The University of the State of New Y ork

REGENTS HIGH SCHOOL EXAMINA TION

MATHEMATICS A

Tuesday , June 22, 1999 — 9:15 a.m. to 12:15 p.m., only

Print Your Name: Imaginary Student (i)

Print Your School's Name: www.jmap.org

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and car efully , tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. All work should be written in pen, except graphs and drawings, which should be done in pencil.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers for the questions in Par ts II, III, and IV directly in this booklet. Clearly indicate the necessary steps you take, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Y our answer paper cannot be accepted if you fail to sign this declaration.

Notice...

A scientific calculator, a straightedge (ruler), and a compass must be available for your use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Record your answers in the spaces provided on the separate answer sheet. [40]

Use this space for computations.

1 A fair coin is thrown in the air four times. If the coin lands with the head up on the first three tosses, what is the probability that the coin will land with the head up on the fourth toss?

(1) 0
(2) $\frac{1}{16}$
(3) $\frac{1}{8}$
(4) $\frac{1}{2}$

A coin has no memory. The probability of getting heads is $\frac{1}{2}$

2 The statement "If x is divisible by 8, then it is divisible by 6" is false if x equals

(1) 6 not ÷ by 8
(2) 14 not ÷ by 8
(3) 32 ÷ 8, but not ÷ 6
(4) 48 ÷ 8 and ÷ 6

3 What is the image of point (2,5) under the translation that shifts (x,y) to $(x + 3, y - 2)$?

(1) (0,3)
(2) (0,8)
(3) (5,3)
(4) (5,8)

(2,5)
$x = 2$
$y = 5$

$(x + 3)\ (y - 2)$
$(2 + 3)\ (5 - 2)$
5 3
x y
(5,3)

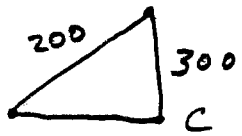
4 The sum of $3x^2 + x + 8$ and $x^2 - 9$ can be expressed as

(1) $4x^2 + x - 1$
(2) $4x^2 + x - 17$
(3) $4x^4 + x - 1$
(4) $3x^4 + x - 1$

$3x^2 + x + 8$
$x^2 - 9$
$4x^2 + x - 1$

5 The direct distance between city A and city B is 200 miles. The direct distance between city B and city C is 300 miles. Which could be the direct distance between city C and city A?

(1) 50 miles $50 + 200 < 300$
(2) 350 miles
(3) 550 miles $200 + 300 < 550$
(4) 650 miles $200 + 300 < 550$



6 Expressed as a single fraction, what is $\frac{1}{x+1} + \frac{1}{x}$, $x \neq 0,-1$?

(1) $\frac{2x+3}{x^2+x}$
(2) $\frac{2x+1}{x^2+x}$
(3) $\frac{2}{2x+1}$
(4) $\frac{3}{x^2}$

$\frac{1}{x+1} + \frac{1}{x}$
$\frac{1(x) + 1(x+1)}{x(x+1)}$
$\frac{x + x + 1}{x(x+1)}$
$\frac{2x+1}{x^2+x}$

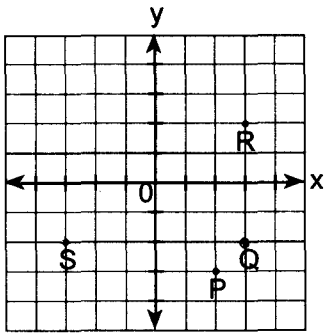
7 How many different three-member teams can be formed from six students?

(1) 20
(2) 120
(3) 216
(4) 720

$6C_3 \Rightarrow \binom{6}{3} \Rightarrow \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = \frac{120}{6} = 20$

8 If $x = -3$ and $y = 2$, which point on the accompanying graph represents $(-x,-y)$?

Use this space for computations.



$X = -3 \quad Y = 2$
$-X = 3 \quad -Y = -2$
$(-X, -Y) = (3, -2)$
$(3, -2)$ is point Q

(1) P
(2) Q
(3) R
(4) S

9 The larger root of the equation $(x + 4)(x - 3) = 0$ is

(1) -4
(2) -3
(3) 3
(4) 4

The roots are the X-axis intercepts.
$x+4=0 \quad x-3=0$
$x=-4 \quad x=3$
3 is larger than -4

10 Linda paid \$48 for a jacket that was on sale for 25% of the original price. What was the original price of the jacket?

(1) \$60
(2) \$72
(3) \$96
(4) \$192

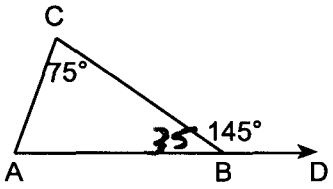
Let x = original price
$.25x = 48$
$x = \frac{48}{.25} \Rightarrow 192$
D(25)

11 The expression $2^3 \cdot 4^2$ is equivalent to

(1) 2^7
(2) 2^{12}
(3) 8^5
(4) 8^6

$2^3 \cdot 4^2$
$8 \cdot 16 = 128 = 2^7$
$2^3 \cdot 4^2$
$2^3 \cdot 16$
$2^3 \cdot 2^4 = 2^{3+4} = 2^7$

12 In the accompanying diagram of $\triangle ABC$, $\overline{AB}$ is extended to D, exterior angle CBD measures 145°, and $m\angle C = 75$.



$75 + 35 + \measuredangle CAB = 180$
$110 + \measuredangle CAB = 180$
$-110 \quad -110$
$\measuredangle CAB = 70$

What is $m\angle CAB$?

(1) 35
(2) 70
(3) 110
(4) 220

13 A total of \$450 is divided into equal shares. If Kate receives four shares, Kevin receives three shares, and Anna receives the remaining two shares, how much money did Kevin receive?

(1) \$100
(2) \$150
(3) \$200
(4) \$250

Use this space for computations.

Kate 4 shares
Kevin 3 shares
Anna 2 shares
Total 9 shares
Kevin received 3/9
$\frac{3}{9}(450) = 150$

14 What is the diameter of a circle whose circumference is 5?

(1) $\frac{2.5}{\pi^2}$
(2) $\frac{2.5}{\pi}$
(3) $\frac{5}{\pi^2}$
(4) $\frac{5}{\pi}$

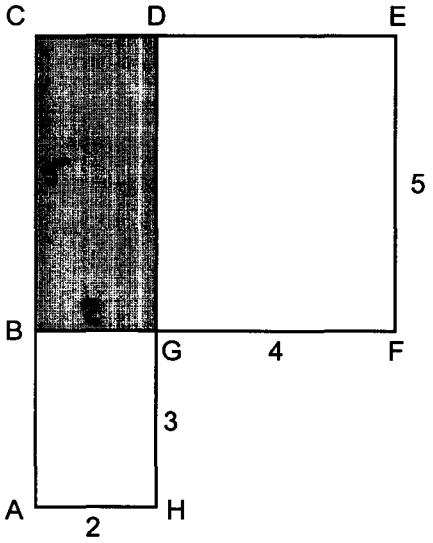
$C = \pi d$
$5 = \pi d$
$\frac{5}{\pi} = d$

15 During a recent winter, the ratio of deer to foxes was 7 to 3 in one county of New York State. If there were 210 foxes in the county, what was the number of deer in the county?

(1) 90
(2) 147
(3) 280
(4) 490

$\frac{\text{Deer}}{\text{Foxes}} \quad \frac{7}{3} = \frac{x}{210} \Rightarrow 7(210) = 3x$
$1470 = 3x$
$490 = x$

16 In the accompanying figure, *ACDH* and *BCEF* are rectangles, $AH = 2$, $GH = 3$, $GF = 4$, and $FE = 5$.



$A = bh$
$A = 2 \times 5 = 10$

What is the area of *BCDG*?

(1) 6
(2) 8
(3) 10
(4) 20

17 If $t^2 < t < \sqrt{t}$, then t could be

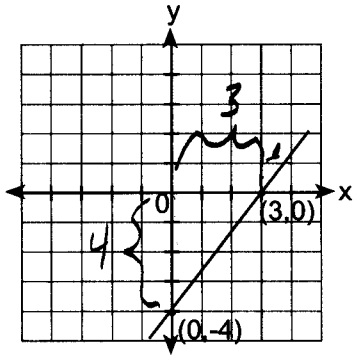
(1) $-\frac{1}{4}$
(2) 0
(3) $\frac{1}{4}$
(4) 4

t	$t^2 < t < \sqrt{t}$
$-\frac{1}{4}$	$\frac{1}{16} < -\frac{1}{4} < \sqrt{-\frac{1}{4}}$
0	$0 < 0 < 0$
$\frac{1}{4}$	$\frac{1}{16} < \frac{1}{4} < \frac{1}{2}$ ✓
4	$16 < 4 < 2$

[4]

18 What is the slope of line ℓ shown in the accompanying diagram?

Use this space for computations.



3 run, 4 rise

slope = $\frac{\text{rise}}{\text{run}} = \frac{4}{3}$

(1) $\frac{4}{3}$ (3) $-\frac{3}{4}$

(2) $\frac{3}{4}$ (4) $-\frac{4}{3}$

19 In a class of 50 students, 18 take music, 26 take art, and 2 take both art and music. How many students in the class are not enrolled in either music or art?

(1) 6 (3) 16

(2) 8 (4) 24

	Music	Not Music	Total
Art	2	24	26
Not Art	16	8	24
Total	18	32	50

20 The expression $\sqrt{27} + \sqrt{12}$ is equivalent to

(1) $5\sqrt{3}$ (3) $5\sqrt{6}$

(2) $13\sqrt{3}$ (4) $\sqrt{39}$

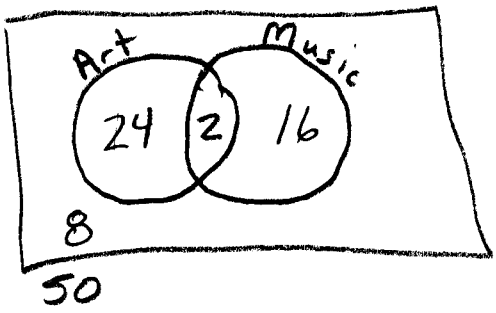
$(\sqrt{27})\sqrt{12}$

$(\sqrt{9})(\sqrt{3})(\sqrt{4})(\sqrt{3})$

$(3\sqrt{3})\,2\,(\sqrt{3})$

$5\sqrt{3}$

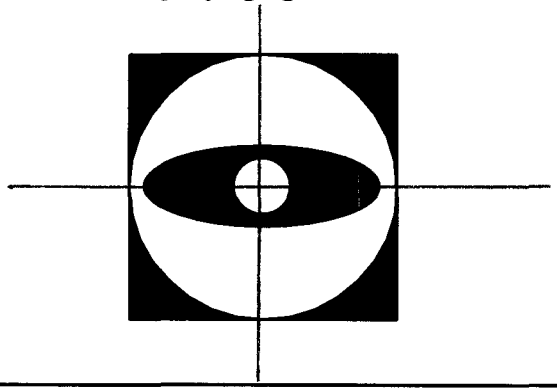
or



Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessar y steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 cr edit. [10]

21 Draw all the symmetry lines on the accompanying figure.



22 Shoe sizes and foot length are related by the formula $S = 3F - 24$, where S represents the shoe size and F represents the length of the foot, in inches.

a Solve the formula for F.

$S = 3F - 24$

$+24 \qquad +24$

$S + 24 = 3F$

$$\frac{S+24}{3} = F$$

b To the *nearest tenth of an inch,* how long is the foot of a person who wears a size $10\frac{1}{2}$ shoe?

$$\frac{S+24}{3} = F$$

$$\frac{10\frac{1}{2}+24}{3} = F$$

$$\frac{34\frac{1}{2}}{3} = F = \frac{34.5}{3} = 11.5 \text{ inches}$$

ratio of 2 integers

23 Which number below is irrational?

$\sqrt{\frac{4}{9}}$, $\sqrt{20}$, $\sqrt{121}$

$\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$ $\quad \sqrt{121} = 11 = \frac{11}{1}$

$\sqrt{20} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$

Why is the number you chose an irrational number?

An irrational number cannot be expressed as the ratio of two integers. Square roots of prime #s are always irrational.

24 Simplify: $\frac{9x^2 - 15xy}{9x^2 - 25y^2}$

$\frac{9x^2 - 15xy}{9x^2 - 25y^2} \Rightarrow \frac{3x(3x-5y)}{(3x+5y)(3x-5y)}$

$\frac{3x}{3x+5y}$

25 Sara's telephone service costs \$21 per month plus \$0.25 for each local call, and long-distance calls are extra. Last month, Sara's bill was \$36.64, and it included \$6.14 in long-distance charges. How many local calls did she make?

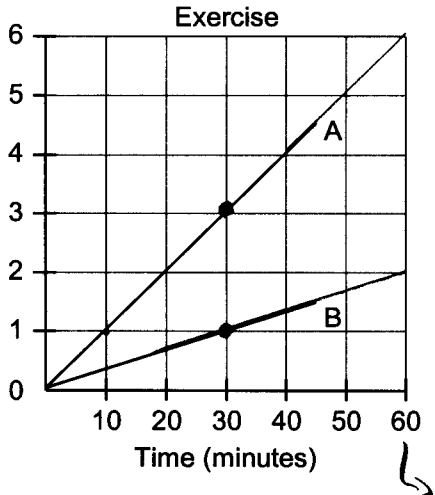
36.64 total
less 21.00 montly cost
15.64 for total calls
-6.14 for long distance calls
9.50 for local calls

$\frac{9.50 \text{ for local calls}}{.25 \text{ for each call}} =$ 38 local calls

Part III

Answer all questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessar y steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [15]

26 During a 45-minute lunch period, Albert (*A*) went running and Bill (*B*) walked for exercise. Their times and distances are shown in the accompanying graph. How much faster was Albert running than Bill was walking, in miles per hour?



Strategy #1
Extend the lines w/ a straightedge
Albert was walking 6 miles per hour
Bill was walking 2 miles per hour
↳ 1 hour The difference is
4 miles per hour

Strategy #2
Albert's rate
Miles / Minutes
$\frac{3}{30} = \frac{x}{60}$
$3(60) = 30x$
$180 = 30x$
$6 = x$

Bill's rate
$\frac{1}{30} = \frac{x}{60}$
$1(60) = 30x$
$60 = 30x$
$2 = x$

Albert is running 4 miles per hour faster than Bill is walking

27 The dimensions of a brick, in inches, are 2 by 4 by 8. How many such bricks are needed to have a total volume of exactly 1 cubic foot?

$V = lwh$

$V_{(brick)} = 2 \cdot 4 \cdot 8 = 64 \text{ inches}^3$

$V_{(1 \text{ cubic foot})} = 12 \cdot 12 \cdot 12 = 1728 \text{ inches}^3$

$\frac{1728}{64} = \#\text{ bricks in 1 cubic foot} = 27$

There are **27 bricks** in one cubic foot

28 A swimmer plans to swim at least 100 laps during a 6-day period. During this period, the swimmer will increase the number of laps completed each day by one lap. What is the *least* number of laps the swimmer must complete on the first day?

Let x = first day's # of laps

$x+1$ = 2nd " "

$x+2$ = 3rd " "

$x+3$ = 4th " "

$x+4$ = 5th " "

$x+5$ = 6th " "

Totals $6x + 15$ = total # of laps over 6 days

then $6x + 15 \geq 100$

$-15 \quad -15$

$6x \geq 85$

$x \geq 14.166666$

the swimmer must complete at least **15 laps** on the first day.

Check

14	15 ✓
15	16
16	17
17	18
18	19
19	20
99	105

Sparky weighs 46
Let Eddie's weight = X
Let Sandy's weight = X

29 The mean (average) weight of three dogs is 38 pounds. One of the dogs, Sparky, weighs 46 pounds. The other two dogs, Eddie and Sandy, have the same weight. Find Eddie's weight.

$$\text{average} = \frac{\text{sum}}{\text{count}} = \frac{46 + X + X}{3}$$

$$38 = \frac{46 + 2X}{3}$$

M(3) $3(38) = 46 + 2X$

$114 = 46 + 2X$

$-46 \quad -46$

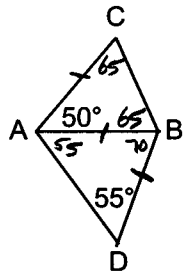
$68 = 2X$

$34 = X$

Eddie weighs 34 pounds

Check $\frac{46 + 34 + 34}{3} = \frac{114}{3} = 38$ ✓

30 In the accompanying diagram, $\triangle ABC$ and $\triangle ABD$ are isosceles triangles with $m\angle CAB = 50$ and $m\angle BDA = 55$. If $AB = AC$ and $AB = BD$, what is $m\angle CBD$?



135°

Reasoning

$\measuredangle ACB = \measuredangle CBA$	Base $\measuredangle$s of isosceles $\triangle$ are equal measure
$2x + 50 = 180$ $2x = 130$ $x = 65$	Sum of $\measuredangle$s in $\triangle$ is 180°
$\measuredangle BAD = \measuredangle BDA$ $\measuredangle BAD = 55° = \measuredangle BDA$	Base $\measuredangle$s of isosceles $\triangle$ are equal in measure
$2(55°) + \measuredangle DBA = 180$ $110 + \measuredangle DBA = 180$ $-110 \quad -110$ $\measuredangle DBA = 70$	Sum of $\measuredangle$s in $\triangle$ is 180°

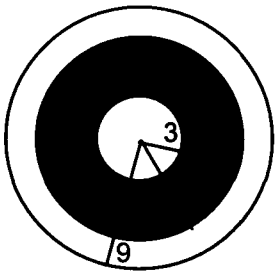
$70° + 65° = 135°$

Part IV

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessar y steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [20]

31 A target shown in the accompanying diagram consists of three circles with the same center. The radii of the circles have lengths of 3 inches, 7 inches, and 9 inches.

$A = \pi r^2$



$A(\text{Big Circle}) = \pi 9^2 = 81\pi \text{ inches}^2$

$A(\text{Middle Circle}) = \pi 7^2 = 49\pi \text{ inches}^2$

$A(\text{Little Circle}) = \pi 3^2 = 9\pi \text{ inches}^2$

a What is the area of the shaded region to the *nearest tenth of a square inch*?

Shaded Area = A(middle circle) − A(Little Circle)

Shaded Area = $49\pi - 9\pi = 40\pi \text{ inches}^2$

$40\pi \text{ inches}^2 = 125.6637061 \text{ inches}^2$

125.7 inches^2

b To the *nearest percent*, what percent of the target is shaded?

$\text{shaded \%} = \frac{\text{shaded area}}{\text{total area (Big Circle)}} = \frac{40\pi}{81\pi} = \frac{40}{81} =$

.4938271605

49%

32 A bookshelf contains six mysteries and three biographies. Two books are selected at random without replacement.

a What is the probability that both books are mysteries?

$P_{(multiple\ events)} = P_A \cdot P_B$

$P_{(first\ book\ is\ mystery)} = \frac{6}{9}$

This leaves 5 mysteries and 8 total books

$P_{(second\ book\ is\ mystery)} = \frac{5}{8}$

$P_{(both\ books\ are\ mysteries)} = \left(\frac{6}{9}\right)\left(\frac{5}{8}\right) = \boxed{\frac{30}{72}}$ Answer

b What is the probability that one book is a mystery and the other is a biography?

$P_{(mystery)} = \frac{6}{9}$ or $P_{(biography)} = \frac{3}{9}$

Leaving eight books for next selection.

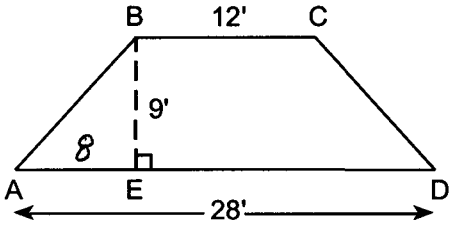
$P_{(biography)} = \frac{3}{8}$ or $P_{(mystery)} = \frac{6}{8}$

$P_{(mystery\ then\ biography)} = \left(\frac{6}{9}\right)\left(\frac{3}{8}\right) = \frac{18}{72}$

$P_{(biography\ then\ mystery)} = \left(\frac{3}{9}\right)\left(\frac{6}{8}\right) = \frac{18}{72}$

Both (mystery then biography) and (biography the mystery) work, so the solution is $\left(\frac{18}{72}\right) + \left(\frac{18}{72}\right) = \boxed{\left(\frac{36}{72}\right)}$ Answer

33 The cross section of an attic is in the shape of an isosceles trapezoid, as shown in the accompanying figure. If the height of the attic is 9 feet, $BC = 12$ feet, and $AD = 28$ feet, find the length of $\overline{AB}$ to the *nearest foot.*



$28 - 12 = 16$

$\frac{16}{2} = 8$

$\therefore AE = 8$

Pythagorean Theorem

$a^2 + b^2 = c^2$

$8^2 + 9^2 = c^2$

$64 + 81 = c^2$

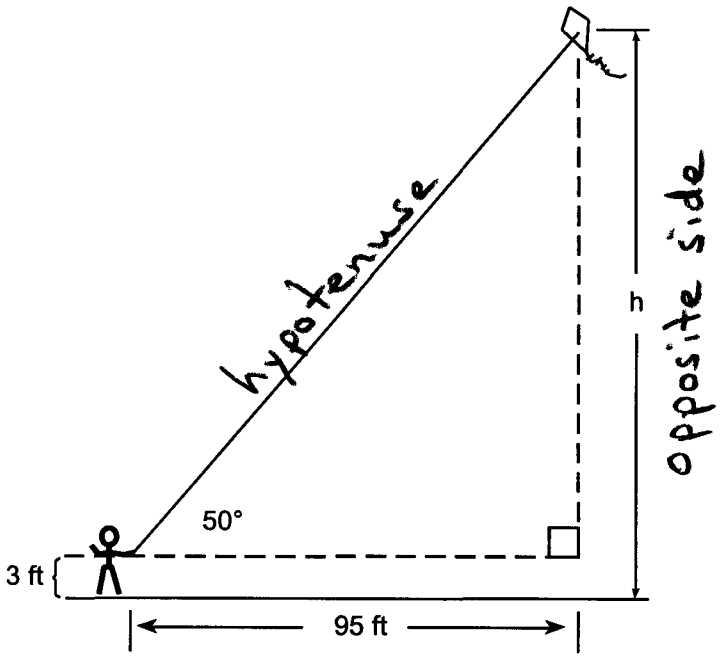
$145 = c^2$

$\sqrt{145} = \sqrt{c^2}$

$12.04159458 = c$

12 ft = $\overline{AB}$

34 Joe is holding his kite string 3 feet above the ground, as shown in the accompanying diagram. The distance between his hand and a point directly under the kite is 95 feet. If the angle of elevation to the kite is 50°, find the height, *h*, of his kite, to the *nearest foot.*



SOH-CAH-TOA

$\sin = \frac{opp}{hyp}$

$\cos = \frac{adj}{hyp}$

$\tan = \frac{opp}{adj}$

adjacent side

$$\tan 50° = \frac{opp}{adj} = \frac{x}{95}$$

$$\tan 50° = \frac{x}{95}$$

$$95(\tan 50°) = x$$

Reminder: Set calculator to degrees mode

$95(\tan 50°) = 113.2165913$

We have to add 3 feet to the 113.2165913

$+3$

116.2165913

The kite is **116 feet** above ground.

35 Solve the following system of equations algebraically *or* graphically for x and y:

$$y = x^2 + 2x - 1$$
$$y = 3x + 5$$

For an algebraic solution, show your work here.

$x^2 + 2x - 1 = 3x + 5$

$-3x \quad -3x$

$x^2 - x - 1 = 5$

$-5 \quad -5$

$x^2 - x - 6 = 0$

One factor must be pos. + one must be neg.

$(x + \_\_)(x - \_\_) = 0$

$(x+2)(x-3) = 0$

$x+2=0 \quad x-3=0$

$x=-2 \quad x=3$

$x=-2$

$y = (-2)^2 + 2(-2) - 1$

$y = 4 - 4 - 1$

$y = -1$

$(-2,-1)$

$x = 3$

$y = 3^2 + 2(3) - 1$

$y = 9 + 6 - 1$

$y = 14$

$(3,14)$

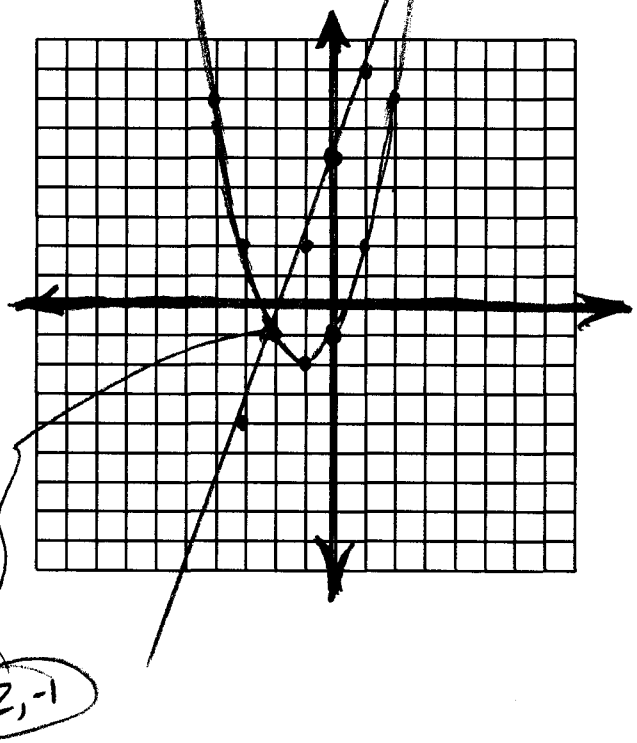
$-2, -1$ and $3, 14$ Answers

For a graphic solution, show your work here.

From Calculator

$y = x^2 + 2x - 1$

x	y
-4	7
-3	2
-2	-1
-1	-2
0	-1
1	2
2	7
3	14



$y = 3x + 5$

$m = \frac{3}{1}$

$b = 5$

The University of the State of New Y ork

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS A

Tuesday , June 22, 1999 — 9:15 a.m. to 12:15 p.m., only

ANSWER SHEET

Pupil ..... Imaginary Student ..... Sex: ☐ Male ☐ Female Grade .....

Teacher ..... Mr. Steve ..... School ..... IHS@PH .....

Your answers to Part I should be r ecor ded on this answer sheet.

Par t I

Answer all 20 questions in this par t.

1	4	11	1
2	3	12	2
3	3	13	2
4	1	14	4
5	2	15	4
6	6	16	3
7	1	17	3
8	2	18	1
9	3	19	2
10	4	20	1

Your answers for Parts II, III, and IV should be written in the test booklet.

The declaration below should be signed when you have completed the examination.

I do hereby af fir m, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

Signatur e