot \frac{5}{8} = \frac{5\pi}{4}$ radians

**22** In the accompanying diagram, point $P(0.6,-0.8)$ is on unit circle $O$.
What is the value of $\theta$, to the *nearest degree*?




$\cos\theta = .6$

$\theta = \cos^{-1}.6$

$= 53°$ and $307°$

$\sin\theta = -.8$

$\theta = -53°$

$\theta = 307°$

**23** A pulley that has a diameter of 8 inches is belted to a pulley that has a diameter of 12 inches. The 8-inch-diameter pulley is running at 1,548 revolutions per minute. If the speeds of the pulleys vary inversely to their diameters, how many revolutions per minute does the larger pulley make?

$s_1 d_1 = s_2 d_2$

$\frac{(1548)(8)}{12} = \frac{(s_2)12}{12}$

$1032 = s_2$

**24** When a current, $I$, flows through a given electrical circuit, the power, $W$, of the circuit can be determined by the formula $W = 120I - 12I^2$. What amount of current, $I$, supplies the maximum power, $W$?

Maximum power occurs at the vertex, which can be found using the axis of symmetry.

$$x = \frac{-B}{2A} = \frac{-120}{2(-12)} = 5$$

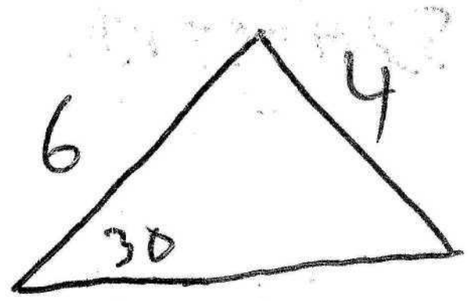
**25** The brightness of the star MIRA over time is given by the equation $y = 2 \sin \frac{\pi}{4}x + 6$, where $x$ represents time and $y$ represents brightness. What is the period of this function, in radian measure?

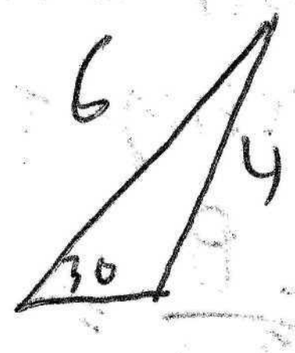
$$\text{period} = \frac{2\pi}{b}$$

$$= \frac{2\pi}{\frac{\pi}{4}}$$

$$= 8 \text{ radians}$$

**26** A landscape designer is designing a triangular garden with two sides that are 4 feet and 6 feet, respectively. The angle opposite the 4-foot side is 30°. How many distinct triangular gardens can the designer make using these measurements?




$\frac{4}{\sin 30} = \frac{6}{\sin C}$

$C \approx 49°$

$30° + 49°$ is possible because less than $180°$

$180 - 49$

$C \approx 131$

$30° + 131°$ is possible because less than $180°$

Two triangles are possible

## Part III

**Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.** [24]

**27** Solve algebraically: $\sqrt{x+5}+1=x$

$\sqrt{x+5} = x-1$

$x+5 = (x-1)^2$

$x+5 = x^2-2x+1$

$x^2-3x-4=0$

$(x-4)(x+1)=0$

$x=4$ ~~$x=-1$~~

CHECK

$\sqrt{4+5}+1=4$

$3+1=4$

$\sqrt{-1+5}+1=-1$

$\sqrt{4}+1=-1$

$3 \neq -1$ extraneous

**28** A board game has a spinner on a circle that has five equal sectors, numbered 1, 2, 3, 4, and 5, respectively. If a player has four spins, find the probability that the player spins an even number *no more than* two times on those four spins.

$n=4$

$r=0,1,2$

$p=\frac{2}{5}$

$q=\frac{3}{5}$

${}_4C_0\left(\frac{2}{5}\right)^0\left(\frac{3}{5}\right)^4 = 1\cdot 1\cdot \frac{81}{625} = \frac{81}{625}$

${}_4C_1\left(\frac{2}{5}\right)^1\left(\frac{3}{5}\right)^3 = 4\cdot \frac{2}{5}\cdot \frac{27}{125} = \frac{216}{625}$

${}_4C_2\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^2 = 6\cdot \frac{4}{25}\cdot \frac{9}{25} = \frac{216}{625}$

$\frac{513}{625}$

**29** The equation for radioactive decay is $p = (0.5)^{\frac{t}{H}}$, where $p$ is the part of a substance with half-life $H$ remaining radioactive after a period of time, $t$.

A given substance has a half-life of 6,000 years. After $t$ years, one-fifth of the original sample remains radioactive. Find $t$, to the *nearest thousand years*.

one-fifth = .2

$p = (0.5)^{\frac{t}{H}}$

$.2 = (.5)^{\frac{t}{6000}}$

$\log .2 = \log (.5)^{\frac{t}{6000}}$

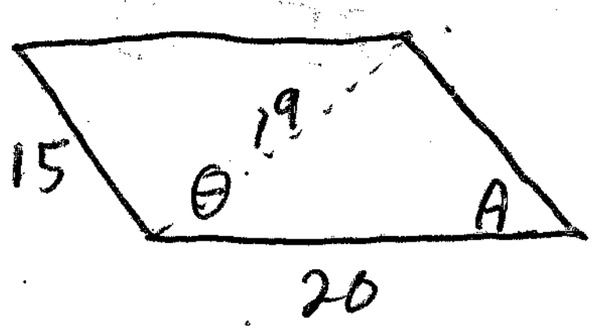
$\log .2 = \frac{t}{6000} \log .5$

$\frac{6000 \log .2}{\log .5} = t$

$14{,}000 \approx t$

**30** One force of 20 pounds and one force of 15 pounds act on a body at the same point so that the resultant force is 19 pounds. Find, to the *nearest degree*, the angle between the two original forces.

Because the resultant is not greater than both forces, the angle between the forces is obtuse.




$a^2 = b^2 + c^2 - 2bc \cos A$

$19^2 = 15^2 + 20^2 - 2(15)(20) \cos A$

$361 = 625 - 600 \cos A$

$-264 = -600 \cos A$

$\frac{-264}{-600} = \cos A$

$64^\circ = A$

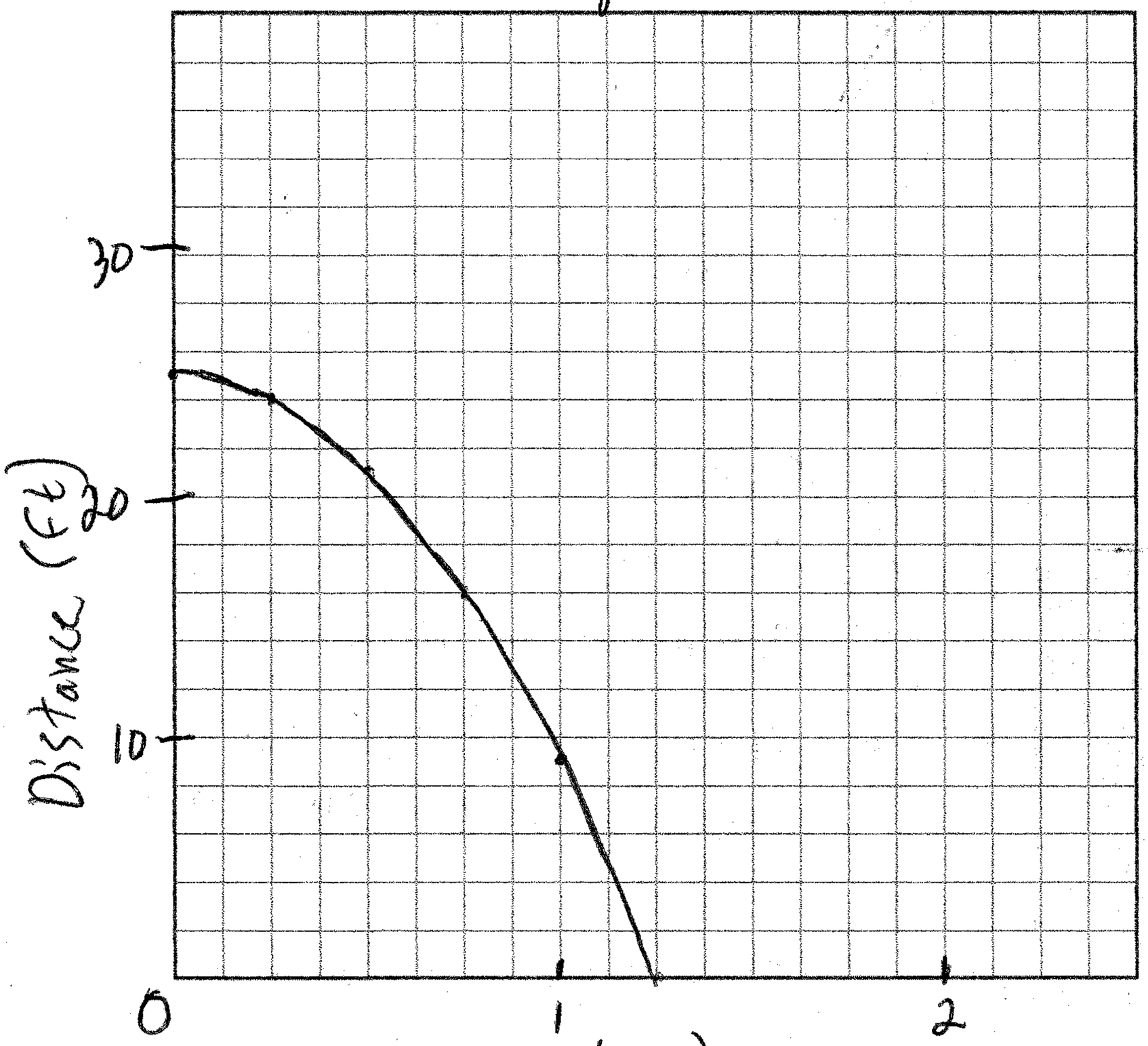
Because $\theta$ is supplementary

$\theta = 180 - 64 = 116^\circ$

**31** An acorn falls from the branch of a tree to the ground 25 feet below. The distance, $S$, the acorn is from the ground as it falls is represented by the equation $S(t) = -16t^2 + 25$, where $t$ represents time, in seconds. Sketch a graph of this situation on the accompanying grid.

Calculate, to the *nearest hundredth of a second*, the time the acorn will take to reach the ground.

A Falling Acorn




$-16t^2 + 25 = 0$

$(-4t+5)(4t+5) = 0$

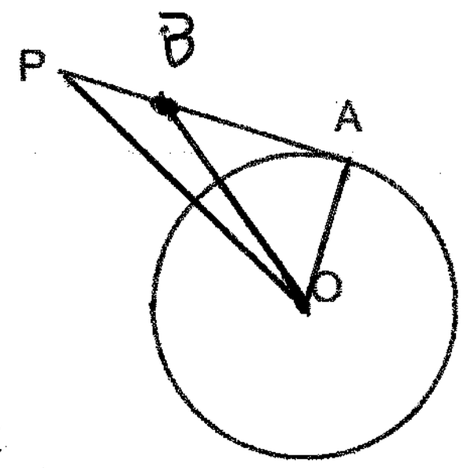
$-4t+5=0$

$t = 1.25$

$4t+5=0$

$t$ is negative

**32** In the accompanying diagram of circle $O$, $\overline{PA}$ is drawn tangent to the circle at $A$. Place $B$ on $\overline{PA}$ anywhere between $P$ and $A$ and draw $\overline{OA}$, $\overline{OP}$, and $\overline{OB}$. Prove that $\overline{OB}$ is *not* perpendicular to $\overline{PA}$.




Prove using an indirect proof:

| | Statement | Reason |
|---|---|---|
| (1) | Circle O, tangent PA | (1) Given |
| (2) | Assume $\overline{OB} \perp \overline{PA}$ | (2) Assumed |
| (3) | $m\angle OBA = 90°$ | (3) Definition of perpendicular lines |
| (4) | $m\angle OAB = 90°$ | (4) A tangent drawn to a radius makes a right angle with the radius |
| (5) | $\triangle ABO$ may not have two right angles | (5) The sum of the angles of a triangle equals 180° |
| (6) | Therefore $\overline{OB} \not\perp \overline{PA}$ | (6) By contradiction. |

## Part IV

**Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.** [12]

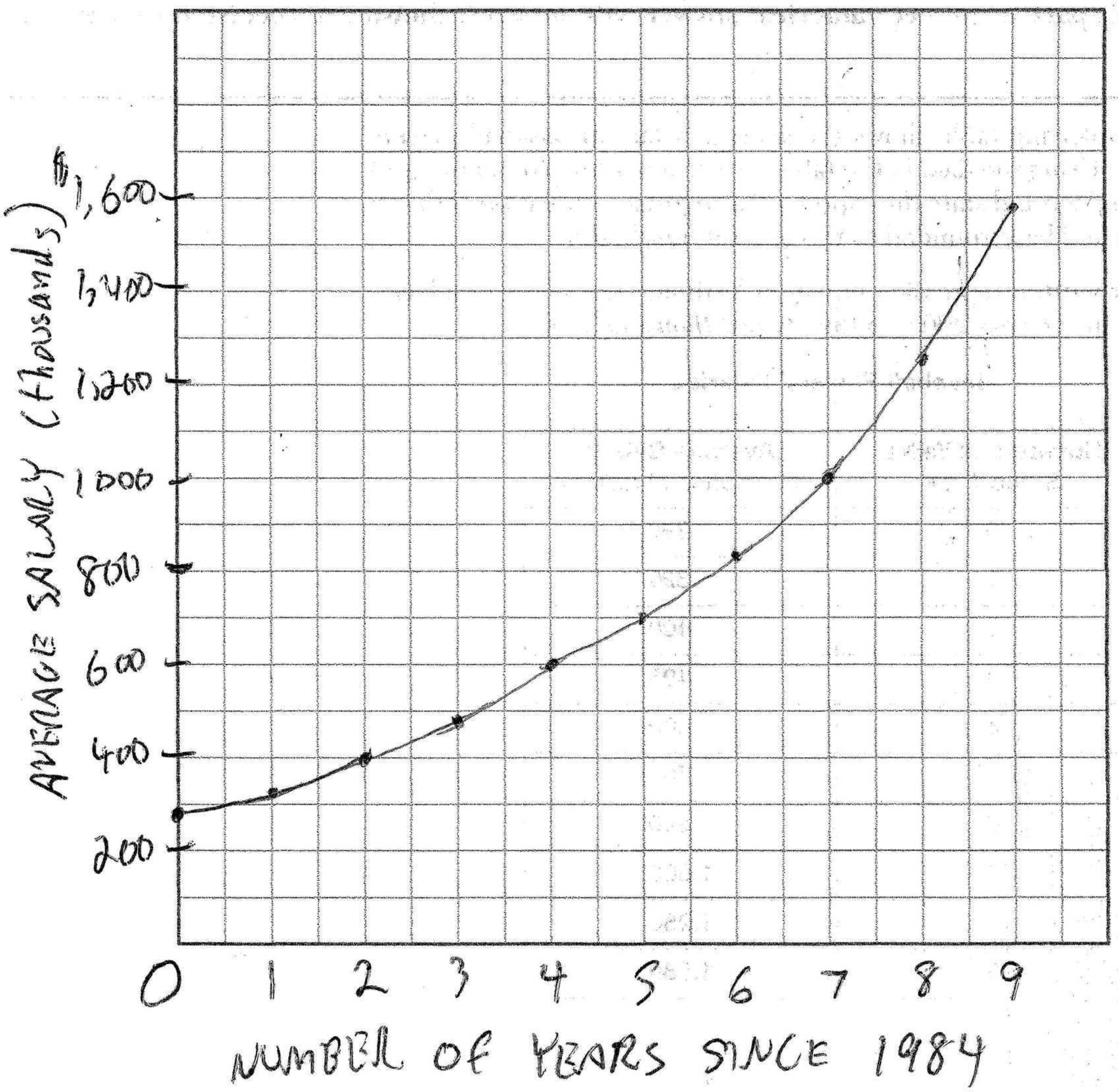
**33** The accompanying table shows the average salary of baseball players since 1984. Using the data in the table, create a scatter plot on the grid *on the next page and state the exponential regression equation with the* coefficient and base rounded to the *nearest hundredth*.

Using your written regression equation, estimate the salary of a baseball player in the year 2005, to the *nearest thousand dollars*.

**Baseball Players' Salaries**

| **Numbers of Years Since 1984** | **Average Salary** (thousands of dollars) |
|---|---|
| 0 | 290 |
| 1 | 320 |
| 2 | 400 |
| 3 | 495 |
| 4 | 600 |
| 5 | 700 |
| 6 | 820 |
| 7 | 1,000 |
| 8 | 1,250 |
| 9 | 1,580 |

**Question 33 continued**




$y = 276.67(1.21)^x$

2005 is 21 years since 1984

$y = 276.67(1.21)^{21}$

$\approx 15,151,000$

**34** Express in simplest form: $\frac{4x+8}{x+1} \cdot \frac{2-x}{3x-15} \div \frac{x^2-4}{2x^2-8x-10}$

$$\frac{4(x+2)}{(x+1)} \cdot \frac{-(x-2)}{3(x-5)} \div \frac{2(x^2-4x-5)}{(x+2)(x-2)}$$

$$\frac{-2(x-5)(x+1)}{3(x+1)(x-5)}$$

$$-\frac{2}{3}$$