MATHEMATICS A

Tuesday, June 17, 2003 — 1:15 to 4:15 p.m., only

Print Your Name: _Imaginary Student_

Print Your School's Name: _www.jmap.org_

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will *not* be scored. All work should be written in pen, except graphs and drawings, which should be done in pencil.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice. . .

A minimum of a scientific calculator, a straightedge (ruler), and a compass must be available for your use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [40]

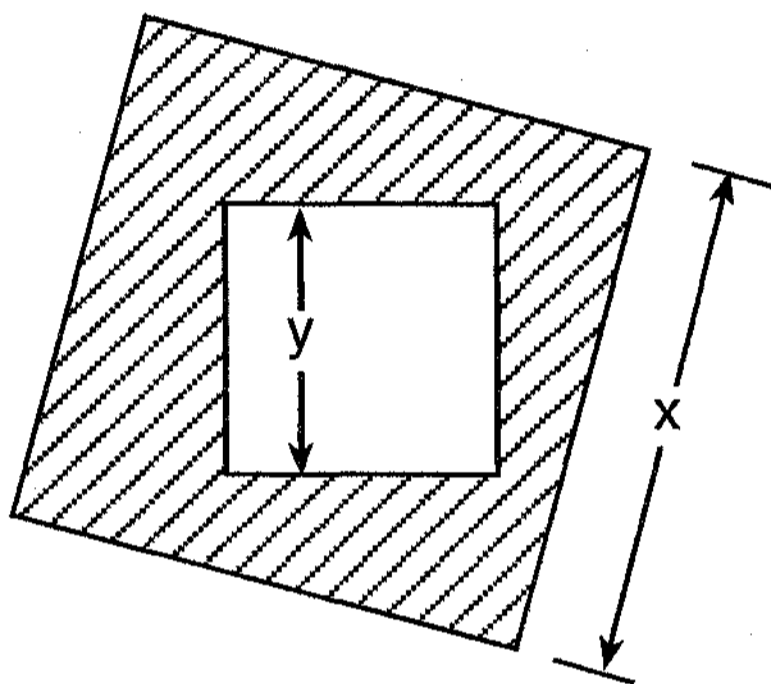1 The number 8.375×10^{-3} is equivalent to

(1) 0.0008375   (3) 0.08375

(2) 0.008375   (4) 8,375

Use this space for computations.

Move the decimal to left ⇒ pos.
" " " right ⇒ neg.

008.375

2 The accompanying diagram shows a square with side y inside a square with side x.

Little square area = y^2
Big square area = x^2
Shaded area = $x^2 - y^2$

Which expression represents the area of the shaded region?

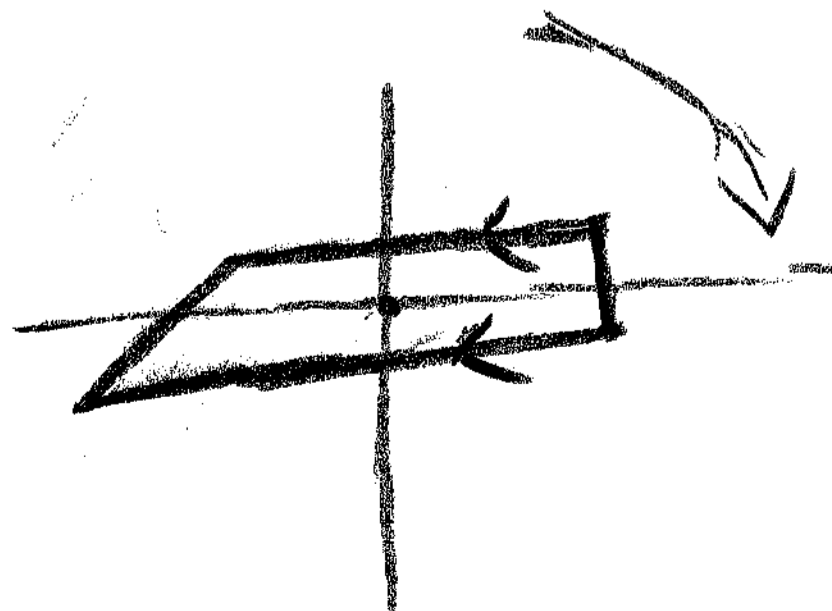(1) x^2 (3) $y^2 - x^2$

(2) y^2 (4) $x^2 - y^2$

3 Which expression represents an irrational number?

(1) $\sqrt{2}$ (3) 0.17

(2) $\frac{1}{2}$ (4) 0

0.17 ⇒ $\frac{0.17}{1}$ ⇒ $\frac{17}{100}$
$\frac{1}{2}$ ⇒ $\frac{1}{2}$
0 ⇒ $\frac{0}{1}$

An irrational # cannot be expressed as the ratio of 2 integers. Rational #s can.

4 Which shape does *not* have rotational symmetry?

(1) trapezoid   (3) circle

(2) regular pentagon   (4) square

5 Bob and Laquisha have volunteered to serve on the Junior Prom Committee. The names of twenty volunteers, including Bob and Laquisha, are put into a bowl. If two names are randomly drawn from the bowl without replacement, what is the probability that Bob's name will be drawn first and Laquisha's name will be drawn second?

(1) $\frac{1}{20} \cdot \frac{1}{20}$

(2) $\frac{1}{20} \cdot \frac{1}{19}$

(3) $\frac{2}{20}$

(4) $\frac{2}{20!}$

$$P_{(A+B)} = P_{(A)} \cdot P_{(B)}$$
First Draw   Second Draw
$\left(\frac{1}{20}\right)$ $\left(\frac{1}{19}\right)$

6 Tori computes the value of 8×95 in her head by thinking $8(100 - 5) = 8 \times 100 - 8 \times 5$. Which number property is she using?

(1) associative

(2) distributive

(3) commutative

(4) closure

$$8(100 - 5) = 8(100) - 8(5)$$

7 A triangle has sides whose lengths are 5, 12, and 13. A similar triangle could have sides with lengths of

(1) 3, 4, and 5

(2) 6, 8, and 10

(3) 7, 24, and 25

(4) 10, 24, and 26

← Dilation factor of 2

5 , 12 , 13
×2   ×2   ×2
10 , 24 , 26

8 Which statement is logically equivalent to "If it is Saturday, then I am not in school"?

(1) If I am not in school, then it is Saturday.

(2) If it is not Saturday, then I am in school.

(3) If I am in school, then it is not Saturday.

(4) If it is Saturday, then I am in school.

Given   If 1, then 2
Inverse   If not 1, then not 2
Converse   If 2, then 1
Contrapositive   If not 2, then not 1

The contrapositive is logically equivalent to the given.
If not I am not in school, then not it is Saturday.

9 A translation moves $P(3,5)$ to $P'(6,1)$. What are the coordinates of the image of point $(-3,-5)$ under the same translation?

(1) $(0,-9)$

(2) $(-5,-3)$

(3) $(-6,-1)$

(4) $(-6,-9)$

X	Y
3 → 6	5 → 1
X + 3	Y - 4
X	Y
-3	-5
0	-9

— Translation rule.

— New position of (3,-5) after translation

10 If $x + y = 9x + y$, then x is equal to

(1) y

(3) 0

(2) $\frac{1}{5}y$

(4) 8

$x + y = 9x + y$
$-x \qquad -x$
$\overline{\qquad\qquad}$
$y = 8x + y$

$y = 8x + y$
$-y \qquad -y$
$\overline{\qquad\qquad}$
$0 = 8x$

$\frac{0}{8} = x$

$0 = x$

11 Which number is in the solution set of the inequality $5x + 3 > 38$?

(1) 5

(3) 7

(2) 6

(4) 8

$5x + 3 > 38$
$\quad -3 \qquad -3$
$\overline{\qquad\qquad}$
$5x \qquad > 35$
$x \qquad > 7$

6 7 8 9
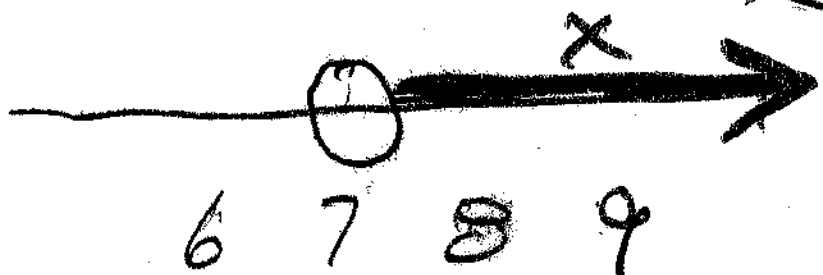
12 The expression $3^2 \cdot 3^3 \cdot 3^4$ is equivalent to
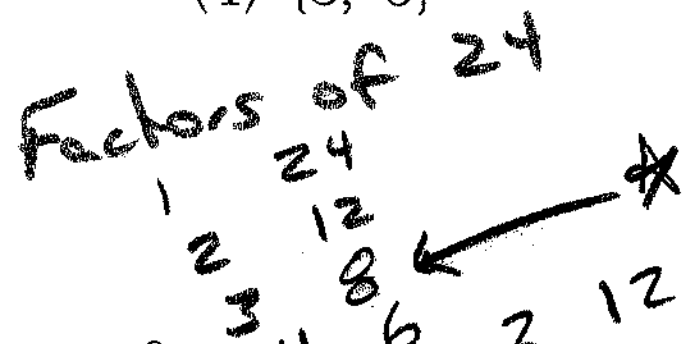
(1) 27^9

(3) 3^9

(2) 27^{24}

(4) 3^{24}

$3^2 \cdot 3^3 \cdot 3^4$
$3^{(2+3+4)}$
3^9

13 What is the solution set of the equation $x^2 - 5x - 24 = 0$?

(1) $\{-3, 8\}$

(3) $\{3, 8\}$

(2) $\{-3, -8\}$

(4) $\{3, -8\}$

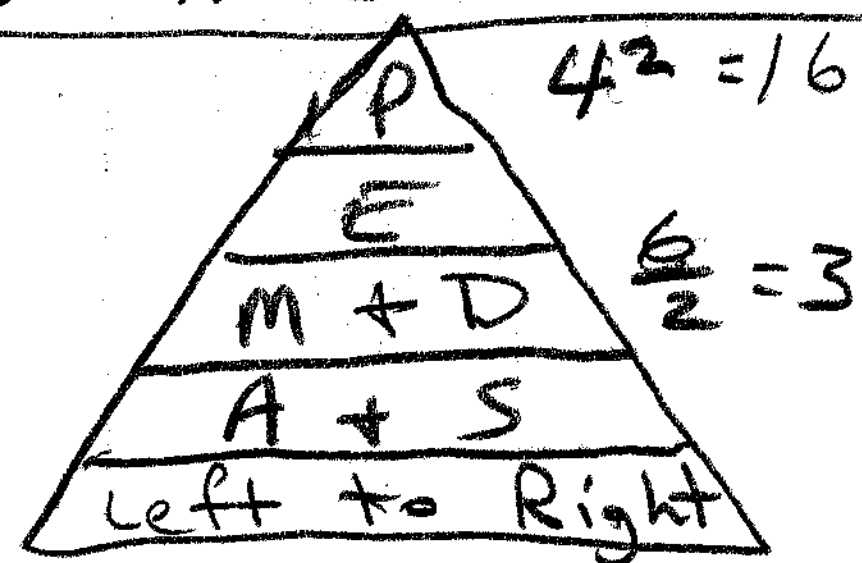$x^2 - 5x - 24 = 0$
$(x + \_)(x - \_) = 0$
$(x + 3)(x - 8) = 0$
$x + 3 = 0 \qquad x - 8 = 0$
$x = -3 \qquad x = 8$

Factors of 24
1   24
2   12
3   8
4   6   2   12

14 If the expression $3 - 4^2 + \frac{6}{2}$ is evaluated, what would be done *last*?

(1) subtracting

(3) adding

(2) squaring

(4) dividing

(this is also a correct answer)

$4^2 = 16$

$\frac{6}{2} = 3$

P
E
M + D
A + S
Left to Right

15 What is the additive inverse of $\frac{2}{3}$?

(1) $-\frac{2}{3}$

(3) $-\frac{3}{2}$

(2) $\frac{1}{3}$

(4) $\frac{3}{2}$

A number and its additive inverse sum to the identity element for addition, which is zero.

$\frac{2}{3} + \left(\frac{-2}{3}\right) = 0$

16 The sum of $\sqrt{18}$ and $\sqrt{72}$ is

   (1) $\sqrt{90}$ (3) $3\sqrt{10}$

   (2) $9\sqrt{2}$ (4) $6\sqrt{3}$

(handwritten computations)
$$\sqrt{18} + \sqrt{72}$$
$$(\sqrt{2})(\sqrt{9}) + (\sqrt{2})(\sqrt{36})$$
$$(\sqrt{2})3 + (\sqrt{2})6$$
$$3\sqrt{2} + 6\sqrt{2}$$
$$9\sqrt{2}$$

17 What is the inverse of the statement "If Julie works hard, then she succeeds"?

   (1) If Julie succeeds, then she works hard.
   (2) If Julie does not succeed, then she does not work hard.
   (3) If Julie works hard, then she does not succeed.
   (4) If Julie does not work hard, then she does not succeed.

(handwritten)
Given: If 1, then 2
Inverse: If not 1, then not 2
Converse: If 2, then 1
Contrapositive: If not 2, then not 1

Inverse
If not Julie works hard, the not she succeeds

18 If one factor of $56x^4y^3 - 42x^2y^6$ is $14x^2y^3$, what is the other factor?

   (1) $4x^2 - 3y^3$ (3) $4x^2y - 3xy^3$
   (2) $4x^2 - 3y^2$ (4) $4x^2y - 3xy^2$

(handwritten)
$$\left(\frac{56x^4y^3}{14x^2y^3}\right) - \left(\frac{42x^2y^6}{14x^2y^3}\right)$$
$$4x^{(4-2)}y^{(3-3)} - 3x^{2-2}y^{6-3}$$
$$4x^2y^0 - 3x^0y^3$$
$$4x^2 - 3y^3$$

19 For which value of x is the expression $\frac{3x-6}{x-4}$ undefined?

   (1) 0   (3) -4
   (2) 2   (4) 4

(handwritten)
The expression is undefined if the numerator is $\emptyset$.
$$x - 4 = 0$$
$$+4 \quad +4$$
$$x = 4$$

20 How many different five-member teams can be made from a group of eight students, if each student has an equal chance of being chosen?

   (1) 40   (3) 336
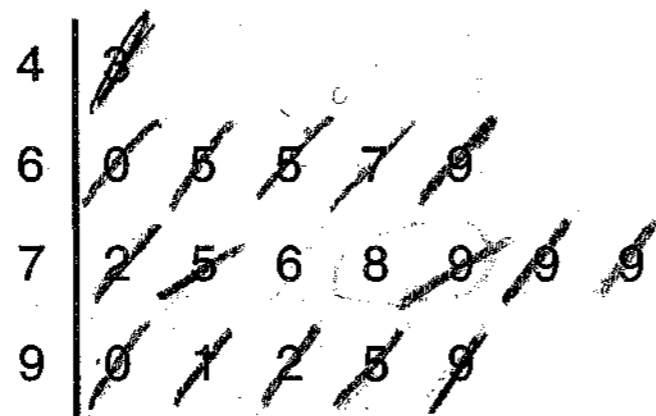   (2) 56   (4) 6,720

(handwritten)
$$_8C_5$$
$$_8C_5 \quad \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{56}{1} = 56$$

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [10]

21 The student scores on Mrs. Frederick's mathematics test are shown on the stem-and-leaf plot below.

```
4 | 8
6 | 0 5 5 7 8
7 | 2 5 6 8 9 9 9
9 | 0 1 1 5 9
```

Key: 4 | 3 = 43 points

Find the median of these scores.

(handwritten work)

43 60 65 65 67 69 72 75 76 78
89 95 96 91 90 79 79 79 78

76 and 78 are in the middle

∴ $\frac{76+78}{2} = 77$

→ middle
78 and 79

The median is 77

22 The lengths of the sides of two similar rectangular billboards are in the ratio 5:4. If 250 square feet of material is needed to cover the larger billboard, how much material, in square feet, is needed to cover the smaller billboard?

(handwritten work)

$A_\square = \ell w$

$A_\square = 250$

$A_\square = \ell w$
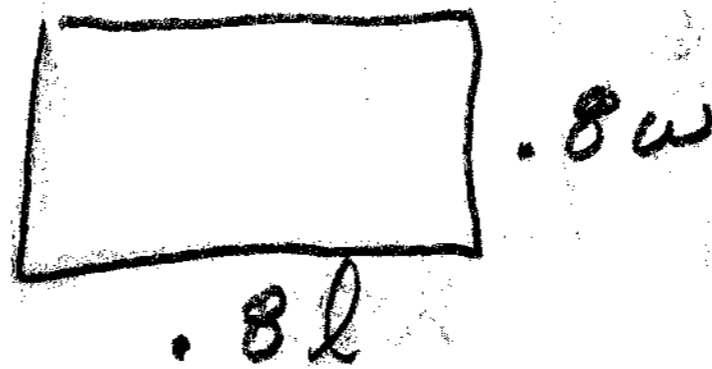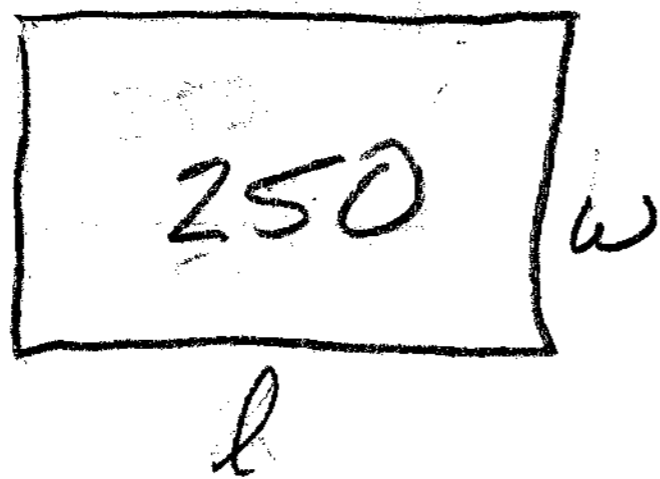
$A_\square = .64 \ell w$

$A_\square = .64(250) \Rightarrow 160 \, ft^2$

23 Solve for m: $0.6m + 3 = 2m + 0.2$

$$.6m + 3 = 2m + .2$$
$$-.6m \qquad\qquad -.6m$$
$$\overline{}$$
$$3 = 1.4m + .2$$
$$-.2 \qquad\qquad -.2$$
$$\overline{}$$
$$2.8 = 1.4m$$
$$\boxed{2 = m}$$

Check
$$.6(2) + 3 = 2(2) + .2$$
$$1.2 + 3 = 4 + .2$$
$$4.2 = 4.2 \checkmark$$

24 In the accompanying diagram, line m is parallel to line p, line t is a transversal, $m\angle a = 3x + 12$, and $m\angle b = 2x + 13$. Find the value of x.
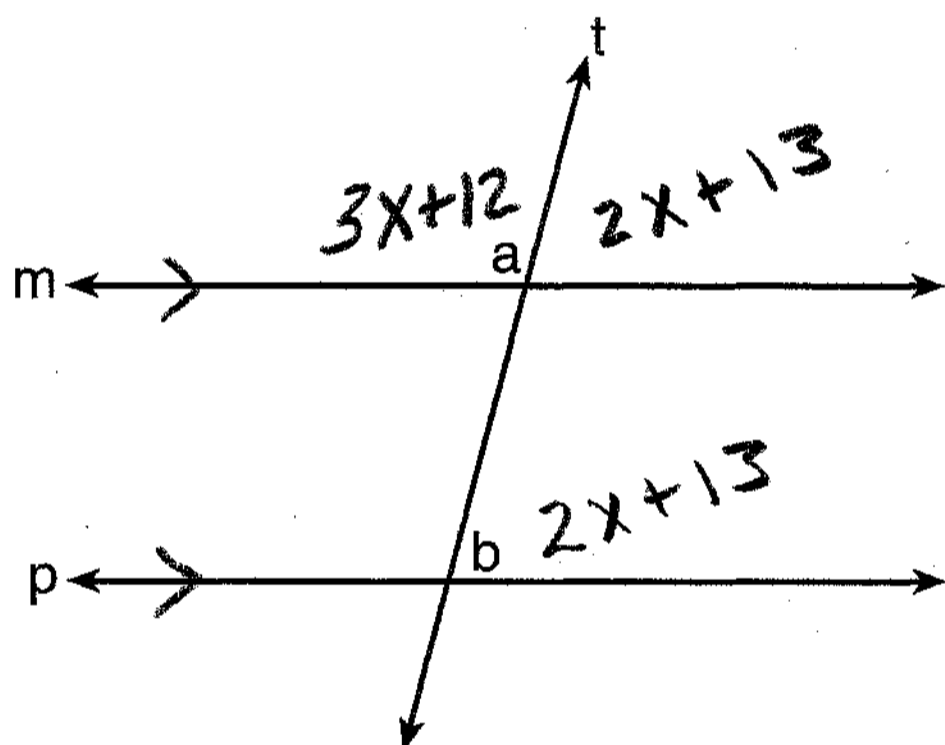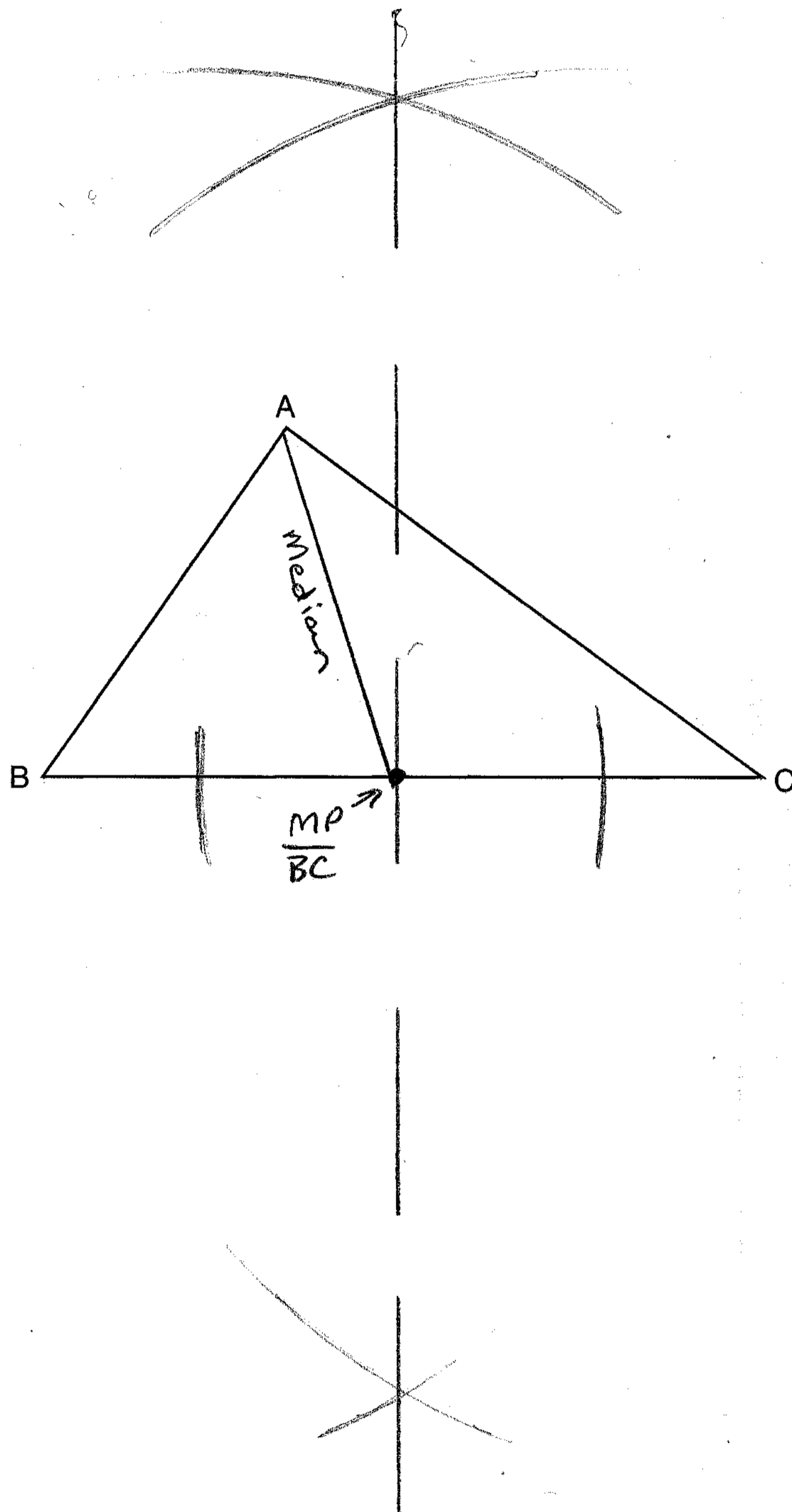
$$3x + 12 + 2x + 13 = 180°$$
$$5x + 25 = 180°$$
$$-25 \qquad\qquad -25$$
$$\overline{}$$
$$5x = 155$$
$$x = 31$$
$$\boxed{x = 31}$$

Check
$$3x + 12 + 2x + 13 = 180$$
$$3(31) + 12 + 2(31) + 13 = 180$$
$$93 + 12 + 62 + 13 = 180$$
$$105 + 75 = 180$$
$$180 = 180 \checkmark$$

25 On the accompanying diagram of $\triangle ABC$, use a compass and a straight-edge to construct a median from A to $\overline{BC}$.

↳ middle

Median

MP↗
$\overline{BC}$

Part III

Answer all questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.   [15]

26 Seth has <u>one less</u> than twice the number of compact discs (CDs) that Jason has. Raoul has <u>53 more</u> CDs than Jason has. If Seth gives Jason 25 CDs, Seth and Jason will have the same number of CDs. How many CDs did *each* of the three boys have to begin with?

let S = Seth's #

let J = Jason's #

let R = Raoul's #

Eq. #1 $-1 + 2J = S$

Eq. #2 $+53 + J = R$

Eq. #3 $J + 25 = S - 25$

Eq #3 $J + 25 = S - 25$

$\; -25 \qquad -25$

$J = S - 50$

Eq. #1 $-1 + 2J = S$

$\; +1 \qquad\qquad +1$

$2J = S + 1 \Rightarrow J = \frac{S+1}{2}$

$S - 50 = \frac{S+1}{2}$

$2S - 100 = S + 1$

$\; -S \qquad -S$

$S - 100 = 1$

$S = 101$

$J = 101 - 50$

$= 51$

$R = 53 + 51$

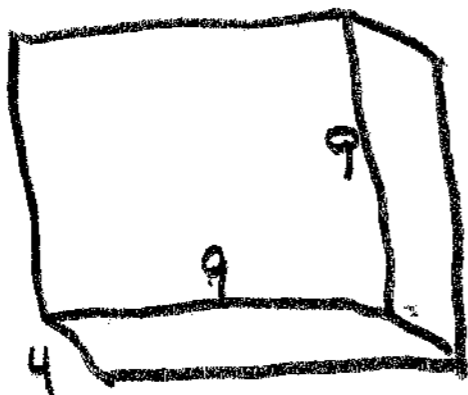$R = 104$

Answer

Seth has 101

Jason has 51

Raoul has 104

27 Tina's preschool has a set of cardboard building blocks, each of which measures 9 inches by 9 inches by 4 inches. How many of these blocks will Tina need to build a wall 4 inches thick, 3 feet high, and 12 feet long?
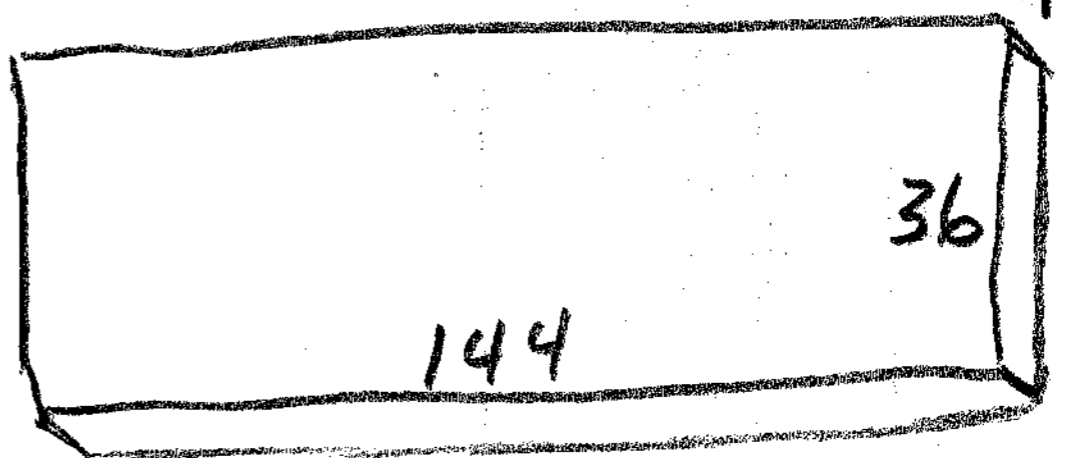
$3 ft = 36 in$

$12 ft = 144 in$

Block

$V = lwh$

$V = 9 \cdot 9 \cdot 4$

$V = 324 in^3$

Wall

$V = 144 \cdot 36 \cdot 4 = 20,736$

$\frac{Wall}{Blocks} \quad \frac{20,736}{324} = \boxed{64 \text{ blocks are needed}}$

28 In a town election, candidates *A* and *B* were running for mayor. There were 30,500 people eligible to vote, and $\frac{3}{4}$ of them actually voted. Candidate *B* received $\frac{1}{3}$ of the votes cast. How many people voted for candidate *B*? What percent of the votes cast, to the *nearest tenth of a percent*, did candidate *A* receive?

$$30,500 \times \frac{3}{4} = \boxed{22,875} \leftarrow \text{\# people who actually voted}$$

$$22,875 \times \frac{1}{3} = \boxed{7,625} \leftarrow \text{\# people who voted for Candidate B}$$

$$\frac{7,625}{22,875} = .\overline{3333} = 33\tfrac{1}{3}\% \quad \text{of the votes were for candidate B}$$

$$100\% - 33\tfrac{1}{3}\% = 66\tfrac{2}{3}\% \Rightarrow \boxed{67\%} \text{ of the votes were cast for Candidate A}$$

29 A certain state is considering changing the arrangement of letters and numbers on its license plates. The two options the state is considering are:

Option 1: three letters followed by a four-digit number <u>with repetition</u> of both letters and digits allowed

Option 2: four letters followed by a three-digit number <u>without repetition</u> of either letters or digits

[Zero may be chosen as the first digit of the number in either option.]

Which option will enable the state to issue more license plates? How many *more* different license plates will that option yield?

26 letters in alphabet

10 digits

Option #1 Letter $\boxed{26}$ Letter $\boxed{26}$ Letter $\boxed{26}$ $\boxed{10}$ $\boxed{10}$ $\boxed{10}$ $\boxed{10}$ = 175,760,000

Option #2 $\boxed{26}$ $\boxed{25}$ $\boxed{24}$ $\boxed{23}$ $\boxed{10}$ $\boxed{9}$ $\boxed{8}$ = 258,336,000

Option #2 produces more plates.
Option #2 yields 82,576,000 more plates.

Answer

30 To get from his high school to his home, Jamal travels 5.0 miles east and then 4.0 miles north. When Sheila goes to her home from the same high school, she travels 8.0 miles east and 2.0 miles south. What is the measure of the shortest distance, to the *nearest tenth of a mile*, between Jamal's home and Sheila's home? [The use of the accompanying grid is optional.]

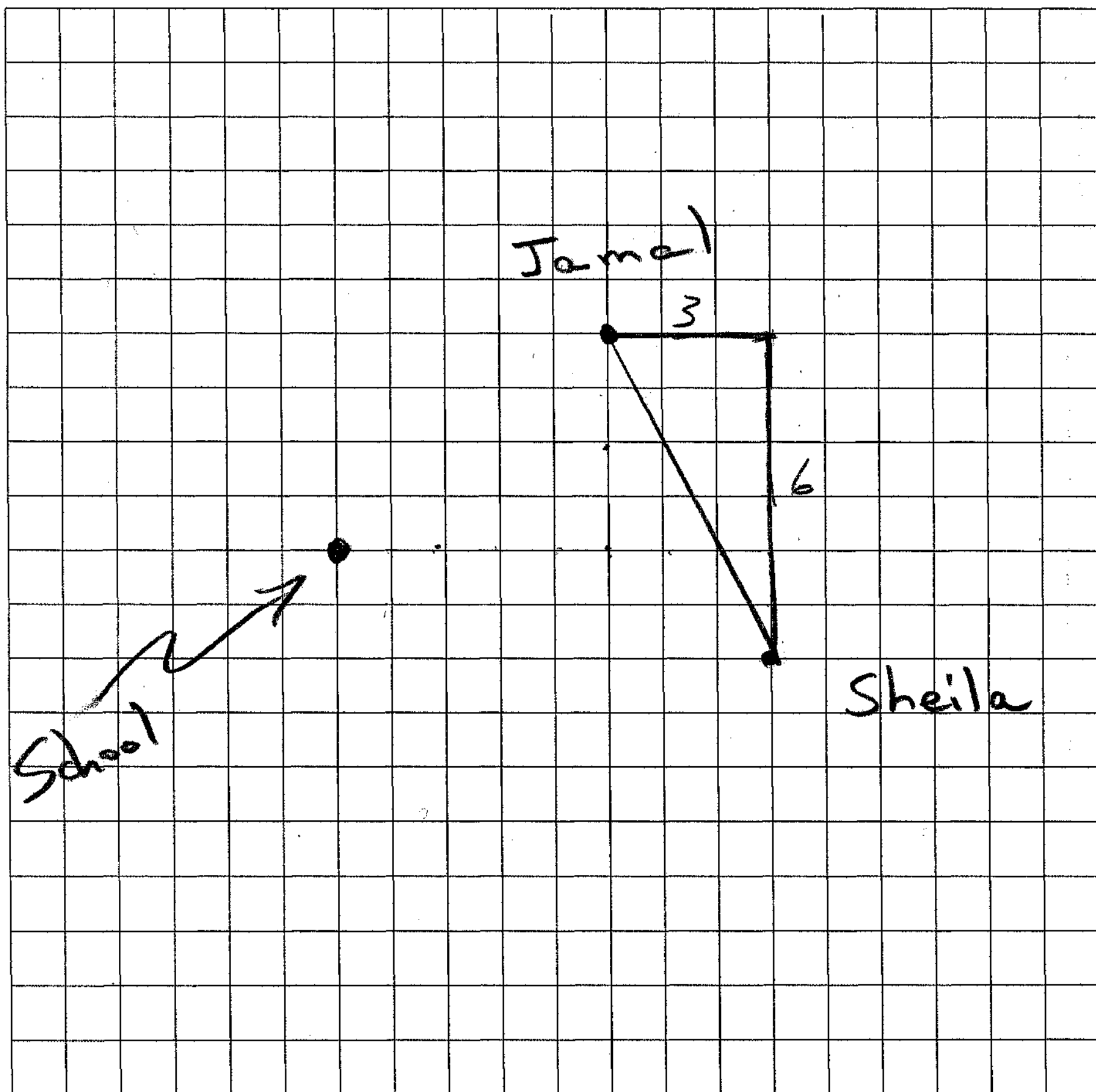$$a^2 + b^2 = C^2$$
$$3^2 + 6^2 = C^2$$
$$9 + 36 = C^2$$
$$45 = C^2$$
$$\sqrt{45} = \sqrt{C^2}$$

$$6.7082303932 = C$$

the shortest distance is 6.7 miles

Jamal
3
6
School
Sheila

N
W ← → E
S

Part IV

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [20]

31 Deborah built a box by cutting 3-inch squares from the corners of a rectangular sheet of cardboard, as shown in the accompanying diagram, and then folding the sides up. The volume of the box is 150 cubic inches, and the longer side of the box is 5 inches more than the shorter side. Find the number of inches in the shorter side of the *original* sheet of cardboard.
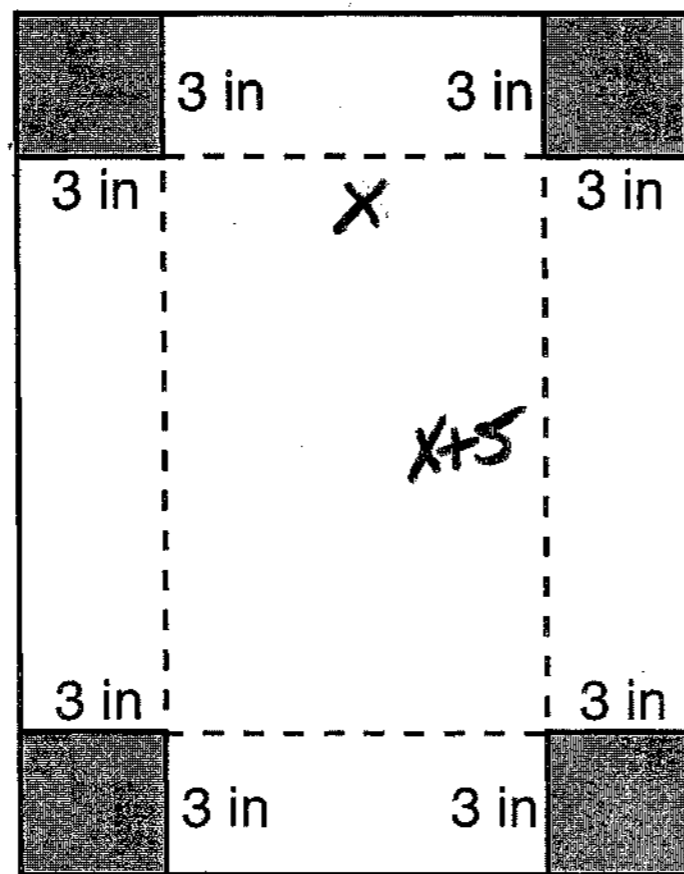
$$V = lwh$$

$$150 = (x+5)(x)\ 3$$

$$150 = (x^2 + 5x)\ 3$$

$$150 = 3x^2 + 15x$$

$$0 = 3x^2 + 15x - 150$$

$$\div 3 \qquad 0 = x^2 + 5x - 50$$

$$0 = (x + 10)(x - 5)$$

$$0 = x + 10 \qquad 0 = x - 5$$

$$-10 = x \qquad 5 = x$$

Factors of 50

1 50
2 25
5 10

$$5 + 3 + 3 = \boxed{11 \text{ inches}}$$
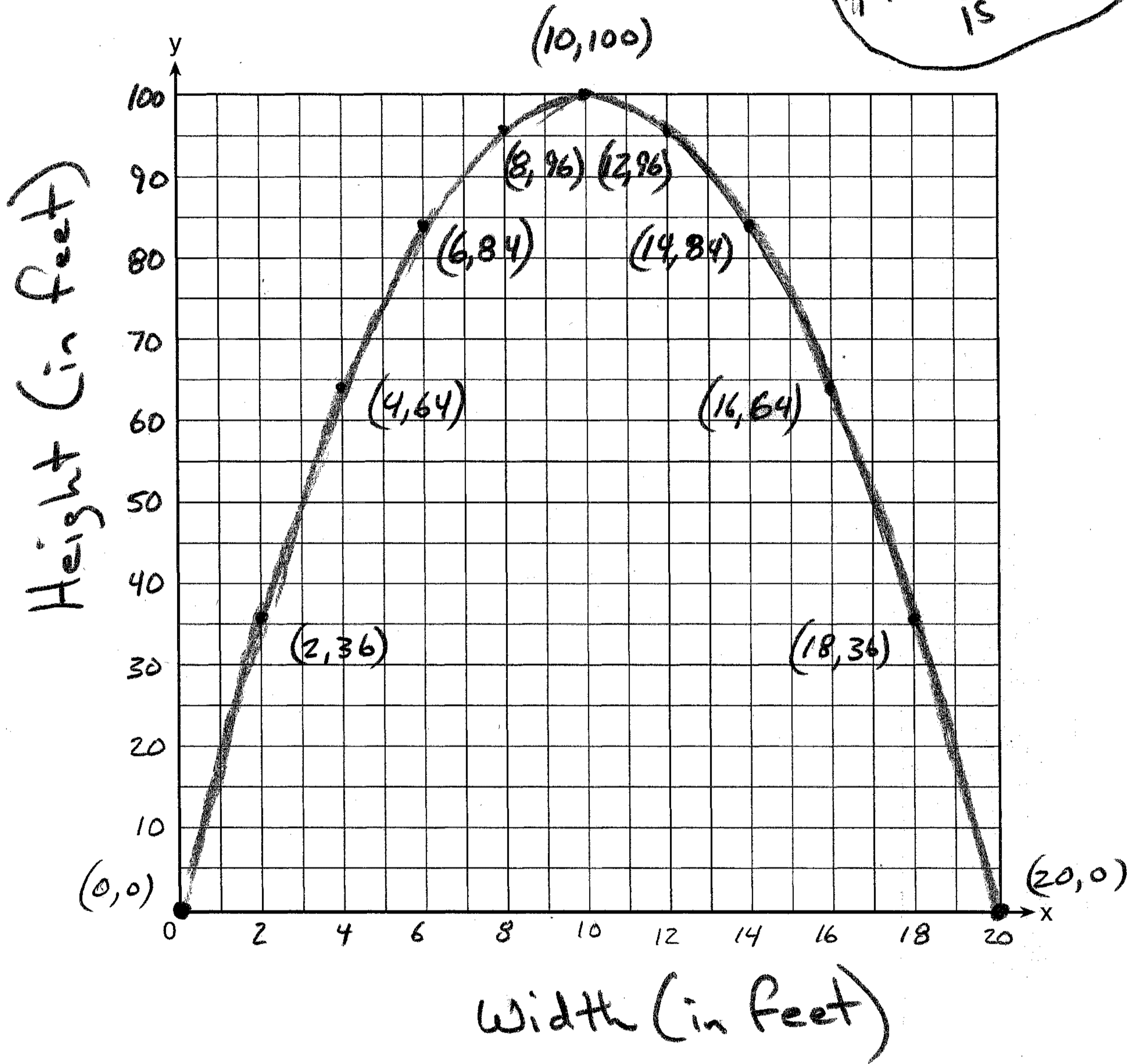
check
$5 \times 10 \times 3 = 150$ ✓

32 A triangular park is formed by the intersection of three streets, Bridge Street, Harbor Place, and College Avenue, as shown in the accompanying diagram. A walkway parallel to Harbor Place goes through the park. A time capsule has been buried in the park in a location that is equidistant from Bridge Street and College Avenue and 5 yards from the walkway. Indicate on the diagram with an **X** *each* possible location where the time capsule could be buried.
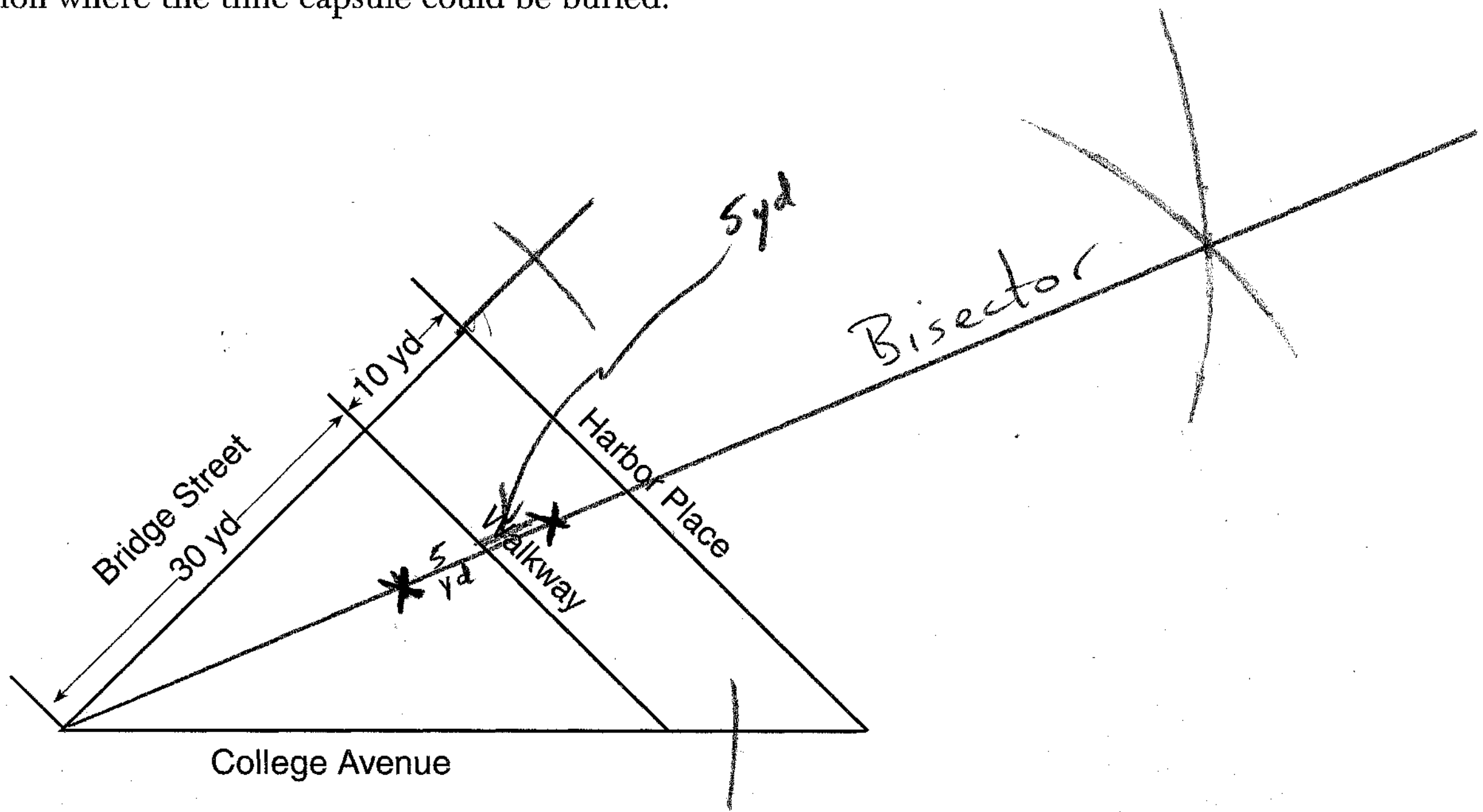
33 An architect is designing a museum entranceway in the shape of a parabolic arch represented by the equation $y = -x^2 + 20x$, where $0 \leq x \leq 20$ and all dimensions are expressed in feet. On the accompanying set of axes, sketch a graph of the arch and determine its maximum height, in feet.

Answer

Max height is 100 ft.

Height (in feet)

Width (in feet)

(10,100)

(8, 96) (12,96)

(6,84) (14,84)

(4,64) (16,64)

(2,36) (18,36)

(0,0) (20,0)

34 A straw is placed into a rectangular box that is 3 inches by 4 inches by 8 inches, as shown in the accompanying diagram. If the straw fits exactly into the box diagonally from the bottom left front corner to the top right back corner, how long is the straw, to the *nearest tenth of an inch*?
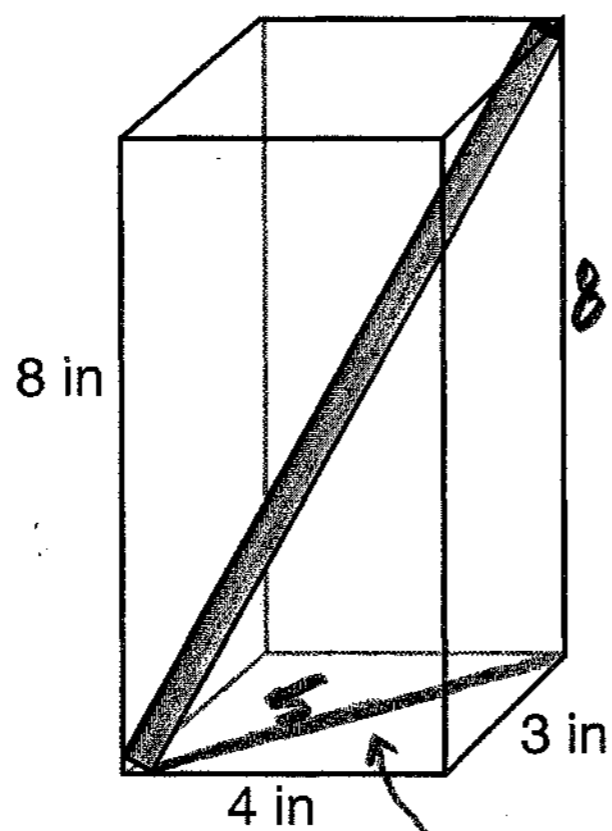
$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = c^2$$
$$9 + 16 = c^2$$
$$25 = c^2$$
$$5 = c$$

$$a^2 + b^2 = c^2$$
$$5^2 + 8^2 = c^2$$
$$25 + 64 = c^2$$
$$89 = c^2$$
$$\sqrt{89} = \sqrt{c^2}$$
$$9.433981132 = c$$

The straw is ⟨ 9.4 inches ⟩ long

Answer

35 The senior class is sponsoring a dance. The cost of a student disk jockey is $40, and tickets sell for $2 each. Write a linear equation and, on the accompanying grid, graph the equation to represent the relationship between the number of tickets sold and the profit from the dance. Then find how many tickets must be sold to break even.

$$Income = \$2 \, (\# \, tickets) - 40$$
$$I = 2t - 40$$
$$y = 2X - 40$$
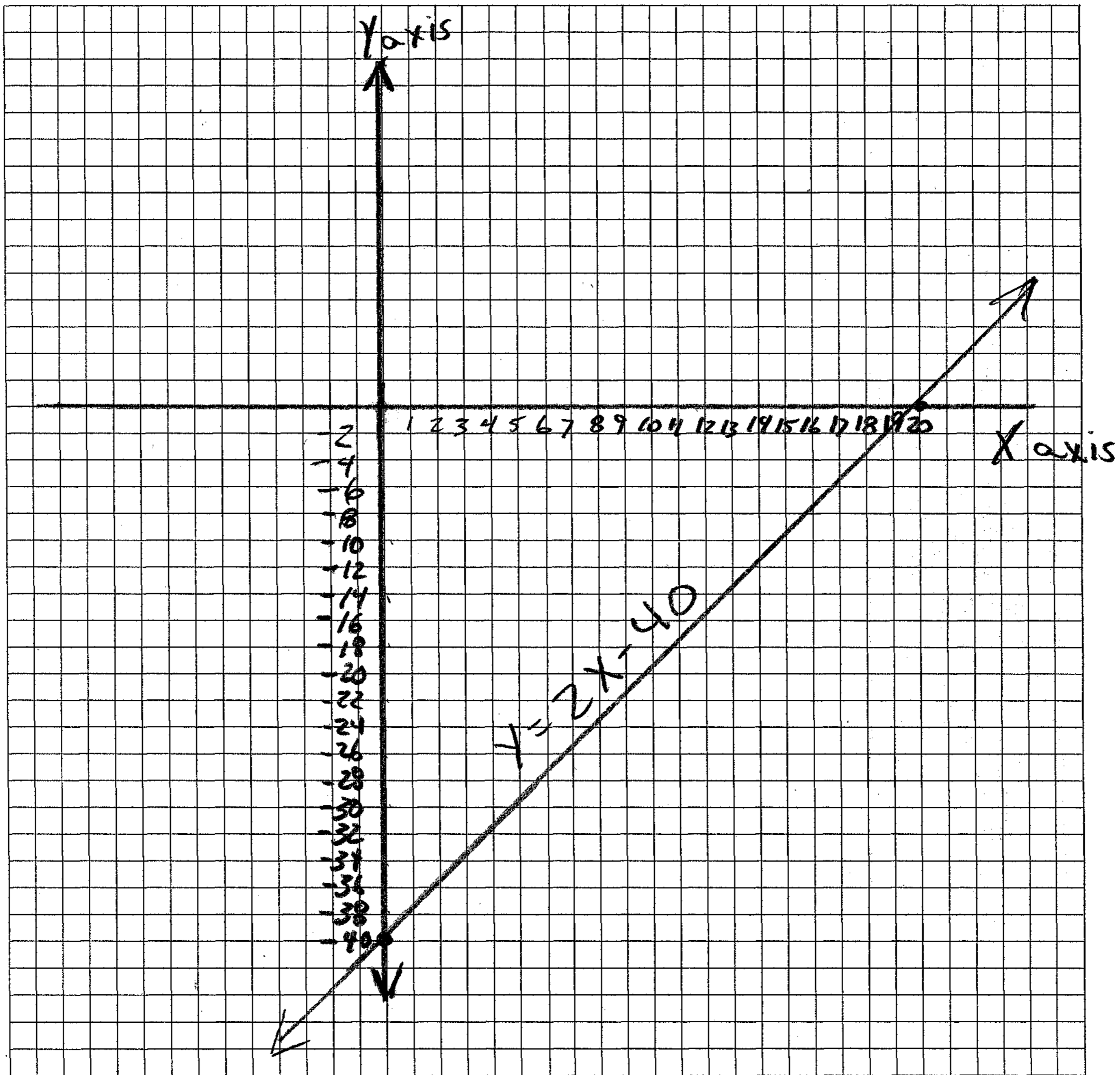
They must sell 20 tickets to break even

$$0 = 2X - 40$$
$$\underline{+40 \qquad\qquad +40}$$
$$40 = 2X$$
$$X = 20$$

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS A

Tuesday, June 17, 2003 — 1:15 to 4:15 p.m., only

ANSWER SHEET

Student ...... *Imaginary Student* ...... Sex: ☐ Male ☐ Female Grade ....

Teacher ...... *Mr. Steve* ...... School ...... *IHS@ PH* ......

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer all 20 questions in this part.

1 2	6 2	11 4	16 2
2 4	7 4	12 3	17 4
3 1	8 3	13 1	18 1
4 1	9 1	14 ... 1 or 3	19 4
5 2	10 3	15 1	20 2

Your answers for Parts II, III, and IV should be written in the test booklet.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

_____
Signature