The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

# MATHEMATICS B

**Friday,** January 25, 2002 — 9:15 a.m. to 12:15 p.m., only

**Print Your Name:** Steve Sibol

**Print Your School's Name:** HSCR

**Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.**

**Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will *not* be scored. All work should be written in pen, except graphs and drawings, which should be done in pencil.**

**This examination has four parts, with a total of 34 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. The formulas that you may need to answer some questions in this examination are found on page 2.**

**When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.**

**Notice. . .**

**A graphing calculator, a straightedge (ruler), and a compass must be available for your use while taking this examination.**

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

## Part I

**Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Record your answers in the spaces provided on the separate answer sheet.** [40]

**1** The roots of a quadratic equation are real, rational, and equal when the discriminant is

(1) –2
(2) 2
(3) 0
(4) 4

**Use this space for computations.**

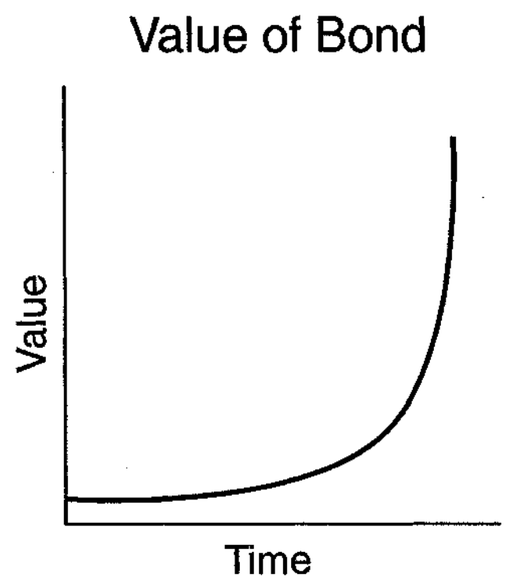
**2** Chad had a garden that was in the shape of a rectangle. Its length was twice its width. He decided to make a new garden that was 2 feet longer and 2 feet wider than his first garden. If $x$ represents the original width of the garden, which expression represents the difference between the area of his new garden and the area of the original garden?

(1) $6x + 4$
(2) $2x^2$
(3) $x^2 + 3x + 2$
(4) 8

ORIGINAL AREA
$(x)(2x) = 2x^2$

NEW AREA
$(x+2)(2x+2)$
$2x^2+6x+4$
$-2x^2$
$6x+4$

**3** The accompanying graph represents the value of a bond over time.




Which type of function does this graph best model?

(1) trigonometric
(2) logarithmic
(3) quadratic
(4) exponential

**4** An object that weighs 2 pounds is suspended in a liquid. When the object is depressed 3 feet from its equilibrium point, it will oscillate according to the formula $x = 3\cos(8t)$, where $t$ is the number of seconds after the object is released. How many seconds are in the period of oscillation?

(1) $\frac{\pi}{4}$
(2) $\pi$
(3) 3
(4) $2\pi$

**Use this space for computations.**

period $= \frac{2\pi}{b} = \frac{2\pi}{8} = \frac{\pi}{4}$

**5** If $\theta$ is an angle in standard position and its terminal side passes through the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ on a unit circle, a possible value of $\theta$ is

(1) 30°
(2) 60°
(3) 120°
(4) 150°

$\cos^{-1}\frac{1}{2} = 60°$

$\sin^{-1}\frac{\sqrt{3}}{2} = 60°$

**6** The expression $\frac{\frac{a}{b}-\frac{b}{a}}{\frac{1}{a}+\frac{1}{b}}$ is equivalent to

(1) $a + b$
(2) $a - b$
(3) $ab$
(4) $\frac{a-b}{ab}$

$\frac{\frac{a^2-b^2}{ab}}{\frac{a+b}{ab}}$

$\frac{a^2-b^2}{ab} \div \frac{a+b}{ab}$

$\frac{(a+b)(a-b)}{ab} \times \frac{ab}{a+b}$

**7** If $f(x) = 5x^2$ and $g(x) = \sqrt{2x}$, what is the value of $(f \circ g)(8)$?

(1) $8\sqrt{10}$
(2) 16
(3) 80
(4) 1,280

$g(8) = \sqrt{2(8)} = 4$

$f(4) = 5(4)^2 = 80$

**8** Which expression is *not* equivalent to $\log_b 36$?

(1) $6\log_b 2$
(2) $\log_b 9 + \log_b 4 = \log_b(9 \times 4)$
(3) $2\log_b 6 = \log_b 6^2$
(4) $\log_b 72 - \log_b 2 = \log_b \frac{72}{2}$

**Use this space for computations.**

**9** If a function is defined by the equation $y = 3x + 2$, which equation defines the inverse of this function?

(1) $x = \frac{1}{3}y + \frac{1}{2}$

(2) $y = \frac{1}{3}x + \frac{1}{2}$

(3) $y = \frac{1}{3}x - \frac{2}{3}$

(4) $y = -3x - 2$

$x = 3y + 2$

$\frac{x-2}{3} = \frac{3y}{3}$

$\frac{1}{3}x - \frac{2}{3} = y$

**10** Which transformation is *not* an isometry?

(1) $r_{y=x}$

(2) $R_{0,90°}$

(3) $T_{3,6}$

(4) $D_2$

**11** Which relation is a function?

(1) $x = 4$

(2) $x = y^2 + 1$

(3) $y = \sin x$

(4) $x^2 + y^2 = 16$

**12** In $\triangle ABC$, $m\angle A = 33$, $a = 12$, and $b = 15$. What is $m\angle B$ to the *nearest degree*?

(1) 41

(2) 43

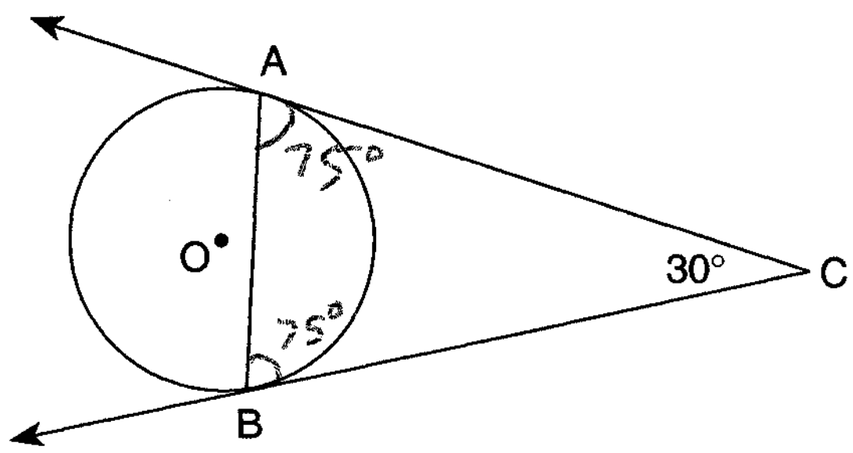
(3) 44

(4) 48

$\frac{15}{\sin B} = \frac{12}{\sin 33}$

$B \approx 43°$

**13** The accompanying diagram represents circular pond $O$ with docks located at points $A$ and $B$. From a cabin located at $C$, two sightings are taken that determine an angle of 30° for tangents $\overrightarrow{CA}$ and $\overrightarrow{CB}$.




$\triangle ABC$ is isosceles

What is $m\angle CAB$?

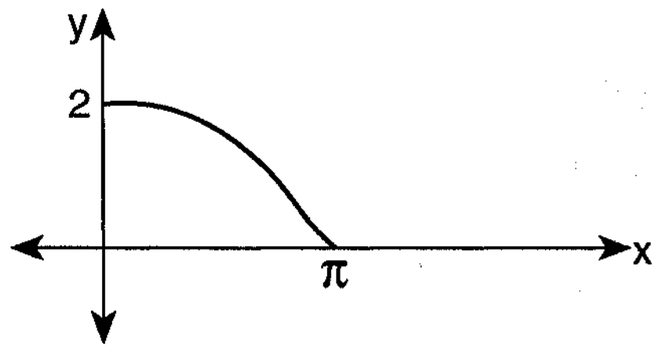
(1) 30

(2) 60

(3) 75

(4) 150

**14** The accompanying diagram shows a section of a sound wave as displayed on an oscilloscope.




Which equation could represent this graph?

(1) $y = 2 \cos \frac{x}{2}$

(2) $y = 2 \sin \frac{x}{2}$

(3) $y = \frac{1}{2} \cos 2x$

(4) $y = \frac{1}{2} \sin \frac{\pi}{2}x$

**Use this space for computations.**

period: $\frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = 4\pi$

**15** Every time the pedals go through a 360° rotation on a certain bicycle, the tires rotate three times. If the tires are 24 inches in diameter, what is the minimum number of complete rotations of the pedals needed for the bicycle to travel at least 1 mile?

(1) 12

(2) 281

(3) 561

(4) 5,280

Since the diameter of a tire is 2" circumference is $2\pi$. To travel 1 mile or 5280', the tire must rotate $840.3 \left(\frac{5280}{2\pi}\right)$ times. The pedals must rotate $281 \left(\frac{840.3}{3}\right)$ times.

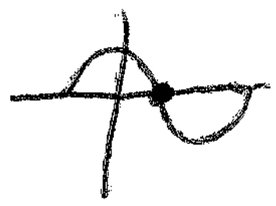
**16** Which type of symmetry does the equation $y = \cos x$ have?

(1) line symmetry with respect to the $x$-axis

(2) line symmetry with respect to $y = x$

(3) point symmetry with respect to the origin

(4) point symmetry with respect to $\left(\frac{\pi}{2},0\right)$



**17** The value of $\left(\frac{3^0}{27^{\frac{2}{3}}}\right)^{-1}$ is $= \frac{27^{2/3}}{3^0} = 27^{2/3} = 9$

(1) –9

(2) 9

(3) $-\frac{1}{9}$

(4) $\frac{1}{9}$

**18** What is the domain of $h(x) = \sqrt{x^2-4x-5}$?

(1) $\{x|x \geq 1 \text{ or } x \leq -5\}$
(2) $\{x|x \geq 5 \text{ or } x \leq -1\}$
(3) $\{x|-1 \leq x \leq 5\}$
(4) $\{x|-5 \leq x \leq 1\}$

$x^2-4x-5 \geq 0$
$(x-5)(x+1) \geq 0$
$x-5 \geq 0$ and $x+1 \geq 0$
$x \geq 5$ and $x \geq -1$

**Use this space for computations.**

or
$x-5 \leq 0$ & $x+1 \leq 0$
$x \leq 5$ & $x \leq -1$

**19** The expression $(-1 + i)^3$ is equivalent to

(1) $-3i$
(2) $-2 - 2i$
(3) $-1 - i$
(4) $2 + 2i$

$(-1+i)(-1+i)(-1+i)$
$(1-i-i+i^2)(-1+i)$
$(1-2i-1)(-1+i)$
$-2i(-1+i)$
$2i-2i^2$
$2i-2(-1)$
$2+2i$

**20** The revenue, $R(x)$, from selling $x$ units of a product is represented by the equation $R(x) = 35x$, while the total cost, $C(x)$, of making $x$ units of the product is represented by the equation $C(x) = 20x + 500$. The total profit, $P(x)$, is represented by the equation $P(x) = R(x) - C(x)$. For the values of $R(x)$ and $C(x)$ given above, what is $P(x)$?

(1) $15x$
(2) $15x + 500$
(3) $15x - 500$
(4) $10x + 100$

$P(x) = R(x) - C(x)$
$= 35x - (20x+500)$
$= 15x - 500$

## Part II

**Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.** [12]

**21** Explain how a person can determine if a set of data represents inverse variation and give an example using a table of values.

In inverse variation, the product of the two variables is constant.

$y = \frac{36}{x}$

| $x$ | $y$ |
|---|---|
| 1 | 36 |
| 2 | 18 |
| 3 | 12 |
| 4 | 9 |
| 6 | 6 |

**22** Solve for $x$ in simplest $a + bi$ form: $x^2 + 8x + 25 = 0$

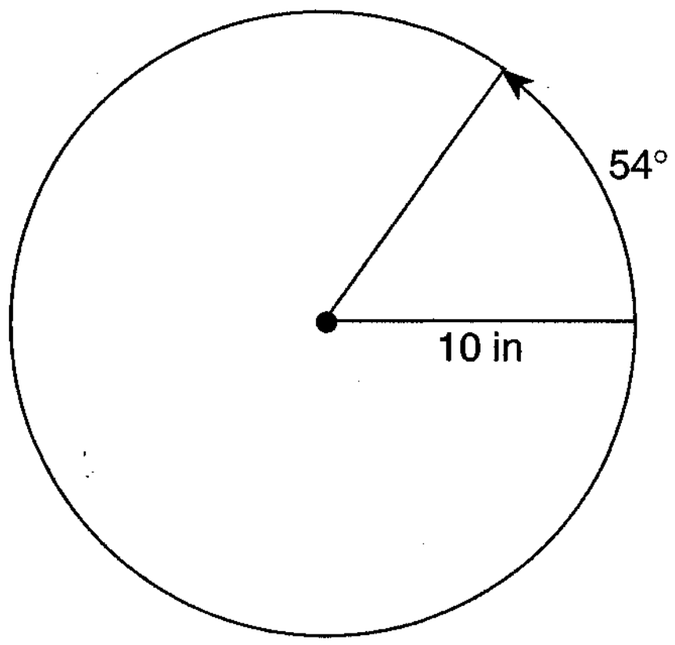
$x^2+8x+16 = -25+16$

$(x+4)^2 = -9$

$x+4 = \pm\sqrt{-9}$

$x = -4 \pm 3i$

**23** A ball is rolling in a circular path that has a radius of 10 inches, as shown in the accompanying diagram. What distance has the ball rolled when the subtended arc is 54°? Express your answer to the *nearest hundredth of an inch.*




$54^\circ \cdot \frac{\pi}{180^\circ} = \frac{3\pi}{10}$ radians

$\theta = \frac{s}{r}$

$\frac{3\pi}{10} = \frac{s}{10}$

$s \approx 9.42$

**24** A rectangle is said to have a golden ratio when $\frac{w}{h} = \frac{h}{w - h}$, where $w$ represents width and $h$ represents height. When $w = 3$, between which two consecutive integers will $h$ lie?

$\frac{3}{h} = \frac{h}{3-h}$

$9 - 3h = h^2$

$h^2 + 3h - 9 = 0$

$a = 1$

$b = 3$

$c = -9$

$h = \frac{-3 \pm \sqrt{3^2 - 4(1)(-9)}}{2(1)}$

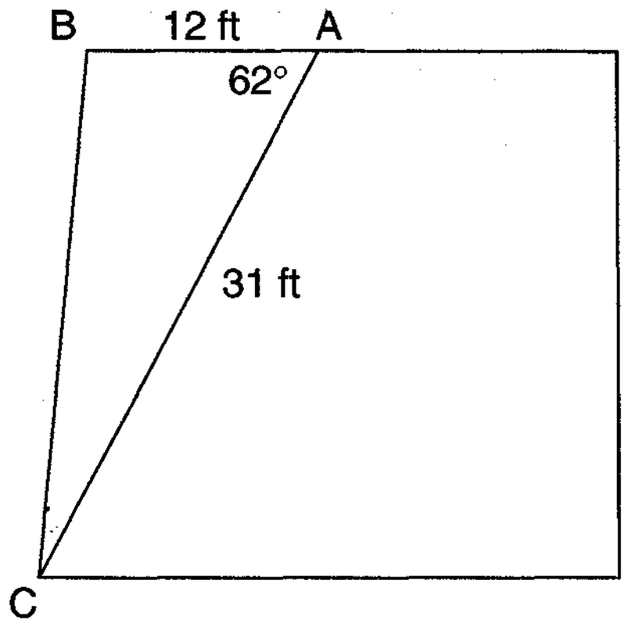
$h = \frac{-3 \pm \sqrt{45}}{2}$

$h = \frac{-3 + \sqrt{45}}{2}$

$h \approx 1.9$

h is between 1 and 2.

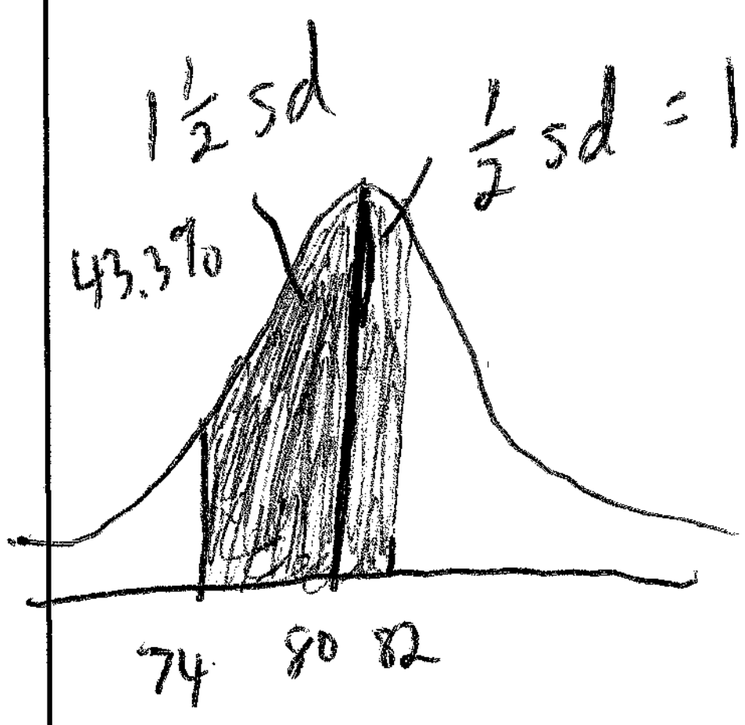
**25** The accompanying diagram shows the floor plan for a kitchen. The owners plan to carpet all of the kitchen except the "work space," which is represented by scalene triangle *ABC*. Find the area of this work space to the *nearest tenth of a square foot.*




Area = $\frac{1}{2}(12)(31)\sin 62$

$\approx 164.2$

**26** A set of normally distributed student test scores has a mean of 80 and a standard deviation of 4. Determine the probability that a randomly selected score will be between 74 and 82.




43.3

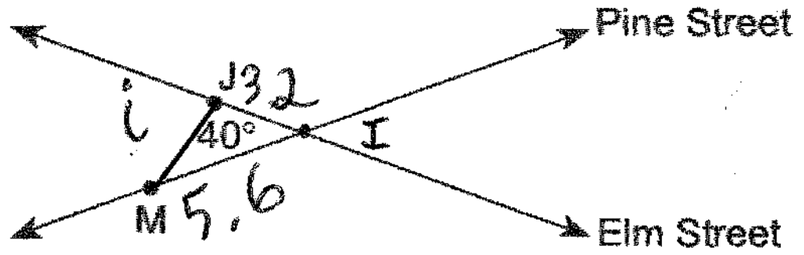
19.1

62.4%

## Part III

**Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.** [24]

**27** Two straight roads, Elm Street and Pine Street, intersect creating a 40° angle, as shown in the accompanying diagram. John's house ($J$) is on Elm Street and is 3.2 miles from the point of intersection. Mary's house ($M$) is on Pine Street and is 5.6 miles from the intersection. Find, to the *nearest tenth of a mile,* the direct distance between the two houses.




Law of Cosines

$i^2 = j^2 + m^2 - 2jm\cos I$

$i^2 = 5.6^2 + 3.2^2 - 2(5.6)(3.2)\cos 40$

$i^2 = 14.1$

$i = 3.8$

**28** At the local video rental store, José rents two movies and three games for a total of \$15.50. At the same time, Meg rents three movies and one game for a total of \$12.05. How much money is needed to rent a combination of one game and one movie?

$3(3m + g = 12.05)$

$2m + 3g = 15.5$

$9m + 3g = 36.15$

$2m + 3g = 15.50$

$7m = 20.65$

$m = 2.95$

$3m + g = 12.05$

$3(2.95) + g = 12.05$

$g = 3.20$

$2.95 + 3.20 = \$6.15$

**29** Team $A$ and team $B$ are playing in a league. They will play each other five times. If the probability that team $A$ wins a game is $\frac{1}{3}$, what is the probability that team $A$ will win *at least* three of the five games?

$n = 5$

$r = 3, 4, 5$

$p = \frac{1}{3}$

$q = \frac{2}{3}$

$P(3) = {}_5C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^2 = \frac{40}{243}$

$P(4) = {}_5C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^1 = \frac{10}{243}$

$P(5) = {}_5C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^0 = \frac{1}{243}$

$\frac{51}{243}$

**30** Depreciation (the decline in cash value) on a car can be determined by the formula $V = C(1 - r)^t$, where $V$ is the value of the car after $t$ years, $C$ is the original cost, and $r$ is the rate of depreciation. If a car's cost, when new, is \$15,000, the rate of depreciation is 30%, and the value of the car now is \$3,000, how old is the car to the *nearest tenth of a year*?

$V = C(1-r)^t$

$\frac{3000}{15000} = \frac{15000(1-.3)^t}{15000}$

$\log .2 = \log .7^t$

$\frac{\log .2}{\log .7} = \frac{t \log .7}{\log .7}$

$t \approx 4.5$

**31** When a baseball is hit by a batter, the height of the ball, $h(t)$, at time $t$, $t \geq 0$, is determined by the equation $h(t) = -16t^2 + 64t + 4$. For which interval of time is the height of the ball greater than or equal to 52 feet?

$-16t^2+64t+4 \geq 52$

$-52 \quad -52$

---

$-16t^2+64t-48 \geq 0$ divide by $-16$

$t^2-4t+3 \leq 0$

$(t-3)(t-1) \leq 0$

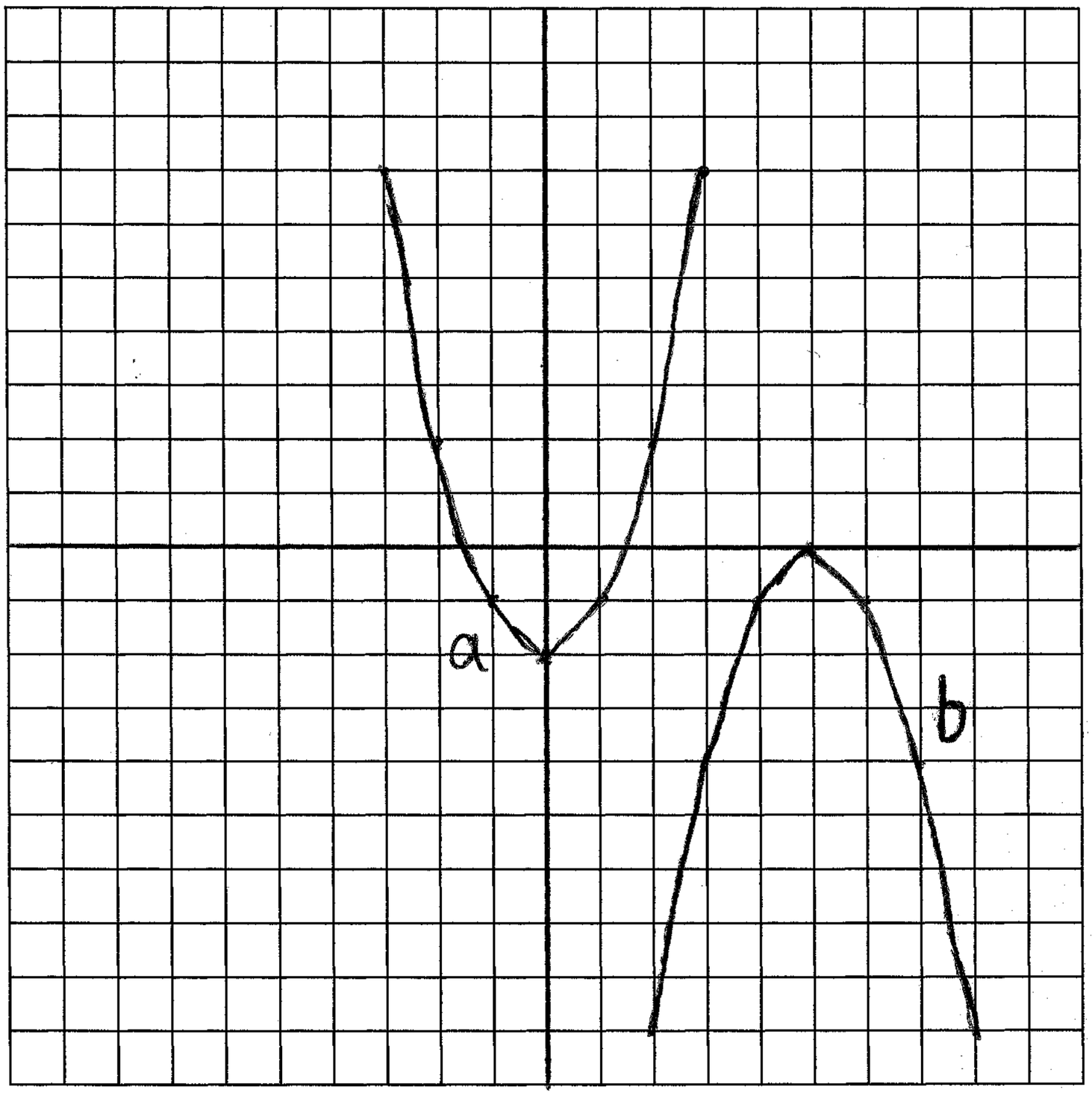
$t \leq 3$ and $t \geq 1$ OR ~~$t \geq 3$ and $t \leq 1$~~

$1 \leq t \leq 3$

**32** *a* On the accompanying grid, graph the equation $2y = 2x^2 - 4$ in the interval $-3 \leq x \leq 3$ and label it *a*.

$y = x^2 - 2$

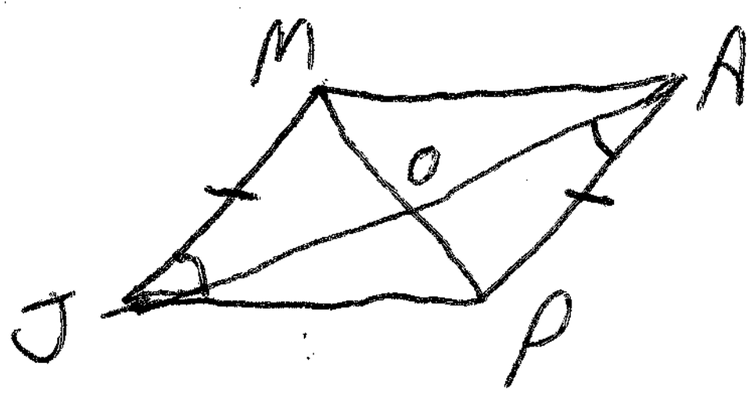
*b* On the same grid, sketch the image of *a* under $T_{5,-2} \circ r_{x\text{-axis}}$ and label it *b*.



## Part IV

**Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.** [12]

**33** Prove that the diagonals of a parallelogram bisect each other.

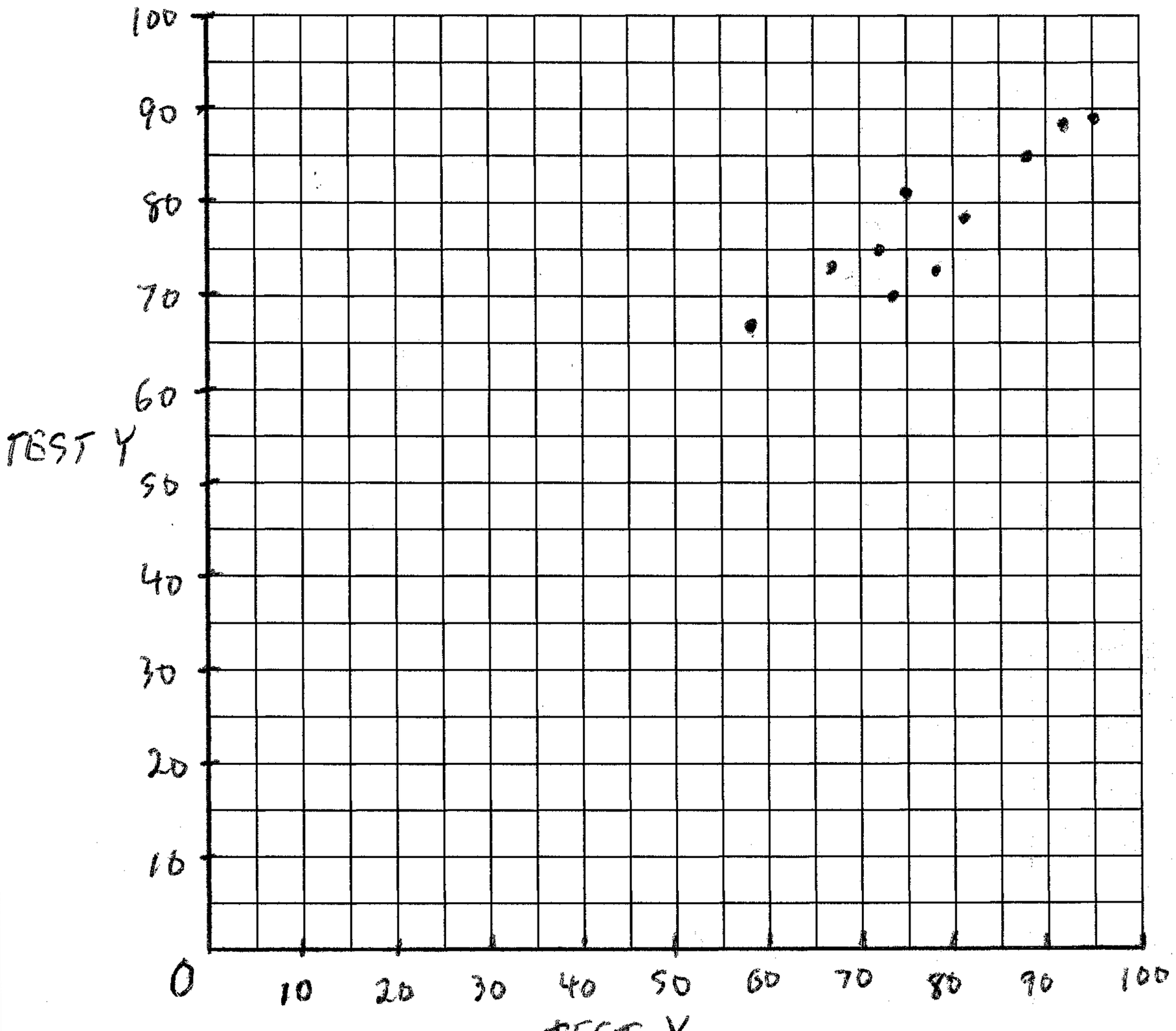



| | STATEMENT | REASON |
|---|---|---|
| (1) | Parallelogram JMAP with diagonals bisecting at O. | (1) Given |
| (2) | $\overline{JM} \cong \overline{AP}$ | (2) Opposite sides of a parallelogram are consistent |
| (3) | $\angle JOM \cong \angle AOP$ | (3) Vertical angles |
| (4) | $\overline{JM} \parallel \overline{AP}$ | (4) Opposite sides of a parallelogram are parallel |
| (5) | $\angle MJO \cong \angle PAO$ | (5) Alternate interior angles formed by parallel lines and a transversal are congruent |
| (6) | $\triangle MJO \cong \triangle PAO$ | (6) AAS |
| (7) | $\overline{JO} \cong \overline{AO}$ $\overline{MO} \cong \overline{PO}$ | (7) CPCTC |

**34** Two different tests were designed to measure understanding of a topic. The two tests were given to ten students with the following results:

| **Test $x$** | 75 | 78 | 88 | 92 | 95 | 67 | 58 | 72 | 74 | 81 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Test $y$** | 81 | 73 | 85 | 88 | 89 | 73 | 66 | 75 | 70 | 78 |

Construct a scatter plot for these scores, and then write an equation for the line of best fit (round slope and intercept to the *nearest hundredth*). $y = .62x + 29.18$




Find the correlation coefficient.

$r \approx .92$

Predict the score, to the *nearest integer*, on test $y$ for a student who scored 87 on test $x$.

$y = .62(87) + 29.18$

$\approx 83$