The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS A

Friday, June 16, 2000 — 9:15 a.m. to 12:15 p.m., only

Print Your Name: Imaginary Student (i)

Print Your School's Name: www.jmap.org

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will *not* be scored. All work should be written in pen, except graphs and drawings, which should be done in pencil.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers for the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps you take, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

Notice...

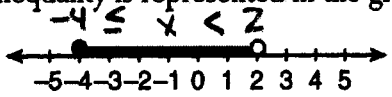
A minimum of a scientific calculator, a straightedge (ruler), and a compass must be available for your use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Record your answers in the spaces provided on the separate answer sheet. [40]

1 Which inequality is represented in the graph below?



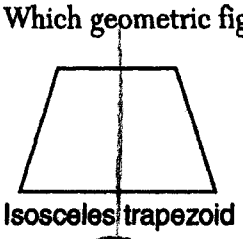
(1) $-4 < x < 2$
(2) $-4 \leq x < 2$
(3) $-4 < x \leq 2$
(4) $-4 \leq x \leq 2$

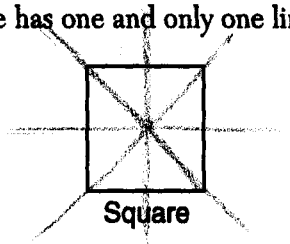
Use this space for computations.

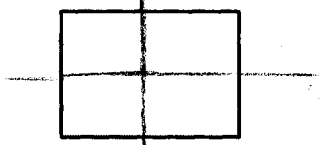
$-4 \leq x < 2$

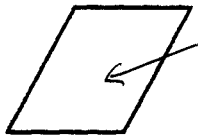
● inequality and equal signs
○ inequality sign only

2 Which geometric figure has one and only one line of symmetry?

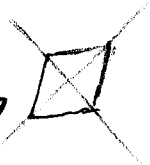

Isosceles trapezoid
(1)


Square
(3)


Rectangle
(2)


Rhombus
(4)

This is not a rhombus. The two horizontal lines are shorter than the other two lines. A rhombus has four sides of equal length.


A better rhombus

3 Which number is rational?

(1) π
(2) $\frac{5}{4}$
(3) $\sqrt{7}$
(4) $\sqrt{\frac{3}{2}}$

A rational number can be expressed as the ratio of two integers.

4 Two numbers are in the ratio 2:5. If 6 is subtracted from their sum, the result is 50. What is the larger number?

(1) 55
(2) 45
(3) 40
(4) 35

Small Part : Large Part | 2 : 5

Sum = 7

$7x - 6 = 50$
$+6 \quad +6$
$7x = 56$

$x = 8$
$5 \times 8 = 40$

5 The quotient of $-\frac{15x^8}{5x^2}$, $x \neq 0$, is

(1) $-3x^4$
(2) $-10x^4$
(3) $-3x^6$
(4) $-10x^6$

$-15 \mid x^8$
$5 \mid x^2$

$-3 \; x^{8-2} \Rightarrow -3x^6$

1 2

6 What is the inverse of the statement "If it is sunny, I will play baseball"?

(1) If I play baseball, then it is sunny.
(2) If it is not sunny, I will not play baseball.
(3) If I do not play baseball, then it is not sunny.
(4) I will play baseball if and only if it is sunny.

If not it is sunny, then not I will play baseball

Given If 1, then 2
Inverse If not 1, then not 2
Converse If 2, then 1
Contrapos. If not 2, then not 1

Use this space for computations.

7 Which ordered pair is the solution of the following system of equations?

$3x + 2y = 4$
$-2x + 2y = 24$

(1) (2,–1)
(2) (2,–5)
(3) (–4,8)
(4) (–4,–8)

$3x + 2y = 4$
$-(-2x + 2y = 24)$
$5x = -20$
$x = -4$

$3x + 2y = 4$
$3(-4) + 2y = 4$
$-12 + 2y = 4$
$+12 \quad +12$
$2y = 16$
$y = 8$

8 Which equation represents a circle whose center is (3,–2)?

(1) $(x + 3)^2 + (y - 2)^2 = 4$
(2) $(x - 3)^2 + (y + 2)^2 = 4$
(3) $(x + 2)^2 + (y - 3)^2 = 4$
(4) $(x - 2)^2 + (y + 3)^2 = 4$

$x^2 + y^2 = 4$
$(3, -2)$
$(x-3)^2 + (y-(-2))^2 = 4$

General form of equation w/ center at origin. $x^2 + y^2 = r^2$
You have to subtract the x and y values when the center is elsewhere

9 The set of integers {3,4,5} is a Pythagorean triple. Another such set is

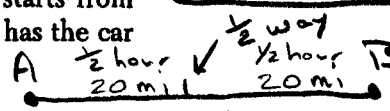
(1) {6,7,8}
(2) {6,8,12}
(3) {6,12,13}
(4) {8,15,17}

Pythagorean triple means the #s could be the lengths of sides of a right Δ.

$6^2 + 7^2 \neq 8^2$ $6^2 + 8^2 \neq 12^2$ $6^2 + 12^2 \neq 13^2$ $8^2 + 15^2 = 17^2$
$64 + 225 = 289$ ✓

10 A truck travels 40 miles from point A to point B in exactly 1 hour. When the truck is halfway between point A and point B, a car starts from point A and travels at 50 miles per hour. How many miles has the car traveled when the truck reaches point B?

(1) 25
(2) 40
(3) 50
(4) 60



after ½ hour, car starts

The car travels ½ hour at 50 mph, or 25 miles

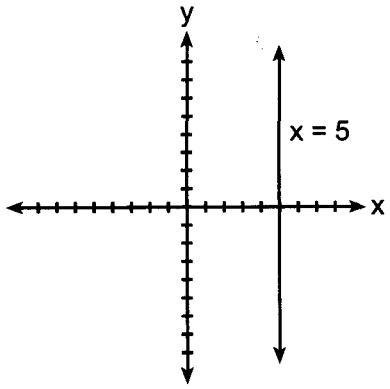
11 If $a \neq 0$ and the sum of x and $\frac{1}{a}$ is 0, then

(1) $x = a$
(2) $x = -a$
(3) $x = -\frac{1}{a}$
(4) $x = 1 - a$

$\frac{1}{a} + ? = 0$

$\frac{1}{a} + \left(-\frac{1}{a}\right) = 0$

12 The accompanying figure shows the graph of the equation $x = 5$.



What is the slope of the line $x = 5$?

(1) 5
(2) –5
(3) 0
(4) undefined

Use this space for computations.

slope = $m = \frac{\text{rise}}{\text{run}} = \frac{\infty}{0}$

Division by zero is undefined.

13 Which transformation does *not* always produce an image that is congruent to the original figure?

(1) translation
(2) dilation
(3) rotation
(4) reflection

congruent = same size, same shape

Makes image bigger or smaller, but not same size.

14 If rain is falling at the rate of 2 inches per hour, how many inches of rain will fall in x minutes?

(1) $2x$
(2) $\frac{30}{x}$
(3) $\frac{60}{x}$
(4) $\frac{x}{30}$

Inches	2	1	4	3	
Time	60	30	120	90	x

It rains 1 inch every 30 minutes
Divide x by 30 to get inches $\Rightarrow \frac{x}{30}$

15 The expression $(x - 6)^2$ is equivalent to

(1) $x^2 - 36$
(2) $x^2 + 36$
(3) $x^2 - 12x + 36$
(4) $x^2 + 12x + 36$

$(x-6)(x-6) = x^2 - 6x - 6x + 36$

$x^2 - 12x + 36$

16 How many different five-digit numbers can be formed from the digits 1, 2, 3, 4, and 5 if each digit is used only once?

(1) 120
(2) 60
(3) 24
(4) 20

Choices

1st Dig. x 2nd Dig. x 3rd Dig. x 4th Dig. x 5th Dig.

5 · 4 · 3 · 2 · 1 = 120

17 For five algebra examinations, Maria has an average of 88. What must she score on the sixth test to bring her average up to exactly 90?

(1) 92
(2) 94
(3) 98
(4) 100

Step One: $\text{avg} = \frac{\text{sum}}{\text{count}}$

$88 = \frac{\text{sum}}{5}$

$5(88) = \text{sum}$

$440 = \text{sum}$

Step Two: $90 = \frac{440 + x}{6}$

$540 = 440 + x$

$-440 \quad -440$

$100 = x$

18 The graphs of the equations $y = x^2 + 4x - 1$ and $y + 3 = x$ are drawn on the same set of axes. At which point do the graphs intersect?

(1) (1,4)
(2) (1,–2)
(3) (–2,1)
(4) (–2,–5)

19 If $2x^2 - 4x + 6$ is subtracted from $5x^2 + 8x - 2$, the difference is

(1) $3x^2 + 12x - 8$
(2) $-3x^2 - 12x + 8$
(3) $3x^2 + 4x + 4$
(4) $-3x^2 + 4x + 4$

20 What is the value of 3^{-2}?

(1) $\frac{1}{9}$
(2) $-\frac{1}{9}$
(3) 9
(4) –9

Use this space for computations.

$5x^2 + 8x - 2$
$-(2x^2 - 4x + 6)$
To subtract, change the signs of the 2nd expression + add
$5x^2 + 8x - 2$
$-2x^2 + 4x - 6$
$3x^2 + 12x - 8$

$y = x^2 + 4x - 1$
$y + 3 = x$
$-3 \quad -3$
$y = x - 3$

$x^2 + 4x - 1 = x - 3$
$-x \quad -x$
$x^2 + 3x - 1 = -3$
$+3 \quad +3$
$x^2 + 3x + 2 = 0$
$(x + \_)(x + \_) = 0$
$(x+2)(x+1) = 0$
$x + 2 = 0 \quad x + 1 = 0$
$x = -2 \quad x = -1$

$y = x - 3$
$y = -2 - 3$
$y = -5$
$(-2, -5)$ Answer

Check
$y = x^2 + 4x - 1$
$-5 = (-2)^2 + 4(-2) - 1$
$-5 = 4 - 8 - 1$
$-5 = -4 - 1$
$-5 = -5$ ✓

A negative exponent means "put the base in the basement" and make the exponent positive.
$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [10]

21 The formula for changing Celsius (C) temperature to Fahrenheit (F) temperature is $F = \frac{9}{5}C + 32$. Calculate, to the *nearest degree*, the Fahrenheit temperature when the Celsius temperature is –8.

$F = \frac{9}{5}C + 32$

$C = -8$

$F = \frac{9}{5}\left(\frac{-8}{1}\right) + 32$

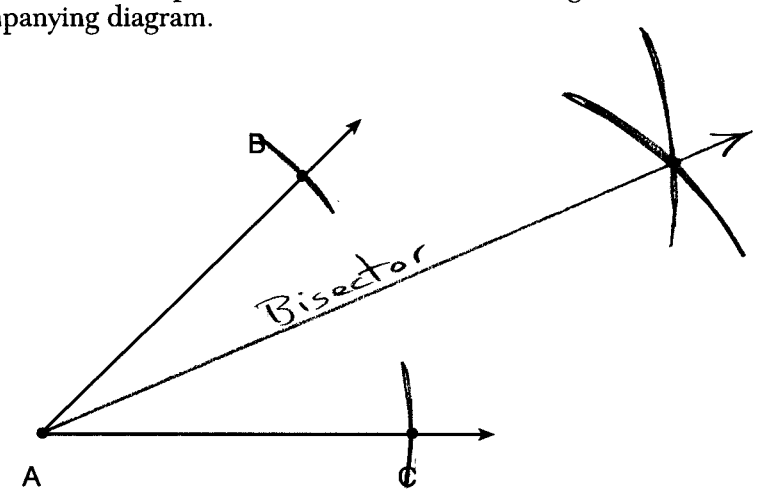
$F = \frac{-72}{5} + 32$

$F = -14\frac{2}{5} + 32$

$F = -14.4 + 32$

$F = 17.6 \Rightarrow$ 18°

22 Using only a ruler and compass, construct the bisector of angle *BAC* in the accompanying diagram.



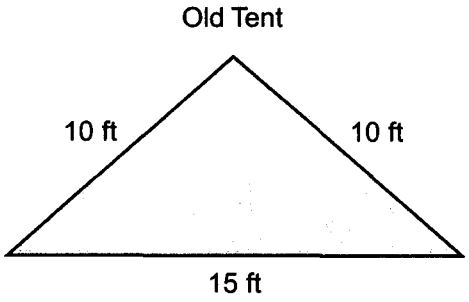
23 All seven-digit telephone numbers in a town begin with 245. How many telephone numbers may be assigned in the town if the last four digits do *not* begin or end in a zero?

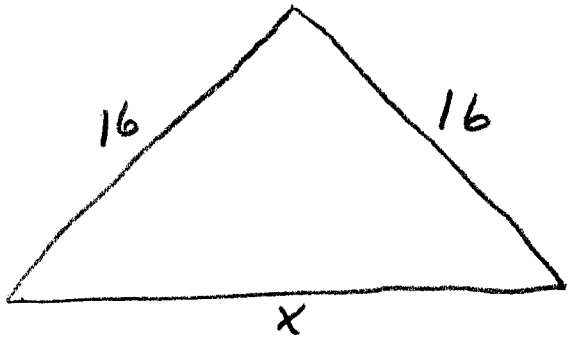
	1st Dig.	2nd Dig.	3rd Dig.	4th Dig.	5th Dig.	6th Dig.	7th Dig.	
# Choices	1	1	1	9	10	10	9	= 8,100
Conditions	2	4	5	Not zero			not zero	

Answer

They can assign 8,100 telephone #s.

24 The Rivera family bought a new tent for camping. Their old tent had equal sides of 10 feet and a floor width of 15 feet, as shown in the accompanying diagram.





If the new tent is similar in shape to the old tent and has equal sides of 16 feet, how wide is the floor of the new tent?

$$\frac{\text{side}}{\text{floor}} \quad \frac{10}{15} = \frac{16}{x}$$

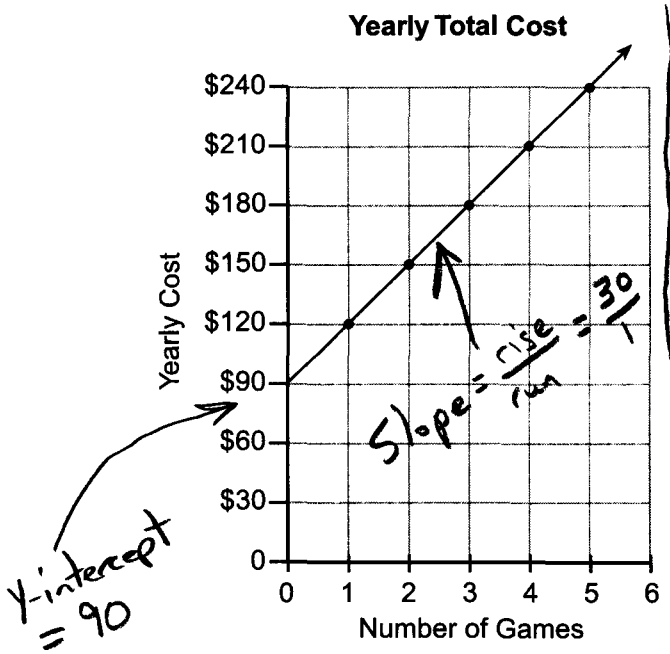
$$10x = 15(16)$$

$$10x = 240$$

$$x = \boxed{24 \text{ feet}}$$

25 The accompanying graph represents the yearly cost of playing 0 to 5 games of golf at the Shadybrook Golf Course. What is the total cost of joining the club and playing 10 games during the year?

$390



Solution #1 Table of Values

X	Y
0	90
1	120
2	150
3	180
4	210
5	240
6	270
7	300
8	330
9	360
10	390

Increase by 30 each time

Solution #2 Equation

$y = mx + b$

(m: slope, b: y-intercept)

$m = 30 \quad b = 90$

$y = 30x + 90$

Let $x = 10$

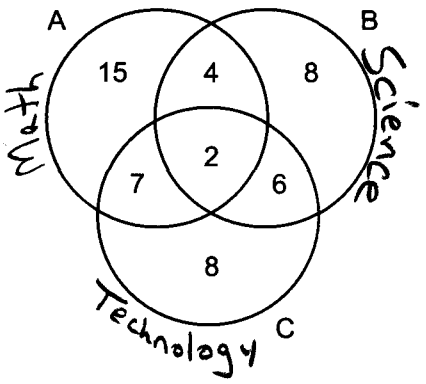
$y = 30(10) + 90$

$y = 300 + 90 \Rightarrow$ $390

Part III

Answer all questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [15]

26 The accompanying Venn diagram shows the number of students who take various courses. All students in circle *A* take mathematics. All in circle *B* take science. All in circle *C* take technology. What percentage of the students take mathematics or technology?



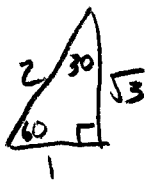
Total # of students =
$15+4+8+7+2+6+8 = 50$

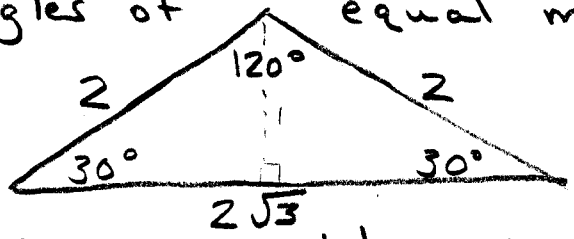
Total taking math or technology
$= 15+4+7+2+6+8$
$= 42$

Percent taking math or tech $= \frac{42}{50} = .84 = \boxed{84\%}$

27 Hersch says if a triangle is an obtuse triangle, then it cannot also be an isosceles triangle. Using a diagram, show that Hersch is incorrect, and indicate the measures of all the angles and sides to justify your answer.

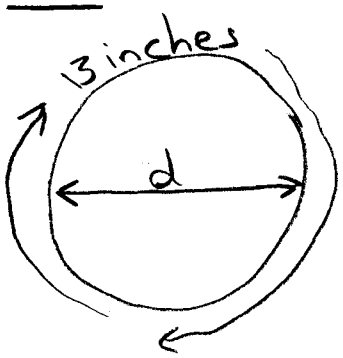
An obtuse $\triangle$ contains 1 angle that is more than 90°. An isosceles $\triangle$ has two base angles of equal measure.





The above $\triangle$ is an obtuse isosceles $\triangle$.

28 Tamika has a hard rubber ball whose circumference measures 13 inches. She wants to box it for a gift but can only find cube-shaped boxes of sides 3 inches, 4 inches, 5 inches, or 6 inches. What is the *smallest* box that the ball will fit into with the top on?



We know circumference. (C)

We need to know diameter. (d)

$C = \pi d$

$13 = \pi d$

$\frac{13}{\pi} = d$

$4.138028852 = d$

The [5 inch box] is the smallest box that can be used.

29 The distance from Earth to the imaginary planet Med is 1.7×10^7 miles. If a spaceship is capable of traveling 1,420 miles per hour, how many days will it take the spaceship to reach the planet Med? Round your answer to the *nearest day*.

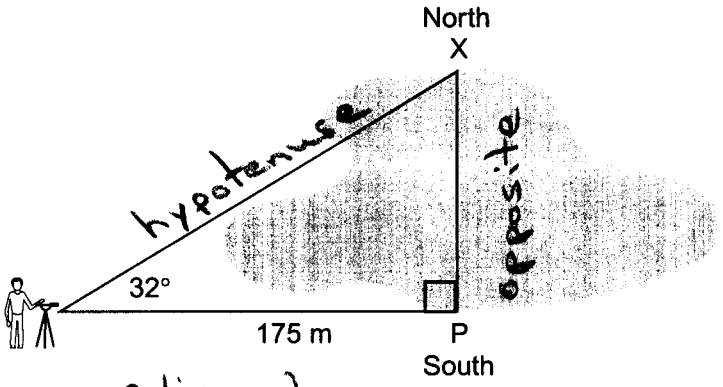
$\frac{1.7 \times 10^7 \text{ miles}}{(1{,}420 \text{ mph})\ 24 \text{ (hours/day)}}$ = # days the spaceship travels

$\frac{1.7 \times 10^7}{34080} \Rightarrow \frac{1.7000000.}{34080}$ (1 2 3 4 5 6 7)

$\frac{17{,}000{,}000}{34080} = 498.82628911$ days

It will take [499 days]

30 A surveyor needs to determine the distance across the pond shown in the accompanying diagram. She determines that the distance from her position to point P on the south shore of the pond is 175 meters and the angle from her position to point X on the north shore is 32°. Determine the distance, PX, across the pond, rounded to the *nearest meter*.



SOH-CAH-TOA

$\sin = \frac{opp}{hyp}$

$\cos = \frac{adj}{hyp}$

$\tan = \frac{opp}{adj.}$

adjacent

$\tan 32° = \frac{opp.}{adj.} = \frac{opp.}{175}$

$\tan 32° = \frac{opp}{175}$

$175 \tan 32° = opp.$

Reminder: Set calculator to degrees mode

$175 \tan 32° = 109.35213 66$

It is **109 meters** across the pond.

Part IV

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [20]

31 The owner of a movie theater was counting the money from 1 day's ticket sales. He knew that a total of 150 tickets were sold. Adult tickets cost $7.50 each and children's tickets cost $4.75 each. If the total receipts for the day were $891.25, how many of *each* kind of ticket were sold?

Answer
There were 65 adult and 85 children tickets sold.

Let A = # adult tickets sold
Let C = # children tickets sold

Equation #1: $A + C = 150 \Rightarrow A + C = 150$, $-C \quad -C$, $A = 150 - C$

Equation #2: $7.5(A) + 4.75C = 891.25$

substitute

$7.5(150 - C) + 4.75C = 891.25$

$1125 - 7.5C + 4.75C = 891.25$

$-1125 \qquad -1125$

$-7.5C + 4.75C = -233.75$

$-2.75C = -233.75$

D(-2.75): $\frac{-2.75C}{-2.75} = \frac{-233.75}{-2.75}$

$C = 85$

$A = 150 - C$

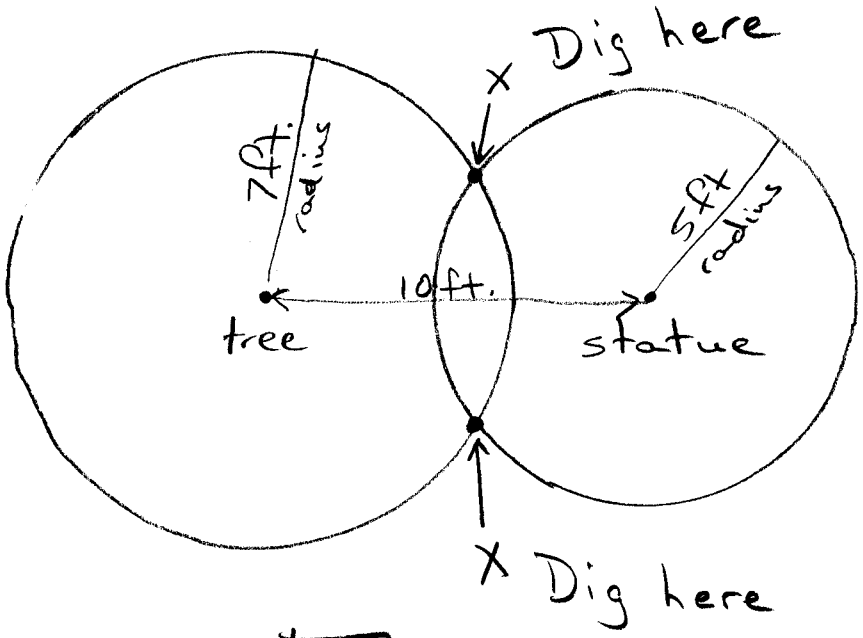
$A = 150 - 85$

$A = 65$

Check
$A + C = 150$
$65 + 85 = 150$
$150 = 150$ ✓

$7.5(A) + 4.75(C) = 891.25$
$7.5(65) + 4.75(85) = 891.25$
$487.5 + 403.75 = 891.25$
$891.25 = 891.25$ ✓

32 A treasure map shows a treasure hidden in a park near a tree and a statue. The map indicates that the tree and the statue are 10 feet apart. The treasure is buried 7 feet from the base of the tree and also 5 feet from the base of the statue. How many places are possible locations for the treasure to be buried? Draw a diagram of the treasure map, and indicate with an **X** *each* possible location of the treasure.

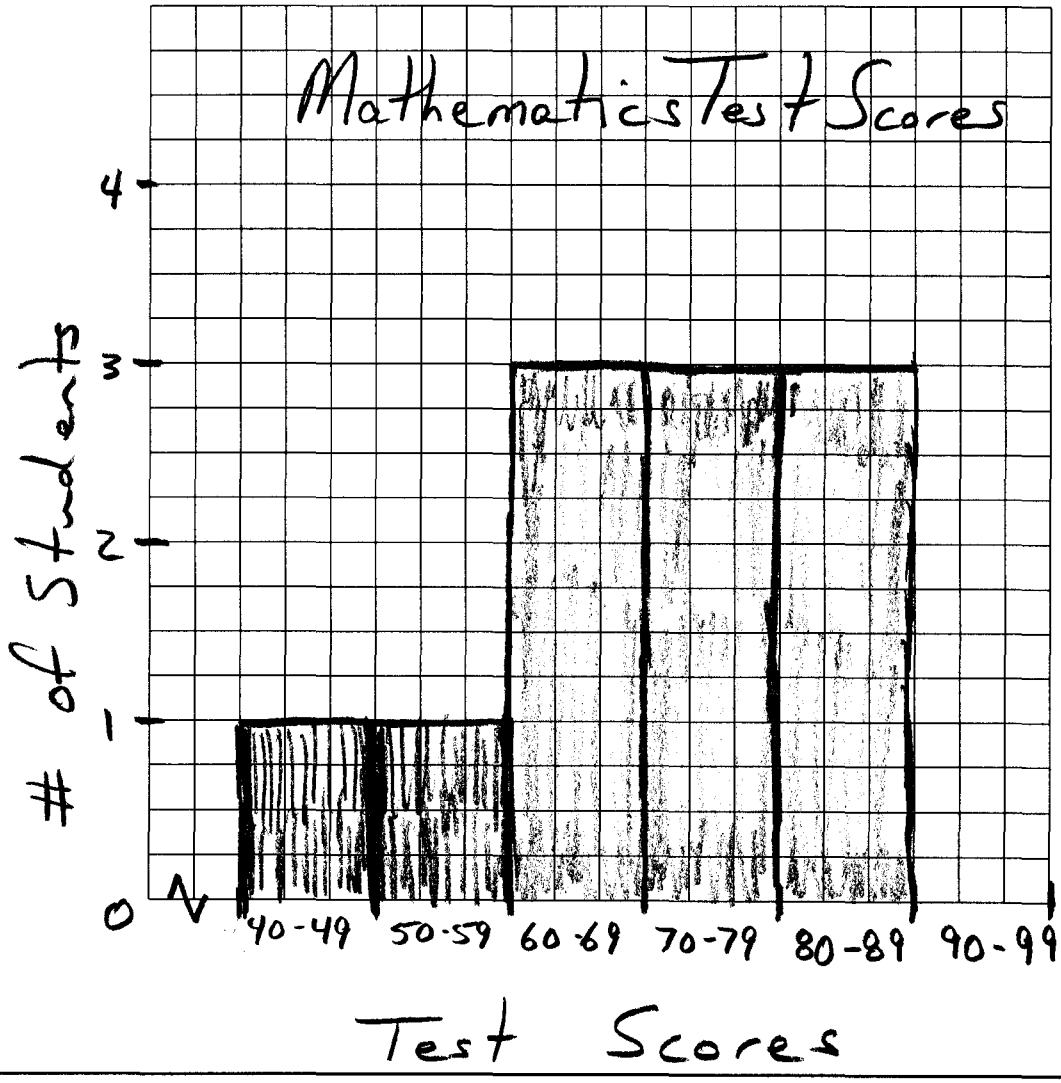


There are 2 possible locations for the treasure.

33 The scores on a mathematics test were 70, 55, 61, 80, 85, 72, 65, 40, 74, 68, and 84. Complete the accompanying table, and use the table to construct a frequency histogram for these scores.

No spaces between bars.

Score	Tally	Frequency
40–49	I	1
50–59	I	1
60–69	III	3
70–79	III	3
80–89	III	3



34 Paul orders a pizza. Chef Carl randomly chooses two different toppings to put on the pizza from the following: pepperoni, onion, sausage, mushrooms, and anchovies. If Paul will not eat pizza with mushrooms, determine the probability that Paul will *not* eat the pizza Chef Carl has made.

$$P_{(\text{event})} = \frac{\#\text{ times event happens}}{\text{total possible outcomes}}$$

$$P\begin{pmatrix}\text{2 topping pizza}\\ \text{w/ mushrooms}\end{pmatrix} = \frac{\#\text{ 2 topping pizzas w/ mushrooms}}{\text{total \# of 2 topping pizzas}}$$

total # of 2 topping pizzas is ${}_5C_2 \Rightarrow \binom{5}{2} = \frac{5\boxed{4}}{\boxed{2}\boxed{1}} = \frac{20}{2} = 10$

↑ # in 1st box in numerator

↳ # boxes + # in first box in denominator

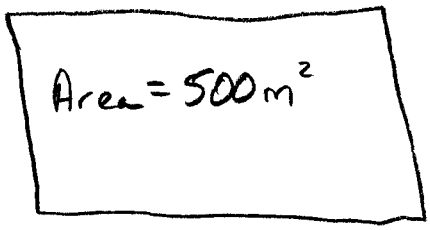
There are 10 possible 2 topping pizzas - We can list them to see how many have mushrooms.

1st topping → 2nd topping

- p → s, m ← won't eat this one (#1), a
- o → s, m ← won't eat this one (#2), a
- s → m ← won't eat this one (#3), a
- m — a ← won't eat this one (#4)

Paul will not eat $\boxed{\frac{4}{10}}$ of the 2 topping pizzas that Chef Carl can randomly make.

35 The area of the rectangular playground enclosure at South School is 500 square meters. The length of the playground is 5 meters longer than the width. Find the dimensions of the playground, in meters. [*Only an algebraic solution will be accepted.*]



$\text{Area} = l\,w$

$500 = (w+5)(w)$

$500 = w^2 + 5w$

$0 = w^2 + 5w - 500$

$w^2 + 5w - 500 = 0$

↖ one positive and one negative factor

factors of 500 must
- multiply to −500
- have a difference of 5

$(w + \_\_)(w - \_\_) = 0$

$(w + 25)(w - 20) = 0$

$w + 25 = 0$ $\quad w - 20 = 0$

$w = -25$ $\quad w = 20$

This one doesn't work because distance/length cannot be negative.

$l = w + 5$

$l = 25$

factors of 500

1 - 500
2 - 250
4 - 125
5 - 100
10 - 50
20 - 25

Difference of 5

Answer

The playground is 25 by 20 meters.

[16]

Check

Area = 500 ✓

length is 5 more than width ✓

Tear Here

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS A

Friday, June 16, 2000 — 9:15 a.m. to 12:15 p.m., only

ANSWER SHEET

Pupil Imaginay Student (i) Sex: ☐ Male ☐ Female Grade .....

Teacher Mr. Steve School IHS @ PH

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer all 20 questions in this part.

1 2	6 2	11 3	16 1
2 1	7 3	12 4	17 4
3 2	8 2	13 2	18 4
4 3	9 4	14 4	19 1
5 3	10 1	15 3	20 1

Your answers for Parts II, III, and IV should be written in the test booklet.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

RSW

Signature

Tear Here