The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS B

Thursday, January 30, 2003 — 9:15 a.m. to 12:15 p.m., only

Print Your Name: Steve Sibol

Print Your School's Name: HSCR

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will *not* be scored. All work should be written in pen, except graphs and drawings, which should be done in pencil.

This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. The formulas that you may need to answer some questions in this examination are found on page 2.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice. . .

A graphing calculator, a straightedge (ruler), and a compass must be available for your use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Record your answers in the spaces provided on the separate answer sheet. [40]

1 A monitor displays the graph $y = 3 \sin 5x$. What will be the amplitude after a dilation of 2?

(1) 5   (3) 7

(2) 6   (4) 10

$3 \times 2 = 6$

2 The probability that Kyla will score above a 90 on a mathematics test is $\frac{4}{5}$. What is the probability that she will score above a 90 on three of the four tests this quarter?
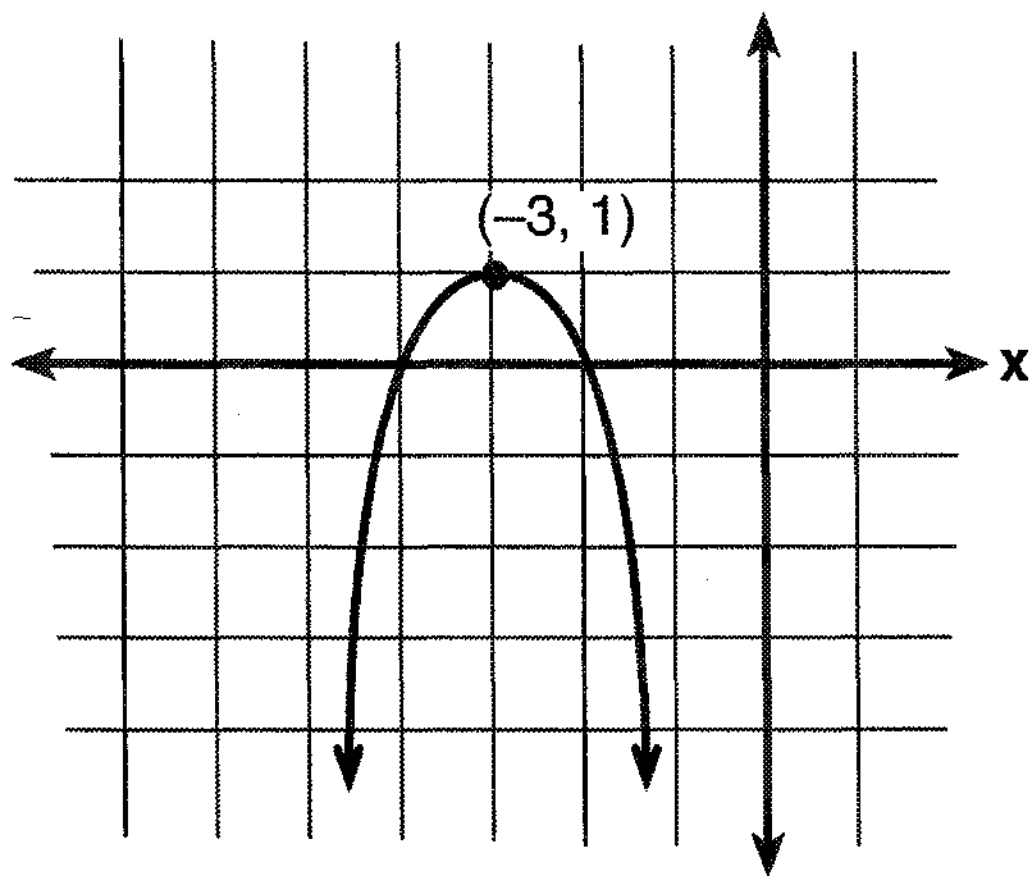
(1) ${}_4C_3 \left(\frac{4}{5}\right)^3 \left(\frac{1}{5}\right)^1$ (3) $\frac{3}{4}\left(\frac{4}{5}\right)^3 \left(\frac{1}{5}\right)^1$

(2) ${}_4C_3 \left(\frac{4}{5}\right)^1 \left(\frac{1}{5}\right)^3$ (4) $\frac{3}{4}\left(\frac{4}{5}\right)^1 \left(\frac{1}{5}\right)^3$

3 Which equation represents the parabola shown in the accompanying graph?

(1) $f(x) = (x + 1)^2 - 3$ (3) $f(x) = -(x + 3)^2 + 1$

(2) $f(x) = -(x - 3)^2 + 1$ (4) $f(x) = -(x - 3)^2 - 3$

4 What is the value of $\sum\limits_{b=0}^{3} \left(2 - (b)i\right)$?

   (1) $2 - 5i$ (3) $8 - 5i$
   (2) $2 - 6i$ (4) $8 - 6i$

(handwritten work:)

b	$2-bi$
0	$2-0i$
1	$2-i$
2	$2-2i$
3	$2-3i$

$8-6i$

5 What is the solution set of the equation $\sqrt{9x + 10} = x$?

   (1) $\{-1\}$ (3) $\{10\}$
   (2) $\{9\}$ (4) $\{10,-1\}$

(handwritten work:)

$9x + 10 = x^2$

$x^2 - 9x - 10 = 0$

$(x - 10)(x+1) = 0$

$x = 10 \qquad x = -1 \leftarrow$ EXTRANEOUS

6 Which statements could be used to prove that $\triangle ABC$ and $\triangle A'B'C'$ are congruent?

   (1) $\overline{AB} \cong \overline{A'B'}$, $\overline{BC} \cong \overline{B'C'}$, and $\angle A \cong \angle A'$ SSA
   (2) $\overline{AB} \cong \overline{A'B'}$, $\angle A \cong \angle A'$, and $\angle C \cong \angle C'$ AAS
   (3) $\angle A \cong \angle A'$, $\angle B \cong \angle B'$, and $\angle C \cong \angle C'$ AAA
   (4) $\angle A \cong \angle A'$, $\overline{AC} \cong \overline{A'C'}$, and $\overline{BC} \cong \overline{B'C'}$ SSA

7 Ileana buys a large circular pizza that is divided into eight equal slices. She measures along the outer edge of the crust from one piece and finds it to be $5\frac{1}{2}$ inches. What is the diameter of the pizza to the *nearest inch*?

   (1) 14   (3) 7
   (2) 8   (4) 4

(handwritten work:)

The central angle of a slice is

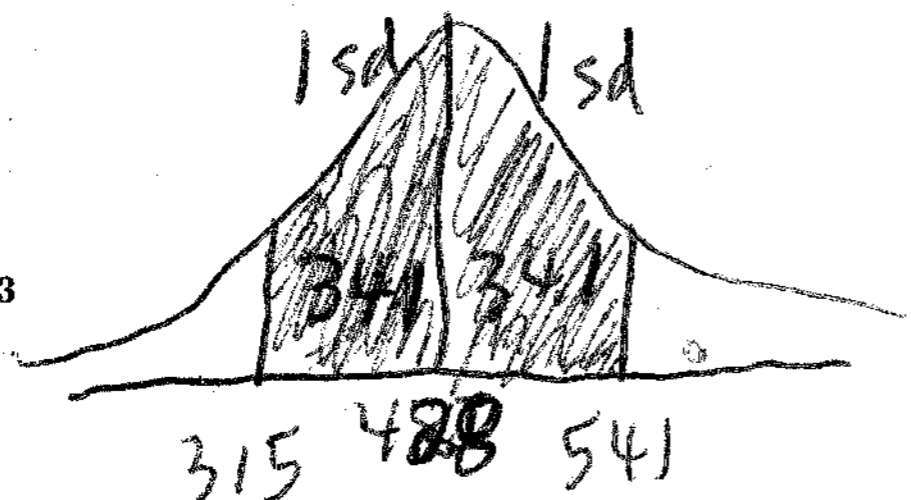$\dfrac{2\pi}{8} = \dfrac{\pi}{4}$

$\theta = \dfrac{S}{R}$

$\dfrac{\pi}{4} = \dfrac{5.5}{R}$

$R \approx 7$

$D \approx 14$

8 The national mean for verbal scores on an exam was 428 and the standard deviation was 113. Approximately what percent of those taking this test had verbal scores between 315 and 541?

   (1) 68.2%   (3) 38.2%
   (2) 52.8%   (4) 26.4%

(handwritten bell curve with 1 sd, 1 sd, 34%, 34%, marked 315, 428, 541)

9 In $\triangle ABC$, if $AC = 12$, $BC = 11$, and $m\angle A = 30$, angle C could be

(1) an obtuse angle, only
(2) an acute angle, only
(3) a right angle, only
(4) either an obtuse angle or an acute angle

$$\frac{11}{\sin 30} = \frac{12}{\sin B}$$

$$B \approx 33 \quad \text{or} \quad 147°$$

$$33 + 30 < 180 \qquad 147 + 30 < 180$$

10 For a rectangular garden with a fixed area, the length of the garden varies inversely with the width. Which equation represents this situation for an area of 36 square units?

(1) $x + y = 36$ (3) $y = \dfrac{36}{x}$

(2) $x - y = 36$ (4) $y = 36x$

11 In which quadrant would the image of point $(5, -3)$ fall after a dilation using a factor of -3?

(1) I   (3) III $(-15, 9)$

(2) II   (4) IV
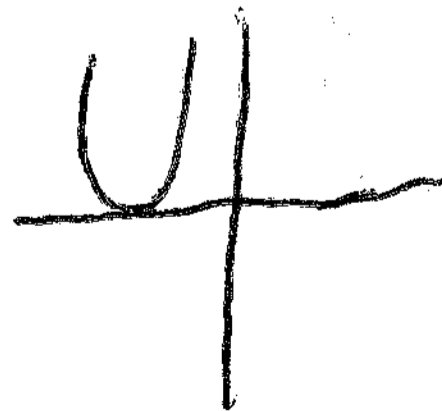
12 The fraction $\dfrac{\frac{x}{y} + x}{\frac{1}{y} + 1}$ is equivalent to

$$\frac{\frac{xy + x}{y}}{\frac{y+1}{y}} = \frac{x(y+1)}{y} \times \frac{y}{y+1} = x$$

(1) $\dfrac{2xy}{1 + y}$ (3) x

(2) $\dfrac{x^2 y}{1 + y}$ (4) $2x$

13 If the roots of $ax^2 + bx + c = 0$ are real, rational, and equal, what is true about the graph of the function $y = ax^2 + bx + c$?

(1) It intersects the x-axis in two distinct points.
(2) It lies entirely below the x-axis.
(3) It lies entirely above the x-axis.
(4) It is tangent to the x-axis.

14 If f(x) = $\dfrac{1}{\sqrt{2x-4}}$, the domain of f(x) is

(1) $x = 2$ (3) $x \geq 2$
(2) $x < 2$ (4) $x > 2$

$\sqrt{2x-4} > 0$

$2x - 4 > 0$

$2x > 4$

$x > 2$

15 What is the sum of $\dfrac{3}{x-3}$ and $\dfrac{x}{3-x}$?

(1) 1 (3) $\dfrac{x+3}{x-3}$
(2) -1 (4) 0

$\dfrac{3}{x-3} + \dfrac{-x}{x-3} = \dfrac{3-x}{x-3} = -1$

16 The expression log $10^{(x+2)}$ − log 10^x is equivalent to

(1) -2 (3) 100
(2) 2 (4) $\dfrac{1}{100}$

$\log \dfrac{10^{x+2}}{10^x} = \log 10^2 = 2$

17 If (sec x − 2)(2 sec x − 1) = 0, then x terminates in

(1) Quadrant I, only
(2) Quadrants I and II, only
(3) Quadrants I and IV, only
(4) Quadrants I, II, III, and IV

$\sec x - 2 = 0$

$\sec x = 2$

$\cos x = \frac{1}{2}$

Q I, IV

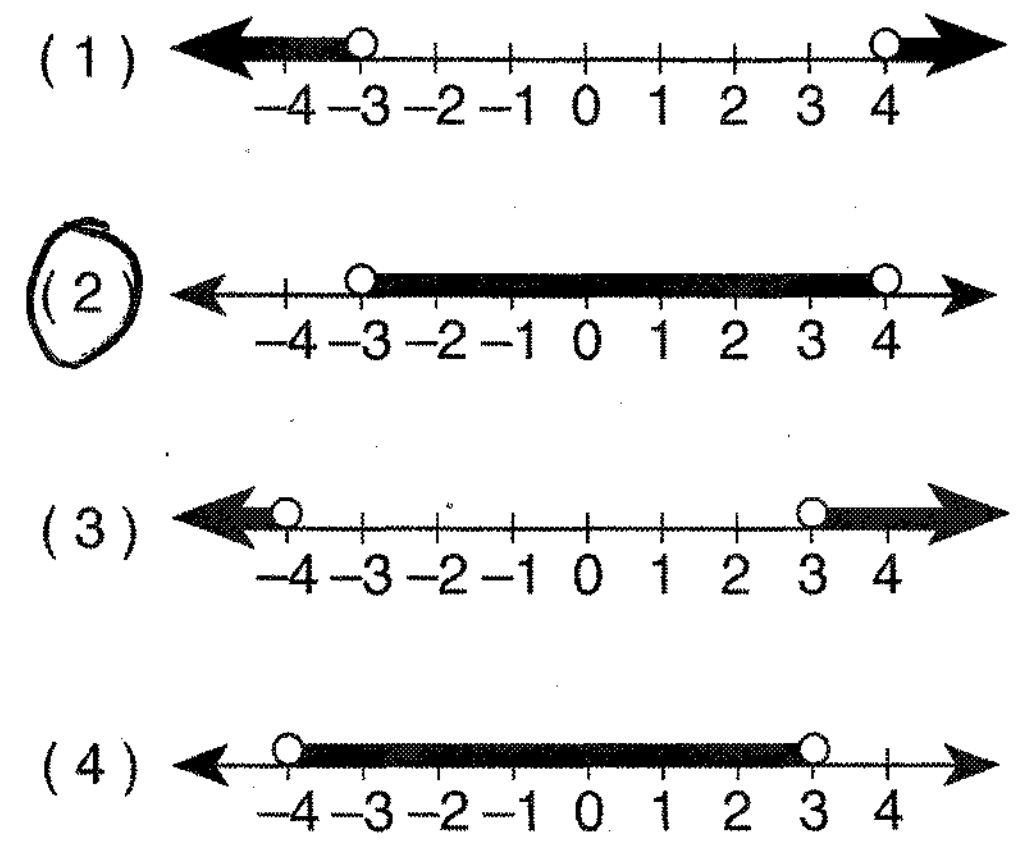$2 \sec x - 1 = 0$

$2 \sec x = 1$

$\sec x = \frac{1}{2}$

$\cos x = 2$

No solution

18 Which graph represents the solution set of $x^2 − x − 12 < 0$?

(1)

(2)

(3)

(4)

$(x-4)(x+3) < 0$

$x - 4 < 0$ $x + 3 > 0$

$x < 4$ $x > -3$

$$\sin^2\theta + \cos^2\theta = 1$$

19 If $\sin\theta = \dfrac{\sqrt{5}}{3}$, then $\cos 2\theta$ equals

(1) $\dfrac{1}{3}$ (3) $\dfrac{1}{9}$

(2) $-\dfrac{1}{3}$ (4) $-\dfrac{1}{9}$

$$\cos 2\theta = 1 - 2\sin^2\theta$$
$$= 1 - 2\left(\frac{\sqrt{5}}{3}\right)^2$$
$$= 1 - 2\left(\frac{5}{9}\right)$$
$$= 1 - \frac{10}{9}$$
$$= -\frac{1}{9}$$

20 If $\sin 6A = \cos 9A$, then $m\angle A$ is equal to

(1) 6 (3) 54

(2) 36 (4) $1\frac{1}{2}$

$$\sin 6(6) = \cos 9(6) \qquad \sin 6(54) = \cos 9(54)$$

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

21 Two social studies classes took the same current events examination that was scored on the basis of 100 points. Mr. Wong's class had a median score of 78 and a range of 4 points, while Ms. Rizzo's class had a median score of 78 and a range of 22 points. Explain how these classes could have the same median score while having very different ranges.

The median is the middle test score and is independent of the range, which depends on the lowest and highest scores.

22 The height of an object, h(t), is determined by the formula h(t) = $-16t^2 + 256t$, where t is time, in seconds. Will the object reach a maximum or a minimum? Explain or show your reasoning.

Because $a < 0$, the graph will have a maximum.

23 A wrecking ball suspended from a chain is a type of pendulum. The relationship between the rate of speed of the ball, R, the mass of the ball, m, the length of the chain, L, and the force, F, is $R = 2\pi\sqrt{\dfrac{mL}{F}}$. Determine the force, F, to the *nearest hundredth*, when $L = 12$, $m = 50$, and $R = 0.6$.

$$R = 2\pi\sqrt{\dfrac{mL}{F}}$$

$$\dfrac{.6 = 2\pi\sqrt{\dfrac{(50)(12)}{F}}}{2\pi}$$

$$\dfrac{.3}{\pi} = \sqrt{\dfrac{600}{F}}$$

$$\dfrac{.09}{\pi^2} = \dfrac{600}{F}$$

$$F \approx 65{,}797.36$$

24 The relationship between the relative size of an earthquake, S, and the measure of the earthquake on the Richter scale, R, is given by the equation $\log S = R$. If an earthquake measured 3.2 on the Richter scale, what was its relative size to the *nearest hundredth*?

$$\log S = R$$

$$\log_{10} S = 3.2$$

$$S = 10^{3.2}$$

$$\approx 1{,}584.89$$

25 In an electrical circuit, the voltage, E, in volts, the current, I, in amps, and the opposition to the flow of current, called impedance, Z, in ohms, are related by the equation $E = IZ$. A circuit has a current of $(3 + i)$ amps and an impedance of $(-2 + i)$ ohms. Determine the voltage in $a + bi$ form.

$$E = IZ$$
$$= (3+i)(-2+i)$$
$$= -6 + 3i - 2i + i^2$$
$$= -6 + i - 1$$
$$= -7 + i$$

26 The inequality $|1.5C - 24| \leq 30$ represents the range of monthly average temperatures, C, in degrees Celsius, for Toledo, Ohio. Solve for C.

$$1.5C - 24 \leq 30$$
$$+24 \quad +24$$
$$\frac{1.5C \leq 54}{1.5 \quad\quad 1.5}$$

$$C \leq 36$$

$$1.5C - 24 \geq -30$$
$$+24 \quad\quad +24$$
$$\frac{1.5C \geq -6}{1.5 \quad\quad 1.5}$$

$$C \geq -4$$

$$-4 \leq C \leq 36$$

Part III

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [24]

27 A shoe manufacturer collected data regarding men's shoe sizes and found that the distribution of sizes exactly fits the normal curve. If the mean shoe size is 11 and the standard deviation is 1.5, find:

a the probability that a man's shoe size is greater than or equal to 11

50%
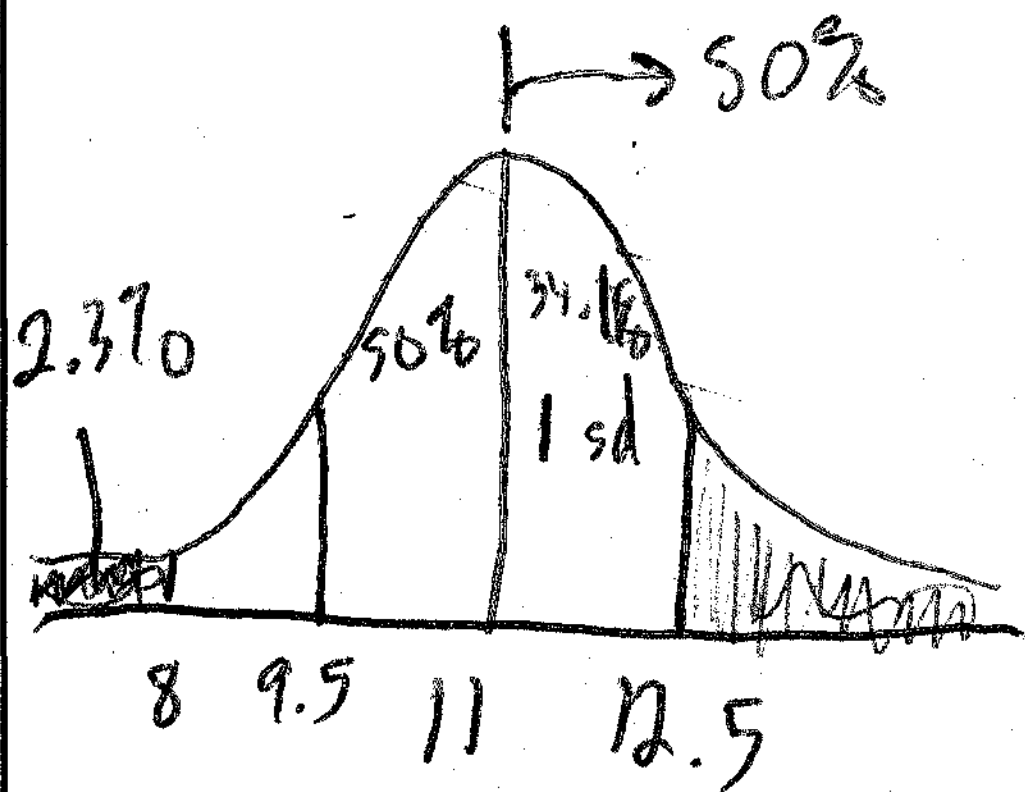
b the probability that a man's shoe size is greater than or equal to 12.5

$$100 - (50 + 34.1) = 15.9\%$$

c $\dfrac{P(\text{size} \geq 12.5)}{P(\text{size} \geq 8)}$ $\dfrac{.159}{(1 - .023)}$; $\dfrac{.159}{.977}$

28 In a mathematics class of ten students, the teacher wanted to determine how a homework grade influenced a student's performance on the subsequent test. The homework grade and subsequent test grade for each student are given in the accompanying table.

Homework Grade (x)	Test Grade (y)
94	98
95	94
92	95
87	89
82	85
80	78
75	73
65	67
50	45
20	40

a Give the equation of the linear regression line for this set of data.

$$y = .834x + 14.65$$

b A new student comes to the class and earns a homework grade of 78. Based on the equation in part *a*, what grade would the teacher predict the student would receive on the subsequent test, to the *nearest integer?*
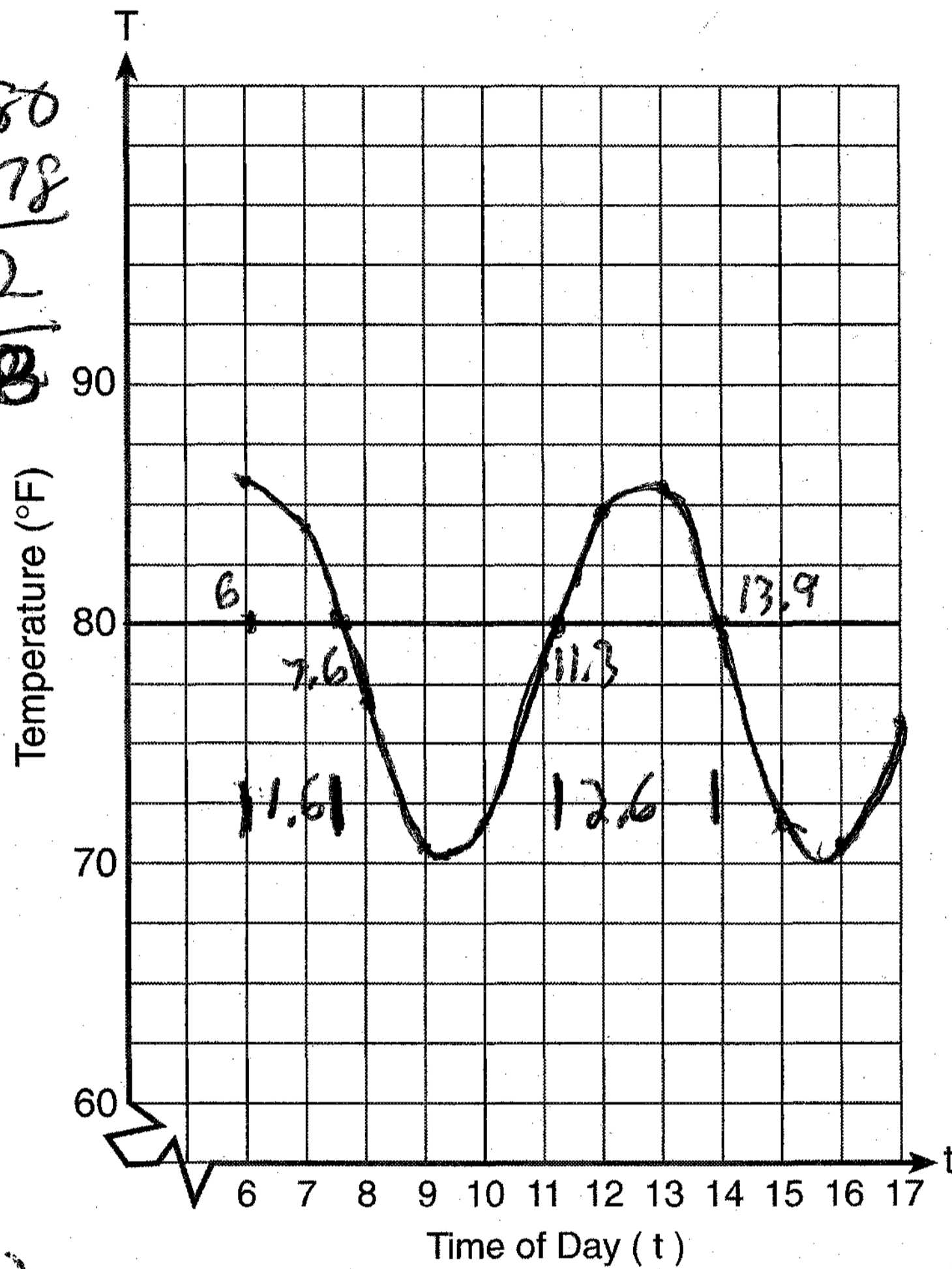
$$y = .834x + 14.65$$
$$.834(78) + 14.65$$
$$\approx 80$$

29 A building's temperature, *T*, varies with time of day, *t*, during the course of 1 day, as follows:

$$T = 8 \cos t + 78$$

The air-conditioning operates when $T \geq 80°F$. Graph this function for $6 \leq t < 17$ and determine, to the *nearest tenth of an hour*, the amount of time in 1 day that the air-conditioning is on in the building.

$8\cos t + 78 \geq 80$

$ -78 \quad\quad 78$

$\dfrac{8\cos t}{8} \geq \dfrac{2}{8}$

$\cos t \geq \dfrac{1}{4}$

$t \geq \cos^{-1}\dfrac{1}{4}$

Time of Day (t)

$\cos^{-1}\left(\tfrac{1}{4}\right) \approx 1.3$
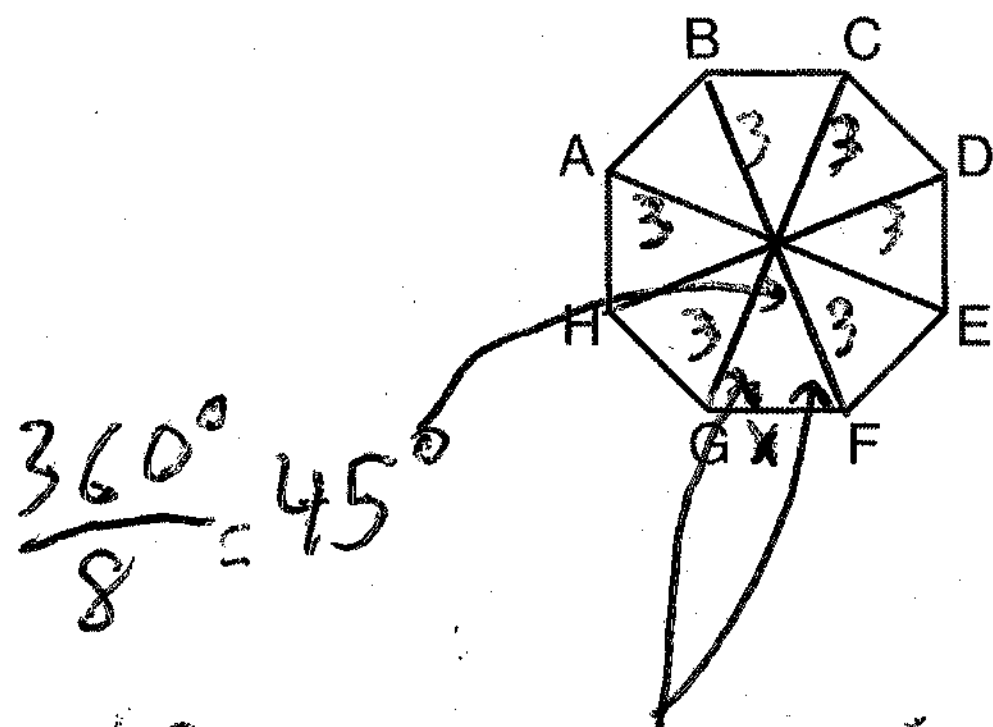
$1.3 + 2\pi \approx 7.6$

$1.3 + 4\pi \approx 13.9$

$4\pi - 1.3 \approx 11.3$

$7.6 - 6 = 1.6$

$\dfrac{13.9 - 11.3 = 2.6}{4.2}$

30 A picnic table in the shape of a regular octagon is shown in the accompanying diagram. If the length of $\overline{AE}$ is 6 feet, find the length of one side of the table to the *nearest tenth of a foot,* and find the area of the table's surface to the *nearest tenth of a square foot.*

$$\frac{360°}{8} = 45°$$

$$\frac{180-45}{2} = 62.5°$$

$$\frac{3}{\sin 67.5} = \frac{x}{\sin 45}$$

$$x \approx 2.3$$

$$\Delta \text{ Area} = \frac{1}{2}(3)(3)\sin 45$$

$$\approx 3.18$$

$$8 \times 3.18 \approx 25.5$$

31 If $f(x) = x^{\frac{2}{3}}$ and $g(x) = 8x^{-\frac{1}{2}}$, find $(f \circ g)(x)$ and find $(f \circ g)(27)$.

$$\left(8x^{-\frac{1}{2}}\right)^{2/3} = 8^{2/3}x^{-2/6} = 4x^{-1/3}$$

$$(f \circ g)(27) = 4(27)^{-1/3} = \frac{4}{3}$$

32 Given: $f(x) = x^2$ and $g(x) = 2^x$

   a The inverse of g is a function, but the inverse of f is not a function.
   Explain why this statement is true.

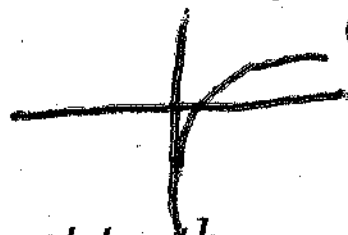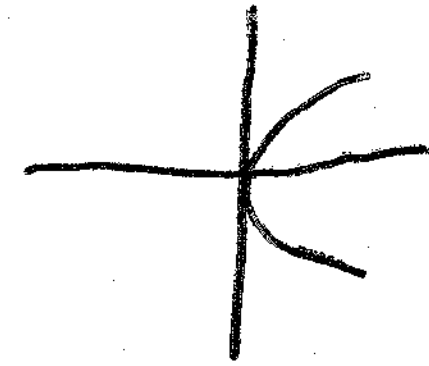$g(x)$

$g'(x)$

$y = 2^x$

$x = 2^y$

$\log x = \log 2^y$

$\dfrac{\log x}{\log 2} = \dfrac{y \log 2}{\log 2}$

$y = \dfrac{\log x}{\log 2}$

PASS VERTICAL LINE TEST

$F(x) = x^2$

$y = x^2$

$F^{-1}(x) \quad x = y^2$

FAILS VERTICAL LINE TEST

   b Find $g^{-1}(f(3))$ to the *nearest tenth*.

$f(3) = 3^2 = 9$

$g^{-1}(9) = \dfrac{\log 9}{\log 2} \approx 3.2$

**Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]**

33 Quadrilateral *KATE* has vertices $K(1,5)$, $A(4,7)$, $T(7,3)$, and $E(1,-1)$.

   a Prove that *KATE* is a trapezoid. [The use of the grid on the next page is optional.]

STATEMENT	REASON
① Quadrilateral KATE with vertices K(1,5), A(4,7), T(7,3), E(1,-1)	① Given
② $M_{AK} = \frac{7-5}{4-1} = \frac{2}{3}$ $M_{ET} = \frac{3-(-1)}{7-1} = \frac{2}{3}$ $M_{EK} = \frac{-1-5}{1-1}$ undefined $M_{AT} = \frac{7-3}{4-7} = -\frac{4}{3}$	② Definition of slope
③ $\overline{AK} \parallel \overline{ET}$, $\overline{EK} \nparallel \overline{AT}$	③ Parallel lines have equal slope
④ KATE is a trapezoid	④ A trapezoid is a quadrilateral that has exactly one set of parallel sides

   b Prove that *KATE* is *not* an isosceles trapezoid.

⑤ $d_{EK} = \sqrt{(1-1)^2 + (5-(-1))^2} = 6$ $d_{AT} = \sqrt{(4-7)^2 + (7-3)^2} = 5$	⑤ Distance Formula
⑥ $\overline{AT} \not\cong \overline{EK}$	⑥ Different lengths
⑦ KATE is not an isosceles trapezoid	⑦ The non-parallel opposite sides are not congruent.
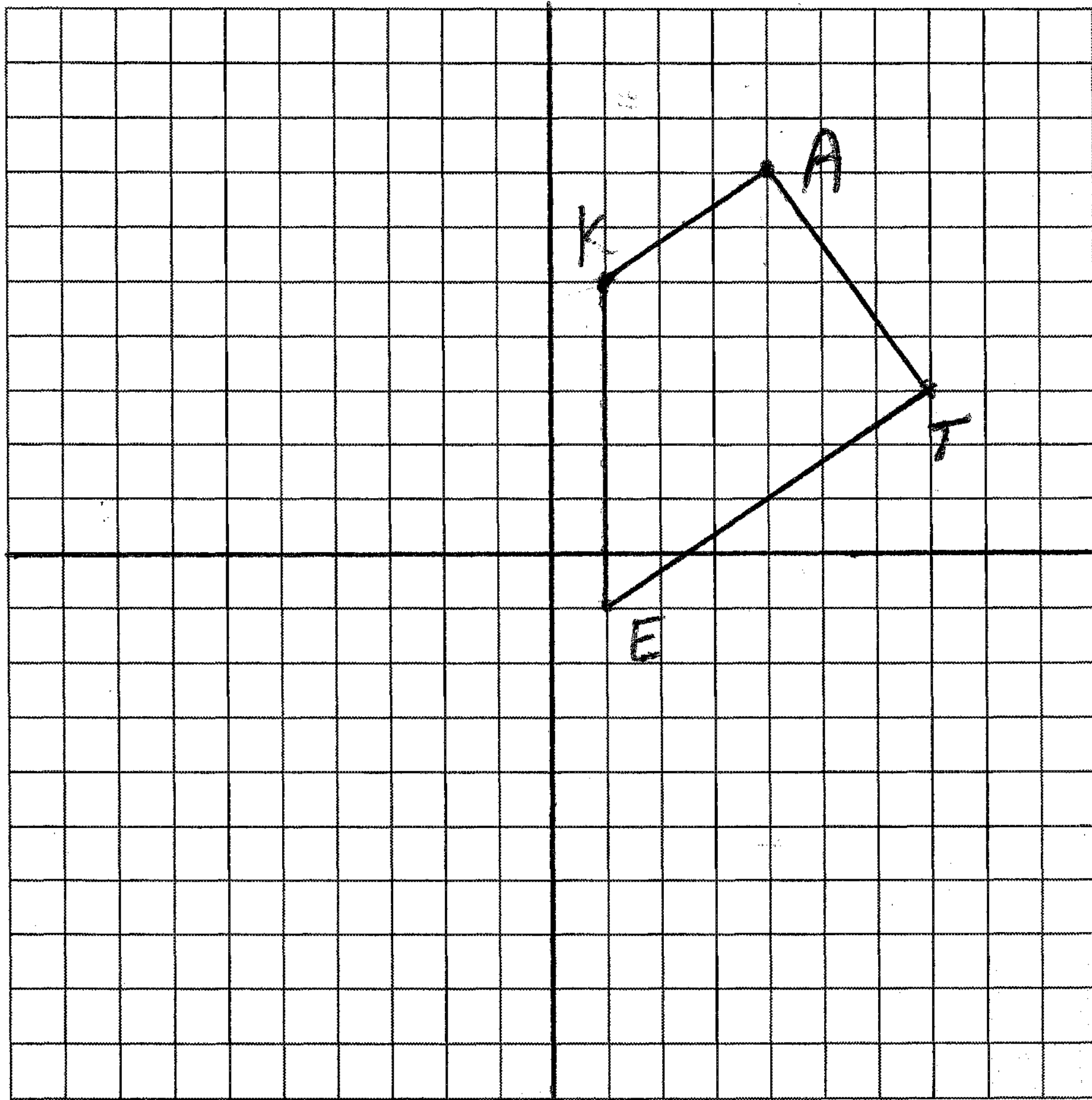
34 A ship captain at sea uses a sextant to sight an angle of elevation of 37° to the top of a lighthouse. After the ship travels 250 feet directly toward the lighthouse, another sighting is made, and the new angle of elevation is 50°. The ship's charts show that there are dangerous rocks 100 feet from the base of the lighthouse. Find, to the *nearest foot,* how close to the rocks the ship is at the time of the second sighting.
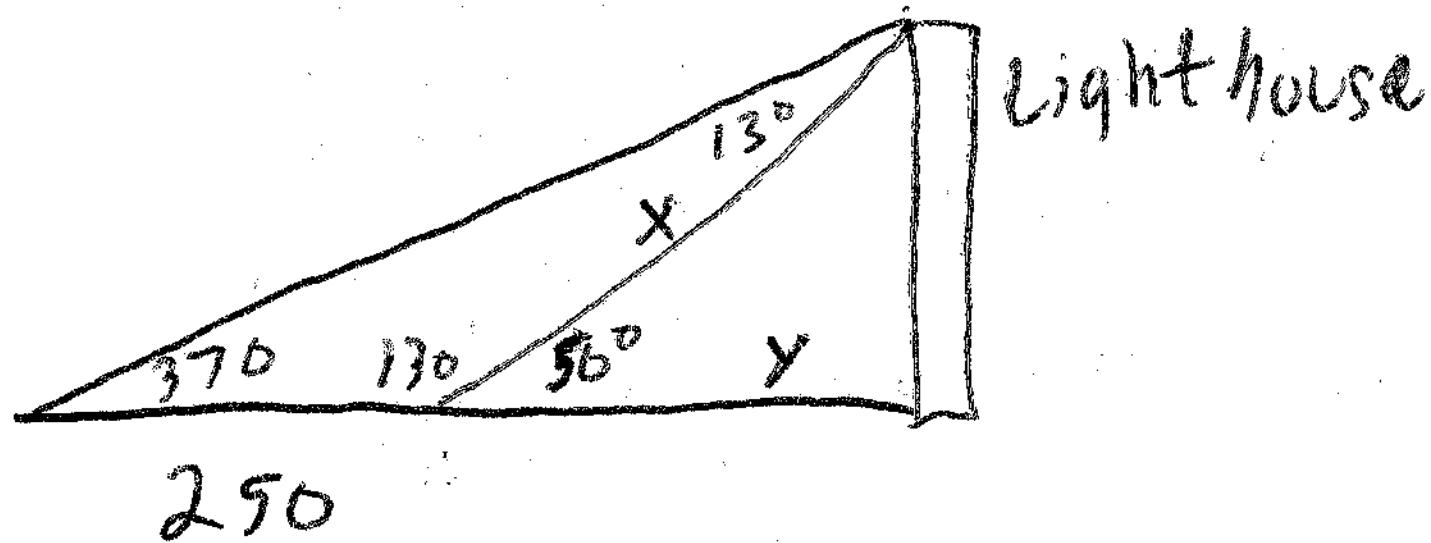
$$\frac{250}{\sin 13} = \frac{X}{\sin 37}$$

$$X \approx 668.8$$

$$\cos 50 \approx \frac{Y}{668.8}$$

$$Y \approx 430$$

Because the rocks are 100 feet from the base of the lighthouse, the ship is 330 (430 - 100) feet from the rocks at the second sighting.