The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS A

Thursday, August 16, 2001 — 8:30 to 11:30 a.m., only

Print Your Name: Imaginary Student

Print Your School's Name: www.jmap.org

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will *not* be scored. All work should be written in pen, except graphs and drawings, which should be done in pencil.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A minimum of a scientific calculator, a straightedge (ruler), and a compass must be available for your use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

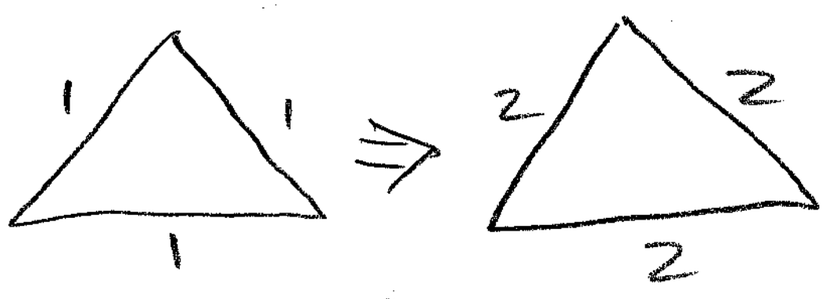
Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Record your answers in the spaces provided on the separate answer sheet. [40]

1 The perimeter of an equilateral triangle varies directly as the length of a side. When the length of a side is doubled, the perimeter of the triangle is

(1) halved
(2) doubled
(3) multiplied by 3
(4) divided by 3

Use this space for computations.



3 ⇒ 6

2 Which expression is rational?

(1) π
(2) $\sqrt{\frac{1}{2}}$
(3) $\sqrt{3}$
(4) $\sqrt{\frac{1}{4}} \Rightarrow \frac{\sqrt{1}}{\sqrt{4}} \Rightarrow \frac{1}{2}$

A rational number can be expressed as the ratio of two integers

3 Written in simplest factored form, the binomial $2x^2 - 50$ can be expressed as

(1) $2(x-5)(x-5)$
(2) $2(x-5)(x+5)$
(3) $(x-5)(x+5)$
(4) $2x(x-50)$

$2x^2 - 50$
$2(x^2 - 25)$
$2(x+5)(x-5)$

4 Which statement is logically equivalent to "If I did not eat, then I am hungry"?

(1) If I am not hungry, then I did not eat.
(2) If I did not eat, then I am not hungry.
(3) If I am not hungry, then I did eat.
(4) If I am hungry, then I did eat.

Given If 1, then 2

Inverse If not 1, then not 2

This is logically equivalent to the Given → Converse If 2, then 1

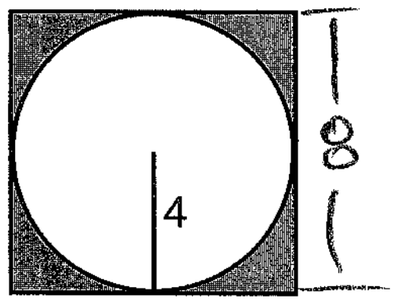
Contrapositive If not 2, then not 1

If (I did not eat)^1, then (I am hungry)^2

If not (I am hungry)^2, then ~~not~~ (I did not eat)^2

double negatives cancel

5 In the accompanying diagram, a circle with radius 4 is inscribed in a square.



What is the area of the shaded region?

(1) $64 - 16\pi$
(2) $16 - 16\pi$
(3) $64\pi - 8\pi$
(4) $16 - 8\pi$

Use this space for computations.

$A_{\square} = 8^2$

$A_{\square} = 64$

$A_o = \pi r^2$

$A_o = \pi 4^2$

$A_o = 16\pi$

$A_{shaded\ Region} = A_{\square} - A_o = 64 - 16\pi$

6 Which letter below has point symmetry, but does *not* have line symmetry?

(1) H
(2) N
(3) A
(4) E

7 The value of 5! is

(1) $\frac{1}{5}$
(2) 5
(3) 20
(4) 120

$5! = 5 \times 4 \times 3 \times 2 \times 1$

$5! = 120$

8 What is the approximate circumference of a circle with radius 3?

(1) 7.07
(2) 9.42
(3) 18.85
(4) 28.27

$r = 3$

$d = 6$

$A_o = \pi d$

$A_o = \pi 6$

$A_o = 6\pi \Rightarrow 6(3.14159...)$

9 The sum of the measures of the interior angles of an octagon is

(1) 180°
(2) 360°
(3) 540°
(4) 1,080°

180° 3 sides
360° 4
540° 5
720° 6
900° 7
1080° 8 sides

By formula
$n - 2(180)$
$8 - 2(180)$
$6(180)$
1080

By reasoning

10 The exact average of a set of six test scores is 92. Five of these scores are 90, 98, 96, 94, and 85. What is the other test score?

(1) 92
(2) 91
(3) 89
(4) 86

Use this space for computations.

$$92 = \frac{90+98+96+94+85+X_6}{6}$$

$$92 = \frac{463+X_6}{6} \Rightarrow 552 = 463 + X_6$$

$$-463 \quad -463$$

$$89 = X_6$$

11 A certain car comes in three body styles with a choice of two engines, a choice of two transmissions, and a choice of six colors. What is the minimum number of cars a dealer must stock to have one car of every possible combination?

(1) 13
(2) 36
(3) 42
(4) 72

Body Style Choices: 3 × Engine Choices: 2 × Transmission Choices: 2 × Color Choices: 6 = 72

12 The operation element @ is determined by the following table:

@	*a*	*b*	*c*
a	*a*	*b*	*c*
b	*b*	*c*	*a*
c	*c*	*a*	*b*

What is the identity element of this operation?

(1) *a*, only
(2) *b*, only
(3) *c*
(4) *a* and *b*

identity element of operation

a @ anything = anything

does not change

When a acts on any other element, the other element does not change.

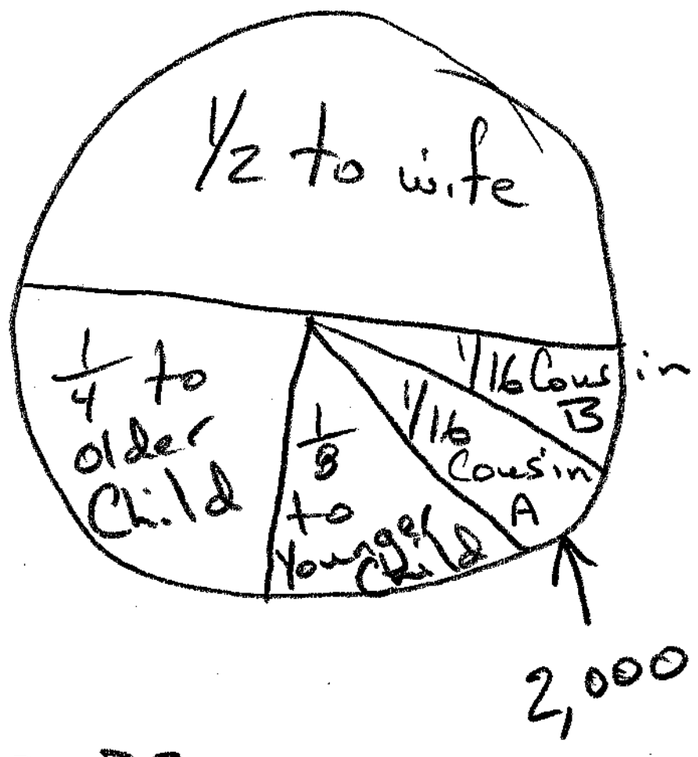
13 If *n* represents an odd number, which computation results in an answer that is an even number?

(1) $2 \times n + 1$
(2) $2 \times n - 1$
(3) $3 \times n - 2$
(4) $3 \times n + 1$

$2 \times n \Rightarrow$ even $3 \times n \Rightarrow$ odd

14 In his will, a man leaves one-half of his money to his wife, one-half of what is then left to his older child, and one-half of what is then left to his younger child. His two cousins divide the remainder equally, each receiving $2,000. What was the total amount of money in the man's will?

(1) $40,000
(2) $32,000
(3) $24,000
(4) $16,000



$\frac{1}{16} = 2000$ $16 \times 2000 = 32{,}000$

15 If $a + b$ is less than $c + d$, and $d + e$ is less than $a + b$, then e is

(1) less than c
(2) equal to c
(3) less than d
(4) greater than d

Use this space for computations.

Part 1: $a+b < c+d$

Part 2: $(d+e) < (a+b) < (c+d)$

$-d \quad -d \quad -d$

$e < a+b-d < c$

$d+e < a+b$

$\therefore (d+e) < (a+b) < (c+d)$

16 Which statement is the converse of "If it is a 300 ZX, then it is a car"?

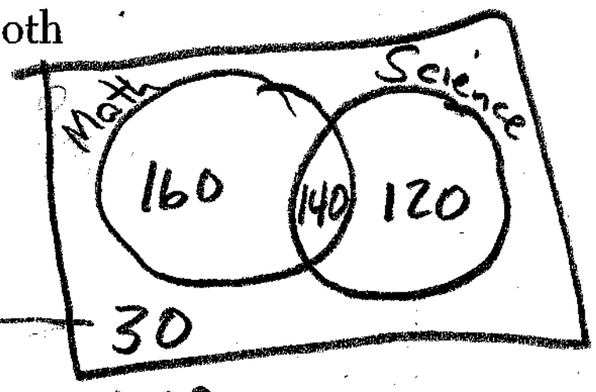
(1) If it is not a 300 ZX, then it is not a car.
(2) If it is not a car, then it is not a 300 ZX.
(3) If it is a car, then it is a 300 ZX.
(4) If it is a car, then it is not a 300 ZX.

Given If 1, then 2
Inverse If not 1, then not 2
We want converse → Converse If 2, then 1
Contrapositive If not 2, then not 1

If (2) it is a car, then (1) it is a 300 ZX.

17 In a class of 450 students, 300 are taking a mathematics course and 260 are taking a science course. If 140 of these students are taking both courses, how many students are *not* taking either of these courses?

(1) 30
(2) 40
(3) 110
(4) 140



18 What is the solution set of $m^2 - 3m - 10 = 0$?

(1) {5,–2}
(2) {2,–5}
(3) {3,–10}
(4) {3,10}

$m^2 - 3m - 10 = 0$

$(m+\_)(m-\_) = 0$

$(m+2)(m-5) = 0$

$m+2=0 \quad m-5=0$

$m=-2 \quad m=5$

Factors of 10
1 10
2 5

19 Which expression is equivalent to $x^{-1} \cdot y^2$?

(1) xy^2
(2) $\frac{y^2}{x}$
(3) $\frac{x}{y^2}$
(4) xy^{-2}

$x^{-1} \cdot y^2$

$\frac{1}{x} \cdot \frac{y^2}{1} \Rightarrow \frac{y^2}{x}$

20 What is the smallest integer greater than 1 that is both the square of an integer and the cube of an integer?

(1) 8
(2) 9
(3) 36
(4) 64

$\sqrt{8} = ?$ $\quad \sqrt[3]{8} = 2$

$\sqrt{9} = 3$ $\quad \sqrt[3]{9} = ?$

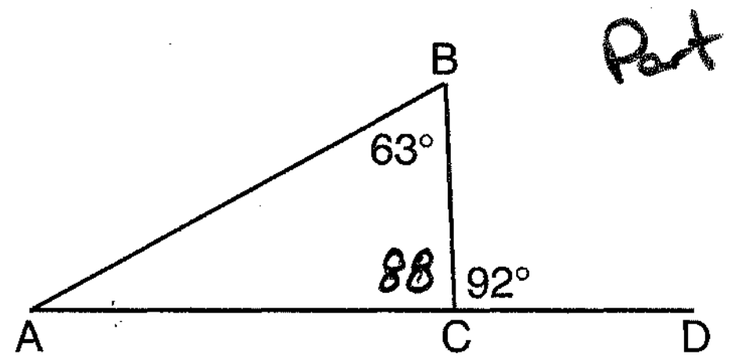
$\sqrt{36} = 6$ $\quad \sqrt[3]{36} = ?$

$\sqrt{64} = 8$ $\quad \sqrt[3]{64} = 4$ ✓

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [10]

21 Triangle ABC, with side $\overline{AC}$ extended to D, is shown in the accompanying diagram. If $m\angle ABC = 63$ and $m\angle BCD = 92$, what is $m\angle BAC$?



Part 1

$\angle BCD + \angle BCA = 180°$

$92° + \angle BCA = 180°$

$-92 \qquad -92$

$\angle BCA = 88$

Part 2

$\angle DCA + \angle ABC + \angle BAC = 180°$

$88° + 63° + \angle BAC = 180°$

$151° + \angle BAC = 180°$

$-151° \qquad -151°$

$\angle BAC = 29°$ Answer

22 How many feet from the base of a house must a 39-foot ladder be placed so that the top of the ladder will reach a point on the house 36 feet from the ground?

Pythagorean Theorem

$a^2 + b^2 = c^2$

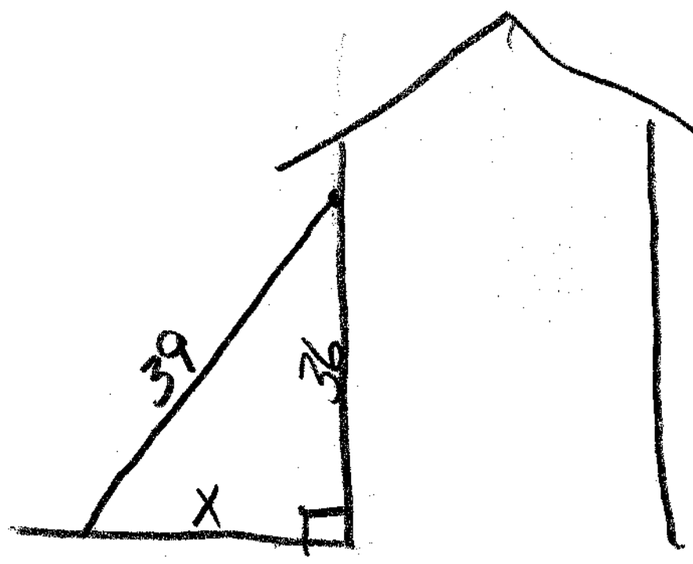
$x^2 + (36)^2 = (39)^2$

$x^2 + 1296 = 1521$

$-1296 \qquad -1296$

$x^2 = 225$

$x = 15$



15 feet

23 Subtract $5x^2 - 7x - 6$ from $9x^2 + 3x - 4$.

$9x^2 + 3x - 4$

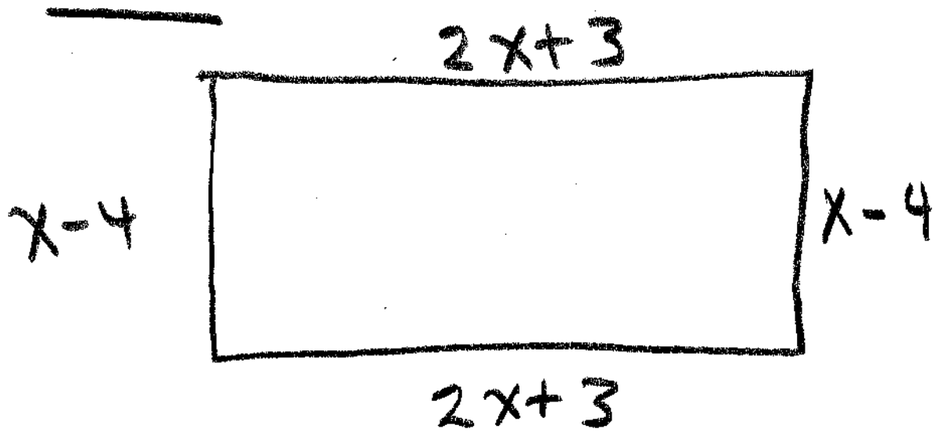
$-(5x^2 - 7x - 6)$

To subtract Change the Signs and add

$-5x^2 + 7x + 6$

$4x^2 + 10x + 2$ Answer

24 An engineer measured the dimensions for a rectangular site by using a wooden pole of unknown length x. The length of the rectangular site is 2 pole measures increased by 3 feet, while the width is 1 pole measure decreased by 4 feet. Write an algebraic representation, in terms of x, for the perimeter of the site.



Perimeter $= (2x+3) + (x-4) + (2x+3) + (x-4)$

$3x-1$ $3x-1$

Perimeter $= 6x-2$

25 Simplify: $\sqrt{50r^2s^4}$

$\sqrt{50r^2s^4}$

$(\sqrt{50})(\sqrt{r^2})(\sqrt{s^4})$

$(\sqrt{2})(\sqrt{25})(r)(s^2)$

$(\sqrt{2})(5)(r)(s^2)$

$5rs^2\sqrt{2}$

Part III

Answer all questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [15]

26 Megan decides to go out to eat. The menu at the restaurant has four appetizers, three soups, seven entrees, and five desserts. If Megan decides to order an appetizer *or* a soup, and one entree, and two different desserts, how many different choices can she make?

Appetizer or Soup Choices (4+3) · Entree Choices · Dessert #1 · Dessert #2

$$7 \cdot 7 \cdot \frac{5 \cdot 4}{2 \cdot 1} = 980$$

She has **490 choices**

27 There are four students, all of different heights, who are to be randomly arranged in a line. What is the probability that the tallest student will be first in line and the shortest student will be last in line?

$$P = \frac{\text{desired outcome}}{\text{total possible outcomes}}$$

$$P_{(A+B)} = P_A \cdot P_B$$

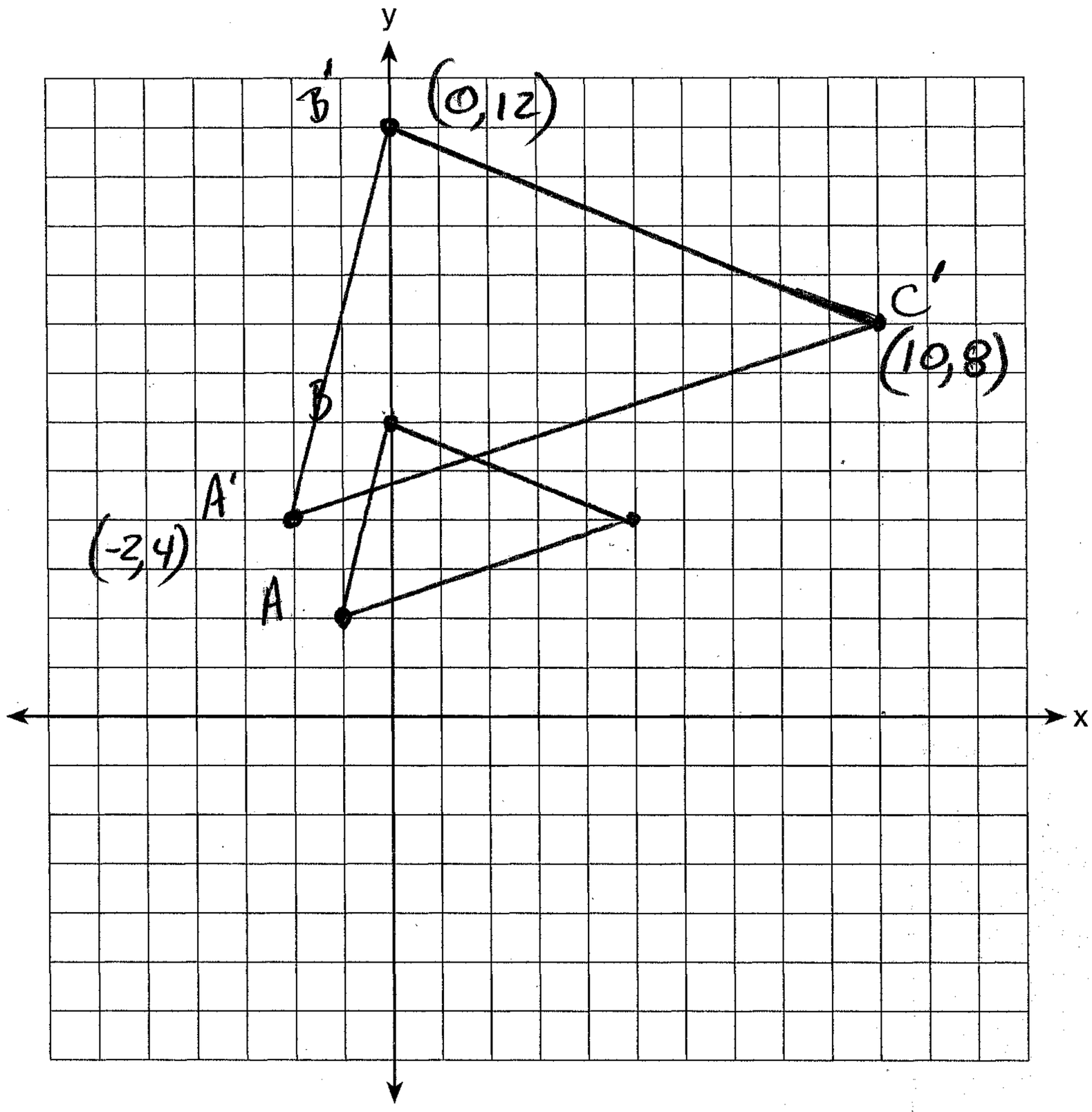
$$P(\text{tallest, either middle, either middle, shortest}) = P_A \cdot P_B \cdot P_C \cdot P_D$$

Tallest · Either Middle · other Middle · Shortest

$$= \frac{1}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{1}$$

$$P = \frac{2}{24}$$

28 On the accompanying set of axes, graph $\triangle ABC$ with coordinates $A(-1,2)$, $B(0,6)$, and $C(5,4)$. Then graph $\triangle A'B'C'$, the image of $\triangle ABC$ after a dilation of 2.



29 Ramón said that the set of integers is *not* closed for one of the basic operations (addition, subtraction, multiplication, or division). You want to show Ramón that his statement is correct.

For the operation for which the set of integers is *not* closed, write an example using:

- a positive even integer and a zero
- a positive and a negative even integer
- two negative even integers

$\frac{0}{2}$ $\frac{+2}{-4}$ $\frac{-2}{-6}$

Be sure to explain why *each* of your examples illustrates that the set of integers is *not* closed for that operation.

None of the examples given can be reduced to a positive or negative whole number, which is the definition of an integer.

30 Shanaya graphed the line represented by the equation $y = x - 6$.

Write an equation for a line that is parallel to the given line.

$Y = X - 8$ (same slope different y-intercept)

Write an equation for a line that is perpendicular to the given line.

$Y = -X - 6$ (slope is the negative reciprocal of $\frac{+1}{}$)

Write an equation for a line that is identical to the given line but has different coefficients.

$2Y = 2X - 12$

or

$3Y = 3X - 18$

Part IV

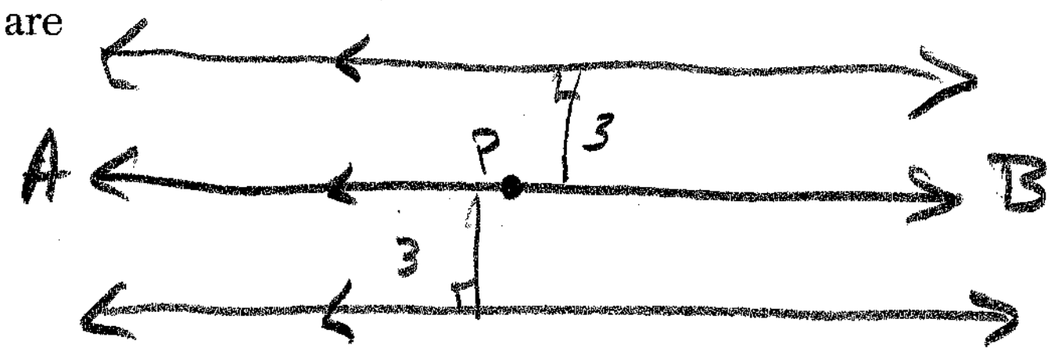
Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [20]

31 Point P is located on $\overleftrightarrow{AB}$.

a Describe the locus of points that are

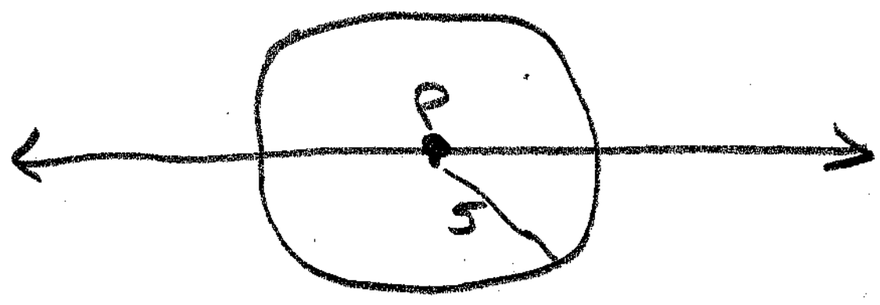
(1) 3 units from $\overleftrightarrow{AB}$

2 Parallel lines 3 units from $\overleftrightarrow{AB}$

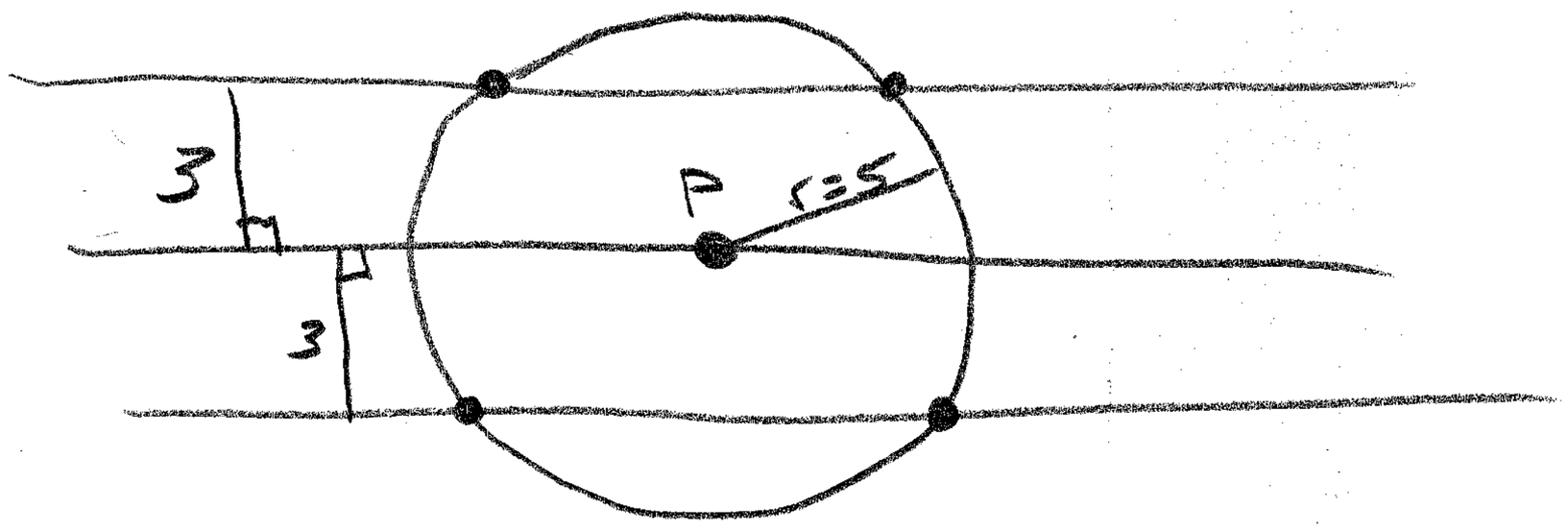


(2) 5 units from point P

A circle w/ center at P and radius = 5



b How many points satisfy both conditions in part *a*?



4 points satisfy both conditions

32 The ninth graders at a high school are raising money by selling T-shirts and baseball caps. The number of T-shirts sold was three times the number of caps. The profit they received for each T-shirt sold was $5.00, and the profit on each cap was $2.50. If the students made a total profit of $210, how many T-shirts *and* how many caps were sold?

Let T = # T-shirts

Let C = # Caps

$T = 3C$

$5T + 2.5C = 210$

$5(3C) + 2.5C = 210$

$15C + 2.5C = 210$

$17.5C = 210$

$C = 12$

$T = 3C$

$T = 3(12)$

$T = 36$

They sold
12 caps
36 T-shirts

Check

$T = 3C$

$36 = 3(12)$

$36 = 36$ ✓

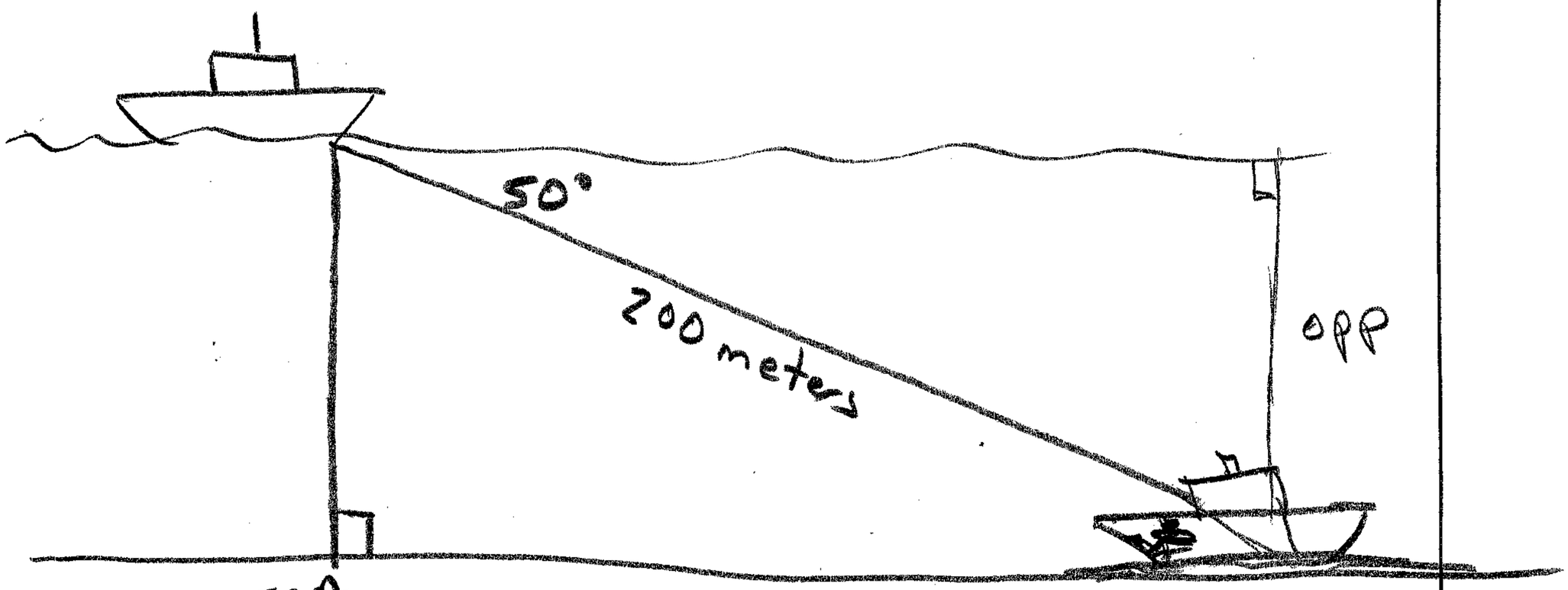
$5T + 2.5C = 210$

$5(36) + 2.5(12) = 210$

$180 + 30 = 210$

$210 = 210$ ✓

33 A ship on the ocean surface detects a sunken ship on the ocean floor at an angle of depression of 50°. The distance between the ship on the surface and the sunken ship on the ocean floor is 200 meters. If the ocean floor is level in this area, how far above the ocean floor, to the *nearest meter*, is the ship on the surface?



SOH-CAH-TOA

$\sin = \frac{opp}{hyp}$ $\cos = \frac{adj}{hyp}$ $\tan = \frac{opp}{adj}$.

$\sin 50° = \frac{opp}{200}$

$200(\sin 50°) = opp$

Set Calculator to Degrees

$153.20888886 = opp.$

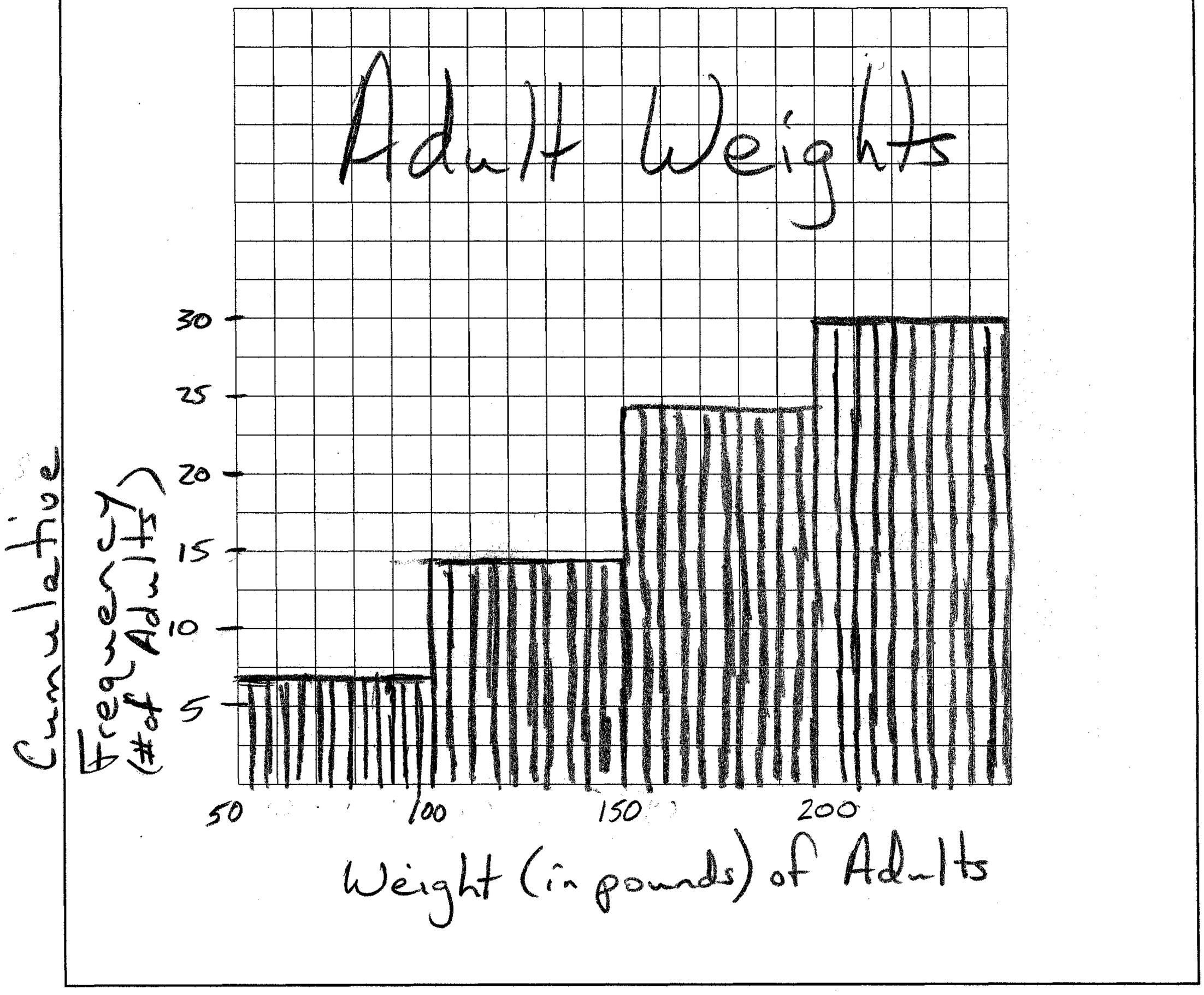
The ship is **153 meters** above the ocean floor.

34 The following data consists of the weights, in pounds, of 30 adults:

~~195~~, ~~206~~, ~~100~~, ~~98~~, ~~150~~, ~~210~~, ~~195~~, ~~106~~, ~~105~~, ~~168~~, ~~180~~, ~~212~~, ~~104~~, ~~195~~, ~~100~~,
~~216~~, ~~195~~, ~~209~~, ~~112~~, ~~99~~, ~~206~~, ~~116~~, ~~195~~, ~~100~~, ~~142~~, ~~100~~, ~~135~~, ~~98~~, ~~160~~, ~~155~~

Using the data, complete the accompanying cumulative frequency table and construct a cumulative frequency histogram on the grid below.

	Interval	Frequency	Cumulative Frequency
11 ++++	51–100	7	7
11 ++++	101–150	7	14
+++ +++	151–200	10	24
1 +++	201–250	6	30



35 Solve the following system of equations algebraically.

$$y = x^2 + 4x - 2$$
$$y = 2x + 1$$

Part 1

$y = x^2 + 4x - 2$

$y = 2x + 1$

$x^2 + 4x - 2 = 2x + 1$

$-2x \quad -2x$

$x^2 + 2x - 2 = 1$

$-1 \quad -1$

$x^2 + 2x - 3 = 0$

$(x + \_)(x - \_) = 0$

$(x+3)(x-1) = 0$

Factors of 3 are 1 and 3

$x + 3 = 0 \quad x - 1 = 0$

$x = -3 \quad x = 1$

Check

$(-3, -5)$ $\quad y = 2x + 1$

$-5 = 2(-3) + 1$

$-5 = -6 + 1$

$-5 = -5$ ✓

Part 2

$y = (-3)^2 + 4(-3) - 2$

$y = 9 - 12 - 2$

$y = -3 - 2$

$y = -5$

One Solution

$(-3, -5)$

$y = (1)^2 + 4(1) - 2$

$y = 1 + 4 - 2$

$y = 5 - 2$

$y = 3$

2nd Solution

$(1, 3)$

$(1, 3)$ $\quad y = 2x + 1$

$3 = 2(1) + 1$

$3 = 2 + 1$

$3 = 3$ ✓

Tear Here

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS A

Thursday, August 16, 2001 — 8:30 to 11:30 a.m., only

ANSWER SHEET

Student Imaginary Student Sex: ☐ Male ☐ Female Grade .....

Teacher Mr. Steve School IHS@PH

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer all 20 questions in this part.

1 2	6 2	11 4	16 3
2 4	7 4	12 1	17 1
3 2	8 3	13 4	18 1
4 3	9 4	14 2	19 2
5 1	10 3	15 1	20 4

Your answers for Parts II, III, and IV should be written in the test booklet.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

RSW

Signature

Tear Here