The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS A

Thursday , August 12, 1999 — 8:30 to 11:30 a.m., only

Print Your Name: _Imaginary Student (i)_

Print Your School's Name: _www.jmap.org_

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and car efully , tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not r equir ed. Any work done on this sheet of scrap graph paper will *not* be scored. All work should be written in pen, except graphs and drawings, which should be done in pencil.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers for the questions in Par ts II, III, and IV directly in this booklet. Clearly indicate the necessary steps you take, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Y our answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Answer all questions in this part. Each correct answer will receive 2 credits. No partial cr   edit will be allowed. Record your answers in the spaces provided on the separate answer sheet.   [40]

1 A roll of candy is shown in the accompanying diagram.

Use this space for computations.

The shape of the candy is best described as a

(1) rectangular solid   (3) cone
(2) pyramid   (4) cylinder

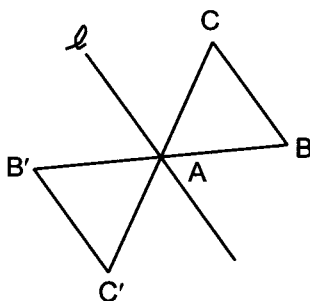**2** The expression $\sqrt{50}$ can be simplified to

(1) $5\sqrt{2}$ (3) $2\sqrt{25}$
(2) $5\sqrt{10}$ (4) $25\sqrt{2}$

$\sqrt{50}$

$\sqrt{25}\ \sqrt{2}$

$5\ \sqrt{2}$

3 The transformation of $\triangle ABC$ to $\triangle AB'C'$ is shown in the accompanying diagram.

Not a line reflection because C' would be here in a line reflection

This transformation is an example of a

(1) line reflection in line ℓ
(2) rotation about point A
(3) dilation — makes bigger or smaller
(4) translation — slides

4 Which expression is equivalent to 6.02×10^{23}?

(1) 0.602×10^{21} (3) 602×10^{21}
(2) 60.2×10^{21} (4) 6020×10^{21}

$602 \times 10^{(23-2)}$
$60.2 \times 10^{(23-1)}$
$.602 \times 10^{(23+1)}$

5 The Pentagon building in Washington, D.C., is shaped like a regular pentagon. If the length of one side of the Pentagon is represented by $n + 2$, its perimeter would be represented by

(1) $5n + 10$ (3) $n + 10$
(2) $5n + 2$ (4) $10n$

regular pentagon has 5 equal sides and 5 equal angles

$5(n+2)$

$5n + 10$

[handwritten: multiply ↗]

6 The product of $4x^2y$ and $2xy^3$ is

(1) $8x^2y^3$

(2) $8x^3y^3$

~~(3) $8x^3y^4$~~

(4) $8x^2y^4$

[handwritten:
$(4x^2y)(2xy^3)$
$(4)(2)(x^2)(x)(y)(y^3)$
$(8)(x^{2+1})(y^{(1+3)})$
$8 x^3 y^4$ *]*

7 Which equation is an illustration of the additive identity property?

(1) $x \cdot 1 = x$

~~(2) $x + 0 = x$~~

(3) $x - x = 0$

(4) $x \cdot \frac{1}{x} = 1$

[handwritten: The identity does not change the value.]

8 The formula $C = \frac{5}{9}(F - 32)$ can be used to find the Celsius temperature (C) for a given Fahrenheit temperature (F). What Celsius temperature is equal to a Fahrenheit temperature of 77°?

(1) 8°

~~(2) 25°~~

(3) 45°

(4) 171°

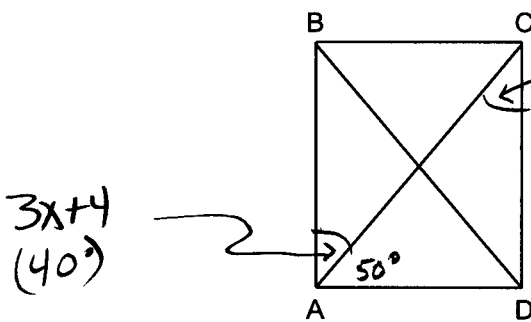*[handwritten:*
$F = 77$
$C = \frac{5}{9}(F - 32)$
$C = \frac{5}{9}(77 - 32)$
$C = \frac{5}{9}(45)$
$C = \frac{5}{9}\left(\frac{45}{1}\right)$
$C = 25$ *]*

9 In the accompanying diagram of rectangle $ABCD$, $m\angle BAC = 3x + 4$ and $m\angle ACD = x + 28$.

[diagram of rectangle ABCD with diagonals, labeled B C top, A D bottom, 50° at A; handwritten annotations: (40°), X+28, 3x+4 (40°)]

[handwritten:
$3x+4 = x+28$
$-x \qquad -x$
$2x+4 = 28$
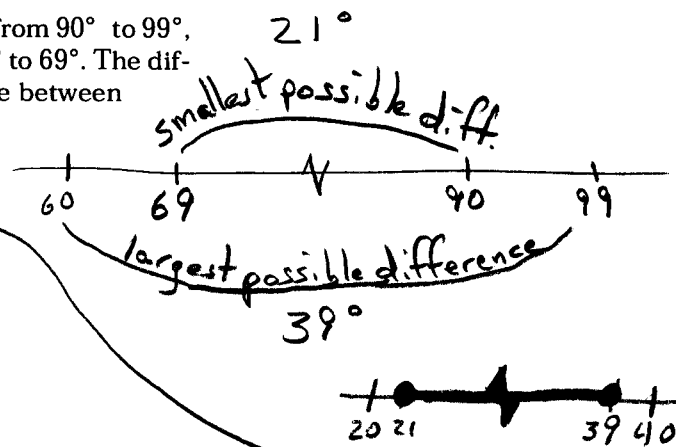$\quad -4 \quad -4$
$2x = 24$
$x = 12$ *]*

What is $m\angle CAD$?

(1) 12

(2) 37

(3) 40

~~(4) 50~~

10 On June 17, the temperature in New York City ranged from 90° to 99°, while the temperature in Niagara Falls ranged from 60° to 69°. The difference in the temperatures in these two cities must be between

(1) 20° and 30°

~~(2) 20° and 40°~~

(3) 25° and 35°

(4) 30° and 40°

[handwritten: 21°, smallest possible diff., number line 60 69 90 99, largest possible difference 39°, number line 20 21 39 40]

11 Which expression is equivalent to $\frac{a}{x} + \frac{b}{2x}$?

~~(1) $\frac{2a+b}{2x}$~~

(2) $\frac{2a+b}{x}$

(3) $\frac{a+b}{3x}$

(4) $\frac{a+b}{2x}$

[handwritten:
$\frac{a}{x} + \frac{b}{2x}$
$\frac{a(2x) + x(b)}{x(2x)}$ *]*

[handwritten:
$\frac{2ax + bx}{x(2x)} = \frac{x(2a+b)}{x(2x)} = \frac{2a+b}{2x}$ *]*

[handwritten: True]

12 What is true about the statement "If two angles are right angles, the angles have equal measure" and its converse "If two angles have equal measure then the two angles are right angles"?

[handwritten: Not always true.]

(1) The statement is true but its converse is false.
(2) The statement is false but its converse is true.
(3) Both the statement and its converse are false.
(4) Both the statement and its converse are true.

13 If 6 and x have the same mean (average) as 2, 4, and 24, what is the value of x?

(1) 5
(2) 10
(3) 14
(4) 36

[handwritten: $\frac{2+4+24}{3} = \frac{30}{3} = 10$]

[handwritten: $\frac{6+x}{2} = 10 \Rightarrow 6+x = 20$; $-6 \quad -6$; $x = 14$]

14 In a hockey league, 87 players play on seven different teams. Each team has at least 12 players. What is the largest possible number of players on any one team?
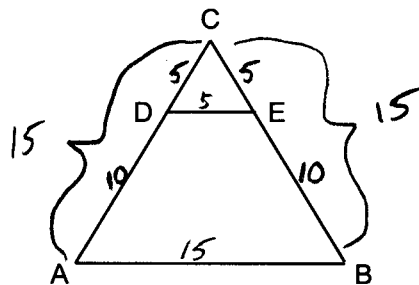
(1) 13
(2) 14
(3) 15
(4) 21

[handwritten: $7 \times 12 = 84$; $87 - 84 = 3$; There are 3 extra players ; If all 3 extra players are on same team, that team would have 15]

[handwritten: Check ✓ ; $\frac{6+14}{2} = \frac{20}{2} = 10$]

15 In the accompanying diagram of equilateral triangle ABC, $DE = 5$ and $\overline{DE} \parallel \overline{AB}$.

[handwritten labels on diagram: C; 5, 5; D, 5, E; 15; 10; 10; A, 15, B; 15]

[handwritten: $\overline{AB} = 15$; $\overline{BE} = 10$; $\overline{ED} = 5$; $\overline{DA} = 10$; 40]

If AB is three times as long as DE, what is the perimeter of quadrilateral $ABED$?

(1) 20
(2) 30
(3) 35
(4) 40

16 At a concert, $720 was collected for hot dogs, hamburgers, and soft drinks. All three items sold for $1.00 each. Twice as many hot dogs were sold as hamburgers. Three times as many soft drinks were sold as hamburgers. The number of soft drinks sold was

(1) 120
(2) 240
(3) 360
(4) 480

[handwritten: Let # hamburgers = H ; Let # hotdogs = 2H ; Let # softdrinks = 3H ; $H + 2H + 3H = 720$; $6H = 720$; $H = 120$; #softdrinks $= 3H = 3(120) = 360$]

17 How many different 6-letter arrangements can be formed using the letters in the word "**ABSENT**," if each letter is used only once?

(1) 6
(2) 36
(3) 720
(4) 46,656

[handwritten: # choices for each letter; 1st letter 2nd letter 3rd letter 4th letter 5th letter 6th letter ; 6 · 5 · 4 · 3 · 2 · 1 = 720]
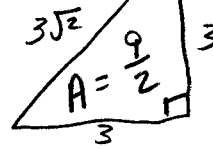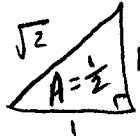
Strategy: Build an example.

18 The ratio of the corresponding sides of two similar squares is 1 to 3. What is the ratio of the area of the smaller square to the area of the larger square?

(1) $1:\sqrt{3}$ (3) 1:6
(2) 1:3   **(4) 1:9**

The area of the bigger △ is 9 times bigger than the area of the small △

19 What is the slope of the line whose equation is $3x - 4y - 16 = 0$?

(1) $\frac{3}{4}$ (3) 3
(2) $\frac{4}{3}$ (4) −4

$y = mx + b$
↳ slope

$y = mx + b$

$3x - 4y - 16 = 0$
$-3x \qquad\qquad -3x$

$-4y - 16 = -3x$
$\qquad +16 \quad +16$

$-4y = -3x + 16$

$D(-4)\dfrac{-4y}{-4} = \dfrac{-3x}{-4} + \dfrac{16}{-4}$

$y = \frac{3}{4}x - 4$

↳ $m = \frac{3}{4}$

20 What is the perimeter of an equilateral triangle whose height is $2\sqrt{3}$?

(1) 6   (3) $6\sqrt{3}$ ↳ 3 equal sides
(2) 12   (4) $12\sqrt{3}$
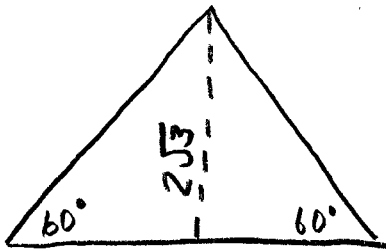
An equilateral triangle can be split into two 30°/60°/90° triangles
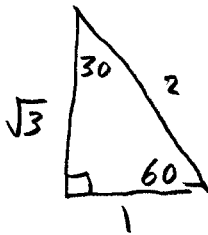
The ratios of the sides of a 30/60/90 triangle are always the same.

The height of the △ in the problem is 2 times the standard ratio, so all the sides are twice the standard ratio.

The perimeter is 12.

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [10]

21 Solve for x: $2(x - 3) = 1.2 - x$

$$2(x-3) = 1.2 - x$$
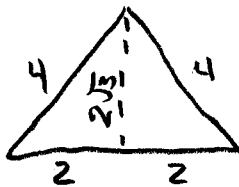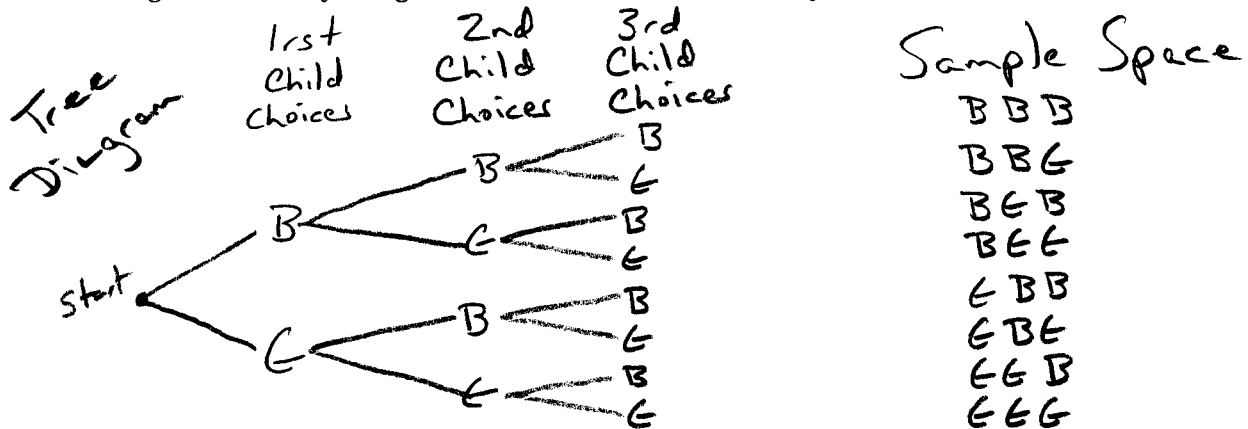$$2x - 6 = 1.2 - x$$
$$+x \qquad\qquad +x$$
$$\overline{}$$
$$3x - 6 = 1.2$$
$$+6 \quad +6$$
$$\overline{}$$
$$3x = 7.2$$
$$\boxed{x = 2.4}$$

Check

$$2(2.4 - 3) = 1.2 - 2.4$$
$$2(-.6) = -1.2$$
$$-1.2 = -1.2 \checkmark$$

22 The Grimaldis have three children born in different years.

   a Draw a tree diagram or list a sample space to show all the possible arrangements of boy and girl children in the Grimaldi family.

Tree Diagram

```
                1rst        2nd      3rd
                Child       Child    Child
                Choices     Choices  Choices
                                      B
                         B
                                      G
                    B
                                      B
                         G
                                      G
          start
                                      B
                         B
                                      G
                    G
                                      B
                         G
                                      G
```

Sample Space
```
B B B
B B G
B G B
B G G
G B B
G B G
G G B
G G G
```

   b Using your information from part *a*, what is the probability that the Grimaldis have three boys?

$$\frac{1}{8}$$

23 Paloma has 3 jackets, 6 scarves, and 4 hats. Determine the number of different outfits consisting of a jacket, a scarf, and a hat that Paloma can wear.

$$\underset{\substack{\text{Jacket}\\ \text{Choices}}}{\boxed{3}} \cdot \underset{\substack{\text{Scarf}\\ \text{Choices}}}{\boxed{6}} \cdot \underset{\substack{\text{Hat}\\ \text{Choices}}}{\boxed{4}} = \boxed{72}$$

Answer

24 In a recent poll, 600 people were asked whether they liked Chinese food. A circle graph was constructed to show the results. The central angles for two of the three sectors are shown in the accompanying diagram. How many people had no opinion?

Check

			?
Like	160°	266⅔	?
Dislike	140°	233⅓	?
No Opinion	60°	100	✓
Total	360°	600	✓

Chinese Food

Answer
100 people had no opinion

Like 160°

No opinion

Dislike 140°

Step #2

$$\frac{Part}{Whole}\bigg| \frac{60°}{360°} = \frac{X}{600}$$

$$60(600) = 360\,X$$
$$36,000 = 360\,X$$
$$\boxed{100 = X}$$

Step #1

No opinion degrees = 360° − (like degrees plus dislike degrees)

" " " = 360° − (160° + 140°)

" " " = 360° − 300°

No opinion degrees = 60°

$$\frac{140}{360} = \frac{X}{600}$$
$$X = 233\tfrac{1}{3}$$
Dislike

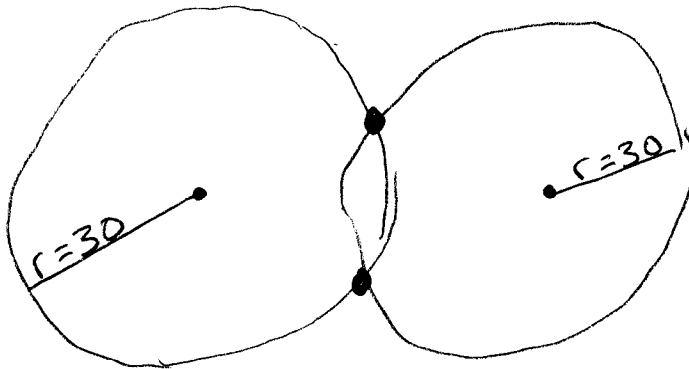$$\frac{160}{360} = \frac{X}{600}$$
$$X = 266\tfrac{2}{3}$$
Like

25 Maria's backyard has two trees that are 40 feet apart, as shown in the accompanying diagram. She wants to place lampposts so that the posts are 30 feet from both of the trees. Draw a sketch to show where the lampposts could be placed in relation to the trees. How many locations for the lampposts are possible?

Side View

Top View

40 ft

r = 30

r = 30
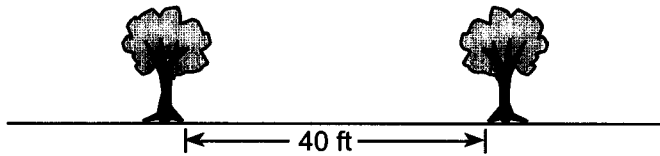
Two lampposts are possible

Part III

Answer all questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 cr   edit.   [15]

---

26 Solve for x: $x^2 + 3x - 40 = 0$ → one factor is pos. one is neg.

$$x^2 + 3x - 40 = 0$$

$$(x + \underline{})(x - \underline{}) = 0$$

$$(x + 8)(x - 5) = 0$$

$$x + 8 = 0 \qquad x - 5 = 0$$

$$x = -8 \qquad x = 5$$

The solutions are

$$\boxed{-8 \text{ and } 5}$$

Factors of 40 must
1) multiply to (-40)
2) sum to $(+3)$

Factors of 40

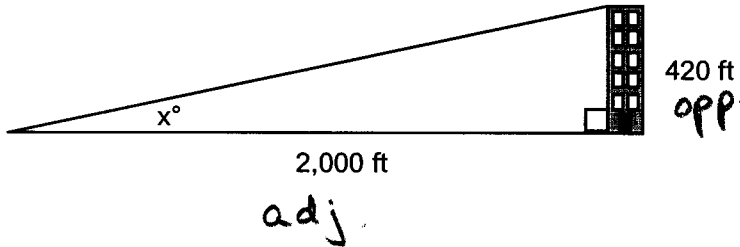|	
1	40
2	20
4	10
5	8

$-5 \qquad 8$

---

27 A person standing on level ground is 2,000 feet away from the foot of a 420-foot-tall building, as shown in the accompanying diagram. To the *nearest degree,* what is the value of x?

420 ft
opp.

2,000 ft
adj

SOH-CAH-TOA

$$\sin = \frac{opp}{hyp}$$

$$\cos = \frac{adj}{hyp}$$

$$\tan = \frac{opp}{adj}$$

$$\tan X = \frac{opp}{adj} = \frac{420}{2,000}$$

$$\tan X = .21$$

Reminder! Set calculator to degree mode

$$\text{arctan } .21 = 11.85977912$$

$$\boxed{12°}$$

2nd TAN (. 21) Enter

28 Bob and Ray are describing the same number. Bob says, "The number is a positive even integer less than or equal to 20." Ray says, "The number is divisible by 4." If Bob's statement is true and Ray's statement is false, what are all the possible numbers?

True	False
Pos. even integers Less than 20	Divisible by 4
18	
~~16~~	not 16
14	
~~12~~	not 12
10	
~~8~~	not 8
6	
~~4~~	not 4
2	

The possible #s are

2, 6, 10, 14, 18

29 Line ℓ contains the points (0,4) and (2,0). Show that the point (−25,81) does *or* does not lie on line ℓ.

Slope Formula $m = \dfrac{rise}{run} = \dfrac{y_2 - y_1}{x_2 - x_1}$

All points on a line are colinear and have same slope

(0,4) and (2,0)

$m = \dfrac{0-4}{2-0} = \dfrac{-4}{2} = -2$

The slope of the line containing (0,4) and (2,0) is −2.

(0,4) and (−25, 81)

$m = \dfrac{81-4}{-25-0} = \dfrac{77}{-25}$

$\dfrac{77}{-25} \neq -2$

∴ (−25, 81) is not on the line.

30 A painting that regularly sells for a price of $55 is on sale for 20% off. The sales tax on the painting is 7%. Will the final total cost of the painting differ depending on whether the salesperson deducts the discount before adding the sales tax or takes the discount after computing the sum of the original price and the sales tax on $55?

Option 1

$$[55 - (55 \cdot .20)](1.07) =$$

↑ Original Price   ↑ Less 20%   ↑ Add 7% tax

$$(55 - 11)(1.07) =$$
$$44(1.07) = 47.08 \text{ total price}$$

Option 2

$$\underset{\text{Price w/ tax}}{(55)(1.07)} = 58.85$$

$$\text{less } (.20)(58.85) \quad \frac{-11.77}{47.08 \text{ total price}}$$

No. The price is the same either way.

Part IV

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [20]

---

31 The profits in a business are to be shared by the three partners in the ratio of 3 to 2 to 5. The profit for the year was $176,500. Determine the number of dollars each partner is to receive.
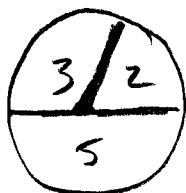
Answers

Partner A $\frac{2}{10}(176,500) = $ $35,300

Partner B $\frac{3}{10}(176,500) = $ $52,950
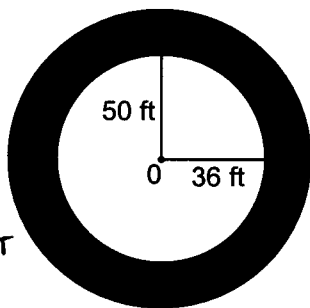
Partner C $\frac{5}{10}(176,500) = $ $88,250

Check: #'s sum to $176,500 ✓

---

32 If asphalt pavement costs $0.78 per square foot, determine, to the nearest cent, the cost of paving the shaded circular road with center O, an outside radius of 50 feet, and an inner radius of 36 feet, as shown in the accompanying diagram.

$A_0 = \pi r^2$

$A(\text{Big Circle}) = \pi 50^2$
$= 2500\pi$

50 ft   0   36 ft

$3,782.477555 \text{ ft}^2$
$\times 0.78 per ft^2
2950.332493

$2950.33

$A(\text{small circle}) = \pi 36^2$
$= 1296\pi$

Shaded Area = $2500\pi - 1296\pi$
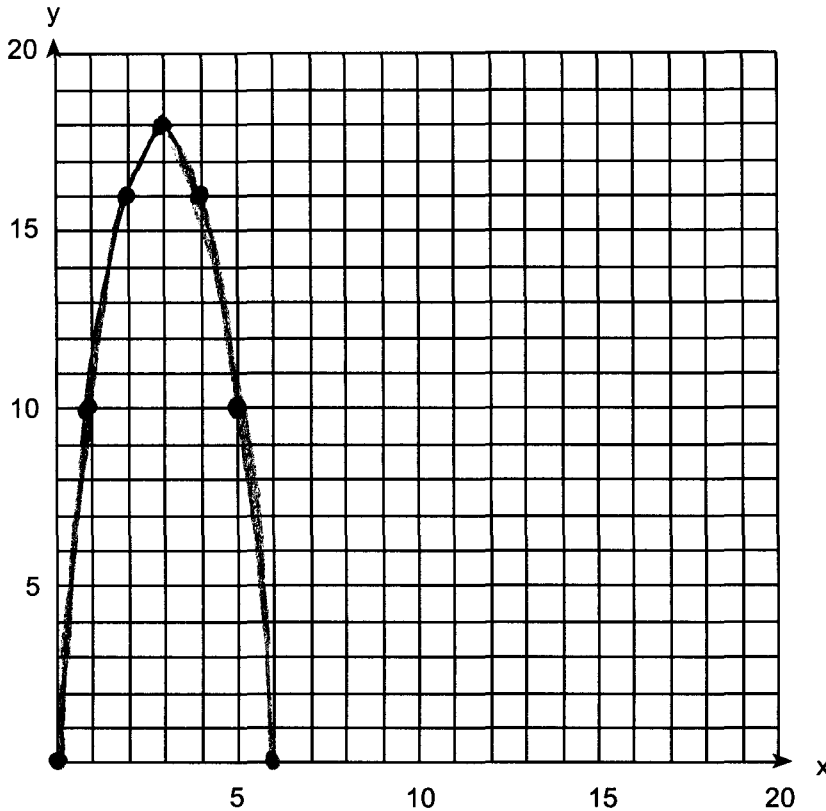Shaded Area = 1204π
Shaded Area = $3,782.477555$

Math. A – Aug. '99   [12]

33 An arch is built so that it is 6 feet wide at the base. Its shape can be represented by a parabola with the equation $y = -2x^2 + 12x$, where y is the height of the arch.

*a* Graph the parabola from $x = 0$ to $x = 6$ on the grid below.

Enter $Y = -2x^2 + 12X$ in Y-editor. Go to table of values

X	Y
0	0
1	10
2	16
3	18
4	16
5	10
6	0

b Determine the maximum height, y, of the arch.

The maximum height is 18 feet

34 Mr. Gonzalez owns a triangular plot of land *BCD* with *DB* = 25 yards and *BC* = 16 yards. He wishes to purchase the adjacent plot of land in the shape of right triangle *ABD*, as shown in the accompanying diagram, with *AD* = 15 yards. If the purchase is made, what will be the total number of square yards in the area of his plot of land, △*ACD*?

D

15 yd   25 yd

A   **2 O**   B   16 yd   C

(not drawn to scale)
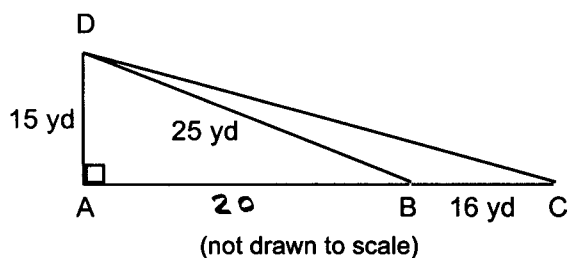
Pythagorean Theorem

$$a^2 + b^2 = c^2$$

△ ABD

$$15^2 + (\overline{AB})^2 = 25^2$$

$$225 + (\overline{AB})^2 = 625$$

$$-225 \qquad\qquad -225$$

$$(AB)^2 = 400$$

$$\sqrt{(AB)^2} = \sqrt{400}$$

$$AB = 20$$

Area △ = ½ bh

Area (△ ACD) = ½ (20 + 16)(15)

= ½ (36)(15)

= (18)(15)

Area (△ ACD) = $\boxed{270 \text{ yards}^2}$

35 Two health clubs offer different membership plans. The graph below represents the total cost of belonging to Club *A* and Club *B* for one year.

Health Club Fees

[Graph: y-axis labeled in dollars from $0 to $250 in increments of 25; x-axis labeled "Month" from 0 to 12. Two lines shown, "Club A" and "Club B", intersecting with handwritten annotation "Same" near the intersection point. Handwritten "Membership Cost for Club A" pointing to the graph.]
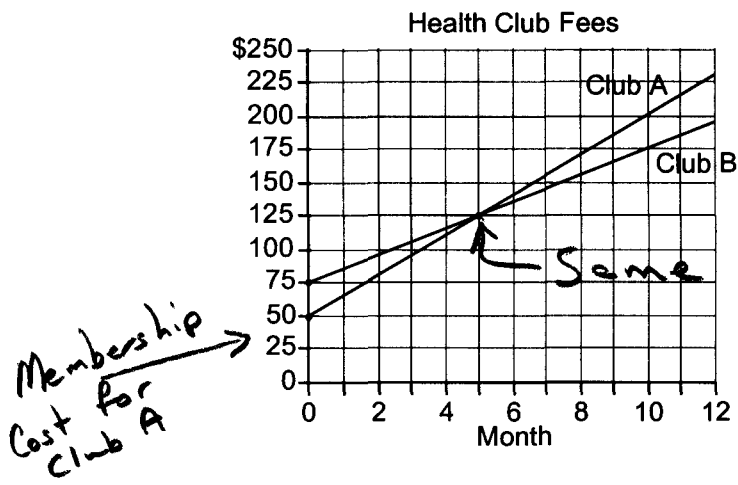
a If the yearly cost includes a membership fee plus a monthly charge, what is the membership fee for Club *A*?

[Answer box: $50]

b (1) What is the number of the month when the total cost is the same for both clubs?

[Answer box: 5]

(2) What is the total cost for Club *A* when both plans are the same?

[Answer box: $125.00]

c What is the monthly charge for Club *B*?

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{125 - 75}{5 - 0} = \frac{50}{5} = 10$$

$(0, 75) \quad (5, 125)$

[Answer box: $10]

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS A

Thursday , August 12, 1999 — 8:30 to 11:30 a.m., only

ANSWER SHEET

Pupil ...*Imaginary Student (L)*... Sex: ☐ Male ☐ Female   Grade ....

Teacher ...*Mr. Steve*... School ...*I HS @ P H*...

Your answers to Part I should be r   ecor ded on this answer sheet.

Par t I

Answer all 20 questions in this par   t.

1	4	11	1
2	1	12	1
3	2	13	3
4	3	14	3
5	1	15	4
6	3	16	3
7	2	17	3
8	2	18	4
9	4	19	1
10	2	20	2

Your answers for Parts II, III, and IV should be written in the test booklet.

The declaration below should be signed when you have completed the examination.

I do hereby af fir m, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

_____
Signatur e

Math. A – Aug. '99