The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS A

Wednesday, August 16, 2000 — 8:30 to 11:30 a.m., only

Print Your Name: Imaginary Student (c)

Print Your School's Name: www.jmap.org

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will *not* be scored. All work should be written in pen, except graphs and drawings, which should be done in pencil.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers for the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps you take, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

Notice...

A minimum of a scientific calculator, a straightedge (ruler), and a compass must be available for your use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Record your answers in the spaces provided on the separate answer sheet. [40]

1 The product of $2x^3$ and $6x^5$ is

(1) $10x^8$
(2) $12x^8$
(3) $10x^{15}$
(4) $12x^{15}$

multiply

$(2x^3)(6x^5)$
$(2)(6)(x^3)(x^5)$
$(12)\,x^{(3+5)} = 12x^8$

Use this space for computations.

2 A hockey team played n games, losing four of them and winning the rest. The ratio of games won to games lost is

(1) $\frac{n-4}{4}$
(2) $\frac{4}{n-4}$
(3) $\frac{4}{n}$
(4) $\frac{n}{4}$

$\frac{\#\text{ won}}{\#\text{ lost}} = \frac{n-4}{4}$

total = n

3 In the coordinate plane, what is the total number of points 5 units from the origin and equidistant from both the x- and y-axes?

(1) 1
(2) 2
(3) 0
(4) 4

circle w/ radius = 5
Equidistant from x + y axes
origin
Equidistant from x + y axes

4 Expressed in decimal notation, 4.726×10^{-3} is

(1) 0.004726
(2) 0.04726
(3) 472.6
(4) 4,726

Move 3 places
00.004.72600
+3+2+1
0.004726

5 Which table does *not* show an example of direct variation?

(1)

x	y
1	4
2	8
3	12
4	16

(3)

x	y
1	$\frac{1}{2}$
2	1
3	$\frac{3}{2}$
4	2

(2)

x	y
2	24
4	12
6	8
8	6

(4)

x	y
−4	−20
−3	−15
−2	−10
−1	−5

Direct variation
- always passes through (0, 0)
- all points make proportions

$\frac{x_1}{y_1} = \frac{x_2}{y_2} = \frac{x_3}{y_3}$

(1) $\frac{1}{4} \stackrel{?}{=} \frac{2}{8} \stackrel{?}{=} \frac{3}{12} \stackrel{?}{=} \frac{4}{16}$ #1 is direct variation
$\frac{1}{4} = \frac{1}{4} = \frac{1}{4} = \frac{1}{4}$

(2) $\frac{2}{24} \stackrel{?}{=} \frac{4}{12} \stackrel{?}{=} \frac{6}{8} \stackrel{?}{=} \frac{8}{6}$ } #(2) is not direct variation
$\frac{1}{12} \neq \frac{1}{3} \neq \frac{3}{4} \neq \frac{4}{3}$

a + c are smaller #s b + d are bigger #s

6 If $a < b$, $c < d$, and a, b, c, and d are all greater than 0, which expression is always true?

(1) $a - c + b - d = 0$
(3) $\frac{a}{d} > \frac{b}{c}$
(2) $a + c > b + d$
(4) $ac < bd$

↳ all positive #s

when multiplying, two small #s will always be less than two big #s

Use this space for computations.

7 The volume of a cube is 64 cubic inches. Its total surface area, in square inches, is

(1) 16
(2) 48
(3) 96
(4) 576

$V_{(cube)} = s^3$ $\sqrt[3]{64} = 4$ $s = 4$

Surface area of one side $= s^2 = 16$

There are six sides in a cube: $6(16) = 96$

8 On an English examination, two students received scores of 90, five students received 85, seven students received 75, and one student received 55. The average score on this examination was

(1) 75
(2) 76
(3) 77
(4) 79

$2 \times 90 = 180$
$5 \times 85 = 425$
$7 \times 75 = 525$
$1 \times 55 = 55$

15 students = 1185 total points

$Avg. = \frac{sum}{count}$

$Avg = \frac{1185}{15} = 79$

9 Which equation represents a line parallel to the line $y = 2x - 5$?

(1) $y = 2x + 5$
(2) $y = -\frac{1}{2}x - 5$
(3) $y = 5x - 2$
(4) $y = -2x - 5$

↳ $m = 2$ → $b = -5$

Parallel lines must have same slope, different y intercept

↳ same slope, different y intercept

10 The operation * for the set $\{p,r,s,v\}$ is defined in the accompanying table. What is the inverse element of r under the operation *?

*	p	r	s	v
p	s	v	p	r
r	v	p	r	(s)
→ s	p	r	s	v
v	r	(s)	v	p

s is the identity element

(1) p
(2) r
(3) s
(4) v

r * v = s

- The inverse under an operation results in the identity element
- The identity element does not change the other elements - it preserves identity.
- s is the identity element by inspection.
- We need to find the element that produces and s when r is input

11 A box contains six black balls and four white balls. What is the probability of selecting a black ball at random from the box?

(1) $\frac{1}{10}$
(2) $\frac{6}{10}$
(3) $\frac{4}{6}$
(4) $\frac{6}{4}$

$P(event) = \frac{\#\text{ times event happens}}{\text{total possible outcomes}}$

$P(black) = \frac{\#\text{ black balls}}{\text{total \# of balls}} = \frac{6}{10}$

12 The solution set for the equation $x^2 - 2x - 15 = 0$ is

(1) {5,3}
(2) {5,–3}
(3) {–5,3}
(4) {–5,–3}

$x^2 - 2x - 15 = 0$ ← requires positive and negative factors

factors must sum to (-2)

$(x + \_)(x - \_) = 0$

$(x + 3)(x - 5) = 0$

$x + 3 = 0$ $x - 5 = 0$

$x = -3$ $x = 5$

Factors of 15 are 1,15 3,5

13 What is the value of y in the following system of equations?

$2x + 3y = 6$
$2x + y = -2$

(1) 1 (3) –3
(2) 2 (4) 4

Use this space for computations.

$2x + 3y = 6$
$2x + y = -2$ ← subtract the second equation from the first.
$0x + 2y = 8$
$y = 4$

14 What is the converse of the statement "If it is sunny, I will go swimming"?

(1) If it is not sunny, I will not go swimming.
(2) If I do not go swimming, then it is not sunny.
(3) If I go swimming, it is sunny.
(4) I will go swimming if and only if it is sunny.

Given: If 1, then 2
Inverse: If not 1, then not 2
Converse: If 2, then 1
Contrapositive: If not 2, then not 1
If I will go swimming, then it is sunny

15 Solve for x: $15x - 3(3x + 4) = 6$

(1) 1 (3) 3
(2) $-\frac{1}{2}$ (4) $\frac{1}{3}$

$15x - 3(3x+4) = 6 \Rightarrow 15x - 9x - 12 = 6$
$+12 \quad +12$
$6x = 18$
$x = 3$

16 The expression $2\sqrt{50} - \sqrt{2}$ is equivalent to

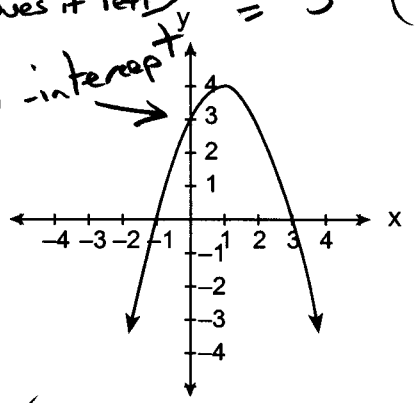
(1) $2\sqrt{48}$ (3) $9\sqrt{2}$
(2) 10 (4) $49\sqrt{2}$

$2\sqrt{50} - \sqrt{2}$
$2\sqrt{25}\sqrt{2} - \sqrt{2}$
$2(5)\sqrt{2} - \sqrt{2}$
$10\sqrt{2} - \sqrt{2}$
$9\sqrt{2}$

17 Which is an equation of the parabola shown in the accompanying diagram?

pos moves it right 3, neg moves it left 3 (same as the # line.)

$ax^2 + bx + c$
c = y-intercept
pos a opens up
neg a opens down



(1) $y = -x^2 + 2x + 3$ (3) $y = x^2 + 2x + 3$ pos. a opens up
(2) $y = -x^2 - 2x + 3$ (4) $y = x^2 - 2x + 3$ pos. a opens up

Any 2 sides of a Δ must sum to more than the 3rd side.

18 If two sides of a triangle are 1 and 3, the third side may be

~~(1) 5~~ $1+3 \not> 5$ (3) 3

~~(2) 2~~ $2+1 \not> 3$ ~~(4) 4~~ $1+3 \not> 5$

Use this space for computations.

⇒ This works!

19 A girl can ski down a hill five times as fast as she can climb up the same hill. If she can climb up the hill and ski down in a total of 9 minutes, how many minutes does it take her to climb up the hill?

(1) 1.8
(2) 4.5
(3) 7.2
(4) 7.5

Time uphill = x
Time downhill = $\frac{x}{5}$
Total Time = 9

20 When $3x^2 - 2x + 1$ is subtracted from $2x^2 + 7x + 5$, the result will be

(1) $-x^2 + 9x + 4$
(2) $x^2 - 9x - 4$
(3) $-x^2 + 5x + 6$
(4) $x^2 + 5x + 6$

$2x^2 + 7x + 5$
$-(3x^2 - 2x + 1)$

to subtract, change signs of 2nd equation and add

$2x^2 + 7x + 5$
$-3x^2 + 2x - 1$
$-x^2 + 9x + 4$

$x + \frac{x}{5} = 9$

$M(5)\ 5x + x = 45$

$6x = 45$

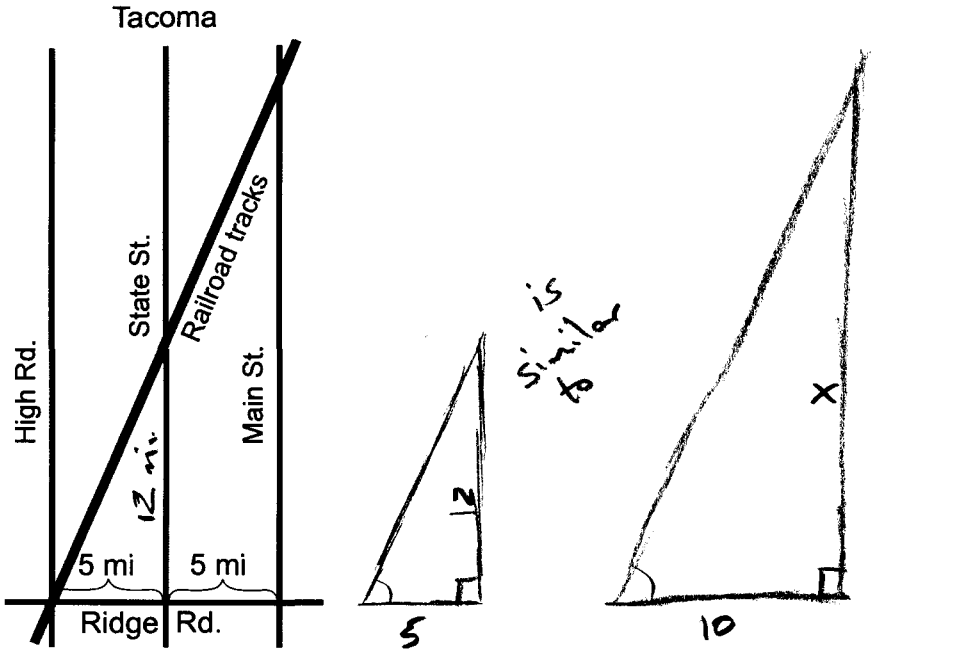
$x = \frac{45}{6}$

$x = 7\frac{1}{2}$

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [10]

21 The accompanying diagram shows a section of the city of Tacoma. High Road, State Street, and Main Street are parallel and 5 miles apart. Ridge Road is perpendicular to the three parallel streets. The distance between the intersection of Ridge Road and State Street and where the railroad tracks cross State Street is 12 miles. What is the distance between the intersection of Ridge Road and Main Street and where the railroad tracks cross Main Street?



bottom / side

$\frac{5}{12} = \frac{10}{X}$

$5X = 120$

$X = 24$ miles

22 Perform the indicated operation and express the result in simplest terms:

$$\frac{x}{x+3} + \frac{3x}{x^2-9}$$

Keep Change Flip

$$\frac{x}{x+3} \times \frac{x^2-9}{3x}$$

$$\frac{(x)\,(x+3)\,(x-3)}{(x+3)\,(3x)}$$

$$\boxed{\frac{x-3}{3}}$$

23 Kerry is planning a rectangular garden that has dimensions of 4 feet by 6 feet. Kerry wants one-half of the garden to have roses, and she says that the rose plot will have dimensions of 2 feet by 3 feet. Is she correct? Explain.

Garden

Area = 24 | 4

6

Roses

Area = 6 | 2

3

Kerry is not correct. The area of the roses needs to be 12 feet^2, because 12 feet^2 is one half of 24 ft^2.

24 The sum of the ages of the three Romano brothers is 63. If their ages can be represented as consecutive integers, what is the age of the middle brother?

let x = age of youngest brother
let $x+1$ = age of middle brother
let $x+2$ = age of oldest brother

$$(x) + (x+1) + (x+2) = 63$$
$$3x + 3 = 63$$
$$3x = 60$$
$$x = 20$$

The middle brother is $x+1$ years old.
$20+1 \Rightarrow$ 21 years old

25 Alan, Becky, Jesus, and Mariah are four students in the chess club. If two of these students will be selected to represent the school at a national convention, how many combinations of two students are possible?

$_4C_2 \Rightarrow$ 4 (starting # in numerator), 2 (# boxes in both numerator and denominator and starting # in denominator)

$\frac{4 \cdot 3}{2 \cdot 1} = \frac{12}{2} = 6$

Order does not matter.

There are 6 possible combinations of 2 students.

Part III

Answer all questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [15]

26 John, Dan, Karen, and Beth went to a costume ball. They chose to go as Anthony and Cleopatra, and Romeo and Juliet. John got the costumes for Romeo and Cleopatra, but not his own costume. Dan saw the costumes for Juliet and himself. Karen went as Anthony. Beth drove two of her friends, who were dressed as Anthony and Cleopatra, to the ball. What costume did John wear?

① John ⇒ not Romeo, not Cleopatra

② Dan ⇒ not Juliet

④ Beth ⇒ not Anthony, not Cleopatra

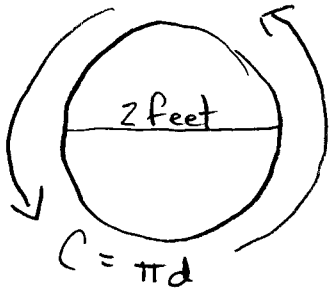
Strategy: Use a binary (Yes/No) table to organize the info.

	Anthony	Cleopatra	Romeo	Juliet
John	N	N	N	
Dan	N			N
Karen	Y			
Beth	N	N		

③ Since Karen is Anthony, every one else is not Anthony

John must have been Juliet, because nobody else was Juliet.

27 To measure the length of a hiking trail, a worker uses a device with a 2-foot-diameter wheel that counts the number of revolutions the wheel makes. If the device reads 1,100.5 revolutions at the end of the trail, how many miles long is the trail, to the *nearest tenth of a mile*?



The circumference of the wheel = 2π

Each revolution measures 2π distance

$(1100.5)(2\pi) = 2201\pi = 6914.645431$ ft

(1100.5 ↙ # revolutions; 2π ↓ distance per revolution)

A mile is 5,280 ft.

$\frac{6914.645431}{5,280} \Rightarrow 1.309591938 \Rightarrow$ **1.3 miles**

28 The coordinates of the endpoints of $\overline{AB}$ are $A(2,6)$ and $B(4,2)$. Is the image $\overline{A''B''}$ the same if it is reflected in the x-axis, then dilated by $\frac{1}{2}$ as the image is if it is dilated by $\frac{1}{2}$, then reflected in the x-axis? Justify your answer. [*The use of the accompanying grid is optional.*]

Option 1 reflect in x axis then dilate by 1/2

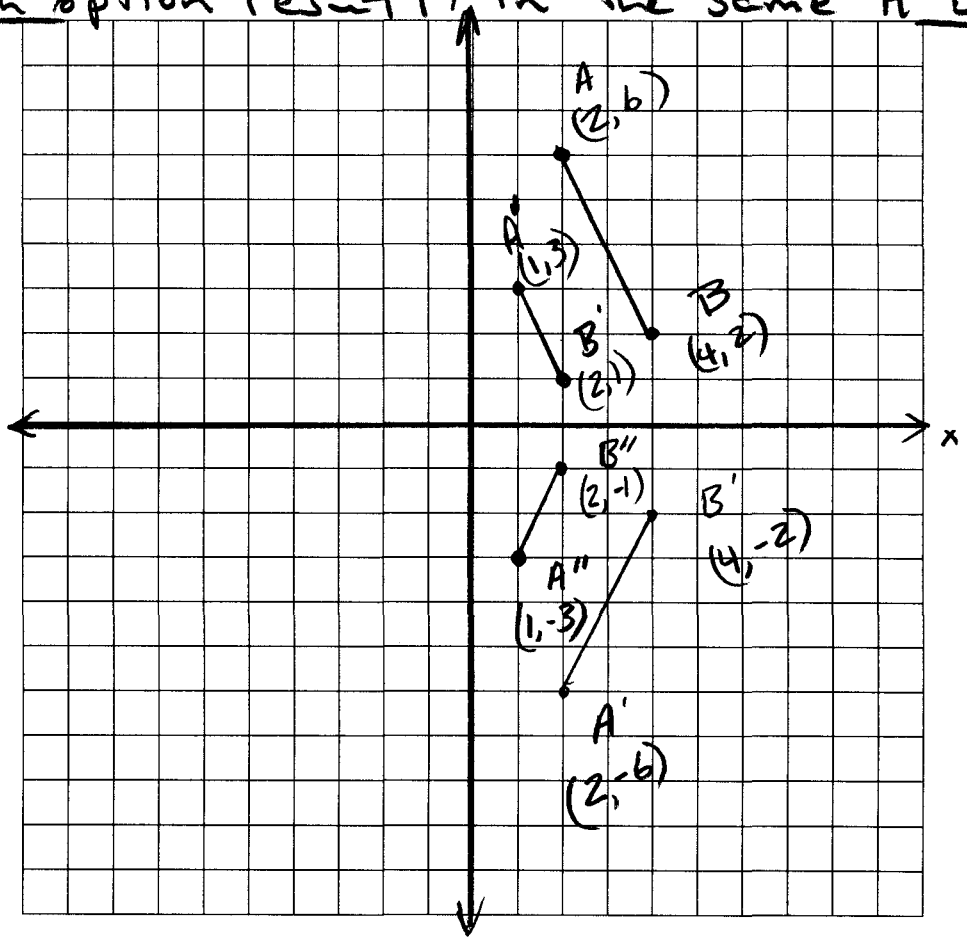
Option 2 Dilate by 1/2 then reflect in x axis

$\overline{AB} \Rightarrow \overline{A'B'} \Rightarrow \overline{A''B''}$

Option #1 $(2,6)(4,2) \Rightarrow (2,-6)(4,-2) \Rightarrow (1,-3)(2,-1)$

Option #2 $(2,6)(4,2) \Rightarrow (1,3)(2,1) \Rightarrow (1,-3)(2,-1)$

Both option resulty in the same $\overline{A''B''}$ image



$\frac{4}{10} + \frac{3}{25}$

$\frac{4(25) + 3(10)}{10(25)}$

$\frac{100 + 30}{250}$

$\frac{130}{250}$

$\frac{13}{25}$

29 After an ice storm, the following headlines were reported in the *Glacier County Times:*

Monday: Ice Storm Devastates County — 8 out of every 10 homes lose electrical power } $\frac{8}{10}$

Tuesday: Restoration Begins — Power restored to $\frac{1}{2}$ of affected homes } $\frac{1}{2}$

Wednesday: More Freezing Rain — Power lost by 20% of homes that had power on Tuesday } $\frac{1}{5}$

Based on these headlines, what fractional portion of homes in Glacier County had electrical power on Wednesday?

On Monday $\frac{8}{10}$ of homes lost power

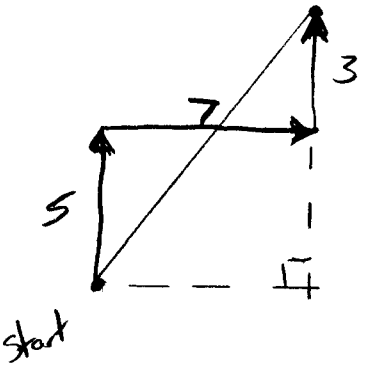
On Tuesday $\frac{1}{2}$ of $\frac{8}{10}$ got their power back, so $\frac{4}{10}$ got power back, leaving $\frac{4}{10}$ still without power. This means $\frac{6}{10}$ had power.

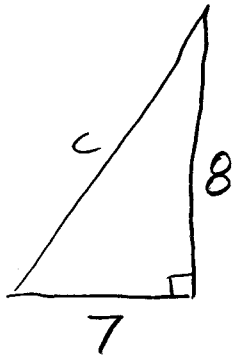
Tuesday Wednesday

On Wednesday, $\frac{1}{5}$ of the $\frac{6}{10}$ that had power lost it, so $\left(\frac{1}{5}\right)\left(\frac{6}{10}\right) = \frac{6}{50} = \frac{3}{25}$ of the homes lost power.

$\frac{4}{10} + \frac{3}{25} = \boxed{\frac{13}{25}}$ of the homes had no power.

30 Katrina hikes 5 miles north, 7 miles east, and then 3 miles north again. To the *nearest tenth of a mile*, how far, in a straight line, is Katrina from her starting point?





Pythagorean Theorem

$a^2 + b^2 = c^2$

$7^2 + 8^2 = c^2$

$49 + 64 = c^2$

$113 = c^2$

$\sqrt{113} = \sqrt{c^2}$

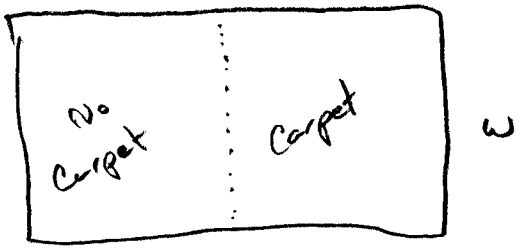
$10.630145\overline{81} = c$

$\boxed{10.6 = c}$

Part IV

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [20]

31 Mr. Santana wants to carpet exactly half of his rectangular living room. He knows that the perimeter of the room is 96 feet and that the length of the room is 6 feet longer than the width. How many square feet of carpeting does Mr. Santana need?



$l = w + 6$

$\text{Perimeter} = 2l + 2w$

$P = 2(w+6) + 2w$

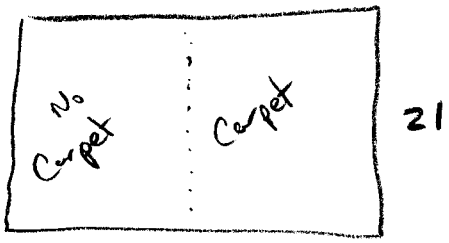
$96 = 2w + 12 + 2w$

$-12 \qquad -12$

$84 = 2w \qquad +2w$

$84 = 4w$

$21 = w$



$\text{Area} = l \cdot w$

$\text{Area} = (27)(21)$

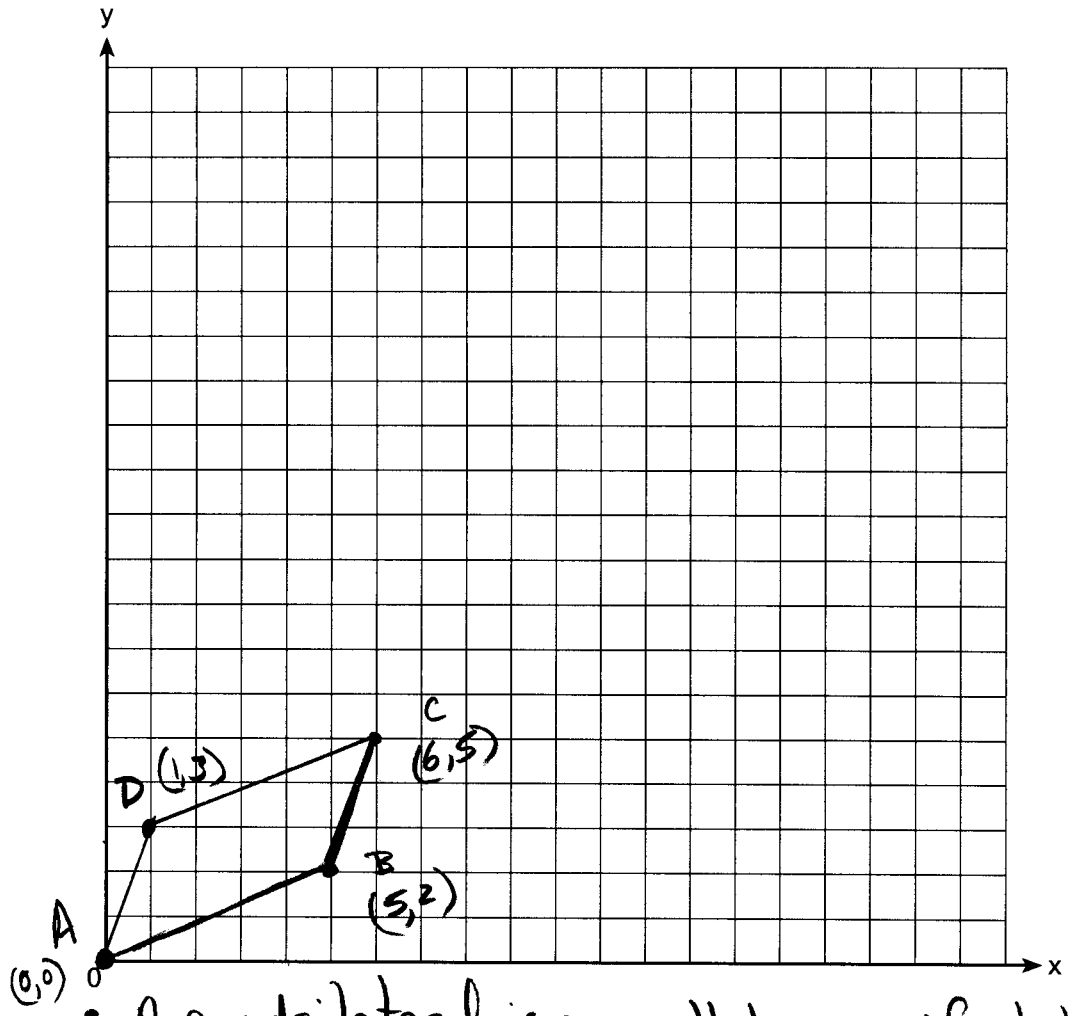
$\text{Area} = 567\ ft^2$

The total area of the living room is $567\ ft^2$

One half of the total area is $\frac{567}{2} = 283.5\ ft^2$

He need $\boxed{283.5\ ft^2}$ of carpet.

32 Ashanti is surveying for a new parking lot shaped like a parallelogram. She knows that three of the vertices of parallelogram $ABCD$ are $A(0,0)$, $B(5,2)$, and $C(6,5)$. Find the coordinates of point D and sketch parallelogram $ABCD$ on the accompanying set of axes. Justify mathematically that the figure you have drawn is a parallelogram.



- A quadrilateral is a parallelogram if it has 2 pair of parallel sides.
- Two lines are parallel if the have same slopes
- $\overline{AD}$ and $\overline{BC}$ both have slopes of $\frac{3}{1}$
  $\therefore$ $\overline{AD}$ and $\overline{BC}$ are parallel lines
- $\overline{AB}$ and $\overline{DC}$ both have slopes of $\frac{2}{5}$
  $\therefore$ $\overline{AB}$ and $\overline{DC}$ parallel lines
- Quadrilateral ABCD has 2 sets of parallel lines
  $\therefore$ Quadrilateral ABCD is a parallelogram.

[13]

[OVER]

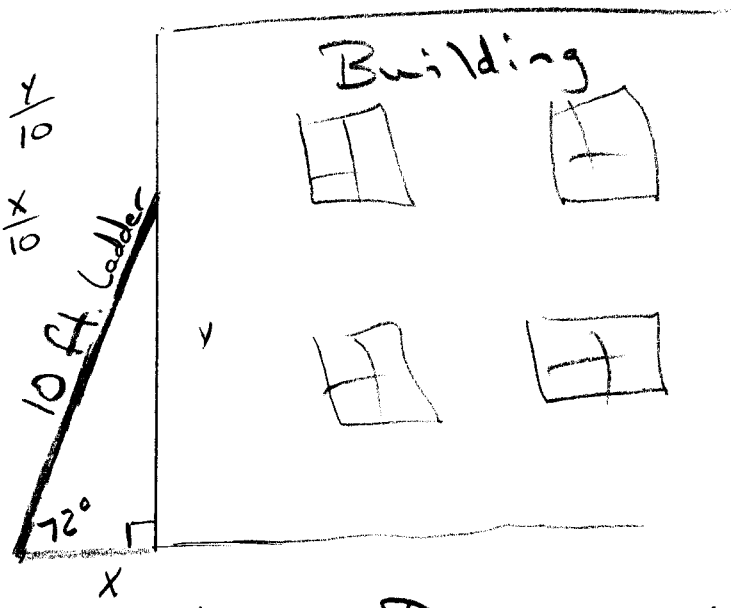
33 A 10-foot ladder is to be placed against the side of a building. The base of the ladder must be placed at an angle of 72° with the level ground for a secure footing. Find, to the *nearest inch*, how far the base of the ladder should be from the side of the building *and* how far up the side of the building the ladder will reach.

SOH-CAH-TOA

$\sin = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{10}$

$\cos = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{10}$

$\tan = \frac{\text{opposite}}{\text{adjacent}}$



Reminder: Set Calculator to Degree Mode

$\sin 72° = \frac{y}{10}$

$10(\sin 72°) = y$

9.510 565 163 feet

× 12 inches/foot

114.126782 inches

114 inches up the wall

y

$\cos 72° = \frac{x}{10}$

$10 \cos 72° = x$

3.090169944 feet

× 12 inches/foot

37.08203932

37 inches from the wall

x

34 The telephone company has run out of seven-digit telephone numbers for an area code. To fix this problem, the telephone company will introduce a new area code. Find the number of new seven-digit telephone numbers that will be generated for the new area code if both of the following conditions must be met:

- The first digit cannot be a zero or a one.
- The first three digits cannot be the emergency number (911) or the number used for information (411).

Strategy: Find the # of choices for each digit and multiply. Then subtract the #s that can't be used.

Step #1

Total Possible Combinations

1st	2nd	3rd	4th	5th	6th	7th
8	10	10	10	10	10	10
Not ∅ Not 1						

The total number of plates that can be made with 7 digits and no 0 or 1 in the first position is 8×10^6

Step #2 #s that cannot be used

Total # of numbers beginning w/ 911

1	1	1	10	10	10	10	$= 1 \times 10^4$
9	1	1	?	?	?	?	

Total # of numbers beginning w/ 411

1	1	1	10	10	10	10	$= 1 \times 10^4$
4	1	1	?	?	?	?	

Step #3 Subtract bad #s from good #s

$8 \times 10^6 - (1 \times 10^4 + 1 \times 10^4)$

$8{,}000{,}000 - (10{,}000 + 10{,}000)$

$8{,}000{,}000 - 20{,}000$

7,980,000

Answer

35 Jack is building a rectangular dog pen that he wishes to enclose. The width of the pen is 2 yards less than the length. If the area of the dog pen is 15 square yards, how many yards of fencing would he need to completely enclose the pen?

Area = 15 yards

$w = l - 2$

l

$\text{Area} = (l)(w)$

$15 = l(l-2)$

$15 = l^2 - 2l$

$0 = l^2 - 2l - 15$

See Problem #12 for how to solve $x^2 - 2x - 15$

$0 = (l-5)(l+3)$

$l - 5 = 0 \qquad l + 3 = 0$

$l = 5 \qquad l = -3$

↑ this is not a possible length.

The length of the dog pen is 5

The width of the dog pen is $5 - 2 = 3$

check ✓

Area = 15 | 3 / 5

Perimeter $= 2(l + w)$

$P = 2(5+3)$

$P = 2(8) = 16$

Jack needs 16 yards of fence

Tear Here

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS A

Wednesday, August 16, 2000 — 8:30 to 11:30 a.m., only

ANSWER SHEET

Pupil Imaginary Student Sex: ☐ Male ☐ Female Grade .....

Teacher Mr. Steve School IHS @ PH

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer all 20 questions in this part.

1 2	6 4	11 2	16 3
2 1	7 3	12 2	17 1
3 4	8 4	13 4	18 3
4 1	9 1	14 3	19 4
5 2	10 4	15 3	20 1

Your answers for Parts II, III, and IV should be written in the test booklet.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

RSW

Signature

Tear Here