The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS A

Tuesday, January 22, 2002 — 1:15 to 4:15 p.m., only

Print Your Name: *Steve Watson*

Print Your School's Name: *IHS @ PH*

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will *not* be scored. All work should be written in pen, except graphs and drawings, which should be done in pencil.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice. . .

A minimum of a scientific calculator, a straightedge (ruler), and a compass must be available for your use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Record your answers in the spaces provided on the separate answer sheet. [40]

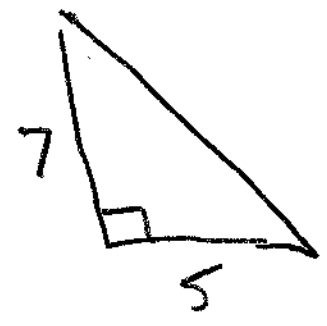**1** Expressed in factored form, the binominal $4a^2 - 9b^2$ is equivalent to

   (1) $(2a - 3b)(2a - 3b)$ (3) $(4a - 3b)(a + 3b)$

   (2) $(2a + 3b)(2a - 3b)$ (4) $(2a - 9b)(2a + b)$

Use this space for computations.

Difference of perfect squares

$(2a + 3b)(2a - 3b)$

2 If the length of the legs of a right triangle are 5 and 7, what is the length of the hypotenuse?

   (1) $\sqrt{2}$ (3) $2\sqrt{6}$

   (2) $2\sqrt{3}$ (4) $\sqrt{74}$

$5^2 + 7^2 = c^2$
$25 + 49 = c^2$
$74 = c^2$
$\sqrt{74} = c$

3 What is the slope of the line whose equation is $2y = 5x + 4$?

   (1) 5   (3) $\frac{5}{2}$

   (2) 2   (4) $\frac{2}{5}$

$y = mx + b$
$2y = 5x + 4$
$y = \frac{5}{2}x + \frac{4}{2}$

4 What is the value of x in the equation $\frac{3}{4}x + 2 = \frac{5}{4}x - 6$?

   (1) -16 (3) -4

   (2) 16   (4) 4

$\frac{3}{4}x + 2 = \frac{5}{4}x - 6$
$-\frac{3}{4}x \quad\quad -\frac{3}{4}x$
$2 = \frac{2}{4}x - 6$
$+6 \quad\quad +6$
$8 = \frac{1}{2}x$
$16 = x$

5 The product of $3x^2y$ and $-4xy^3$ is

   (1) $-12x^3y^4$ (3) $-12x^2y^3$

   (2) $12x^3y^4$ (4) $12x^2y^3$
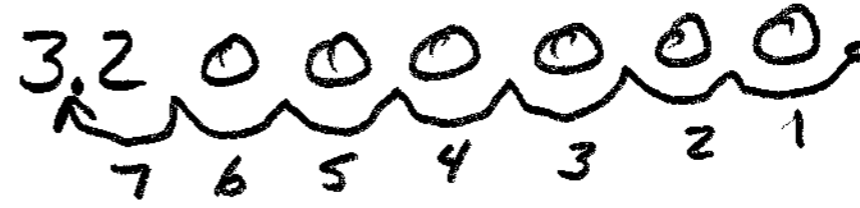
$(3x^2y)(-4xy^3)$
$(3)(-4)(x^{2+1})(y^{1+3})$
$-12x^3y^4$

Math. A – Jan. '02   [2]

6 The approximate number of seconds in a year is 32,000,000. When this number is written in scientific notation, the numerical value of the exponent is

(1) −7
(2) 6
(3) 7
(4) 8

Handwritten: 3,2 0 0 0 0 0 0, 7 6 5 4 3 2 1

7 Which expression must be added to $3x - 7$ to equal 0?

(1) 0
(2) $3x + 7$
(3) $-3x - 7$
(4) $-3x + 7$

Handwritten:
$$3X - 7$$
$$-3X + 7$$
$$0 + 0$$

8 What is the greatest possible number of points of intersection of a triangle and a circle?

(1) 6
(2) 2
(3) 3
(4) 4
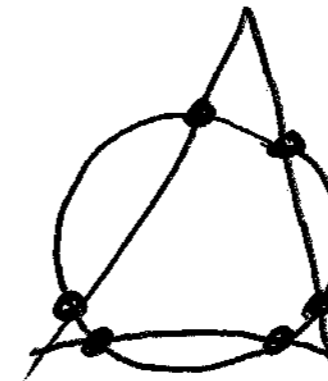
9 A fair coin is tossed three times. What is the probability that the coin will land tails up on the second toss?

(1) $\frac{1}{3}$
(2) $\frac{1}{2}$
(3) $\frac{2}{3}$
(4) $\frac{3}{4}$

Handwritten: A coin has no memory

10 There are 357 seniors in Harris High School. The ratio of boys to girls is 7:10. How many boys are in the senior class?

(1) 210
(2) 147
(3) 117
(4) 107

Handwritten:
$$\frac{Boys}{Girls} = \frac{7}{1}$$
$$\frac{Boys}{Total} = \frac{7}{17}$$
$$\frac{7}{17} = \frac{X}{357}$$
$$7(357) = 17X$$
$$2499 = 17X$$
$$X = 147$$

Math. A – Jan. '02   [3]   [OVER]

11 If x and y are defined as indicated by the accompanying table, which equation correctly represents the relationship between x and y?

x	y
2	1
3	3
5	7
7	11

(1) $y = x + 2$
(2) $y = 2x + 2$
(3) $y = 2x + 3$
(4) $y = 2x - 3$

Handwritten work:

No
$y = x + 2$
$1 \neq 2 + 2$

No
$y = 2x + 2$
$1 \neq 2(2) + 2$

No
$y = 2x + 3$
$1 \neq 2(2) + 3$

Yes
$y = 2x - 3$
$1 = 2(2) - 3$
$3 = 2(3) - 3$
$7 = 2(5) - 3$
$11 = 2(7) - 3$

12 What is the area of a square whose perimeter is represented by $12x$?

(1) $6x\sqrt{2}$
(2) $9x^2$
(3) $12x^2$
(4) $144x^2$

Handwritten work:

$P = 4s$
$12x = 4s$
$3x = s$

$A = s^2$
$A = (3x)^2$
$A = 9x^2$

13 Which inequality is true if $x = \frac{3.04}{1.48}$, $y = 1.99 + 0.33$, and $z = (1.3)^3$?

Handwritten: $2.054\cdots$ 2.32 2.197

(1) $y < z < x$
(2) $y < x < z$
(3) $x < z < y$
(4) $x < y < z$

Handwritten work:

$x < z < y$
$2.054 < 2.197 < 2.32$

14 Frank, George, and Hernando are a plumber, a cabinet maker, and an electrician, though not necessarily in that order. Each can do all work appropriate to his own field, but no work in other fields. Frank was not able to install a new electric line in his home. Hernando was not able to make cabinets. George is also a building contractor who hired one of the other people to do his electrical work. Which statement must be true?

(1) Hernando is an electrician.
(2) George is a cabinet maker.
(3) Frank is a plumber.
(4) Frank is an electrician.

Handwritten chart:

	Plumber	Cabinets	Electrician
Frank			No
George			No
Hernando		No	

15 What is the solution set of the equation $3x^2 = 48$?

(1) $\{-2,-8\}$
(2) $\{2,8\}$
(3) $\{4,-4\}$
(4) $\{4,4\}$

Handwritten work:

$3x^2 = 48$
$x^2 = 16$
$x = \pm 4$

16 In the accompanying diagram, $\overrightarrow{ABCD}$ is a straight line, and angle E in triangle BEC is a right angle.
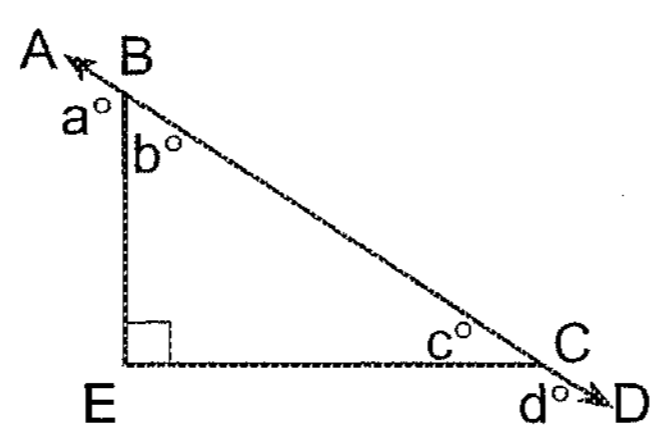
What does $a° + d°$ equal?

(1) 135°

(2) 160°

(3) 180°

(4) 270°

Handwritten computations:

$\angle b + \angle c = 90°$

$\angle a + \angle b = 180°$

$\angle c + \angle d = 180°$

$\angle a + \angle b + \angle c + \angle d = 360°$

Subtract $\quad \angle b + \angle c \quad = 90°$

$\angle a \quad\quad\quad + \angle d = 270°$

17 Which set is closed under division?

(1) {1}

(2) counting numbers

(3) integers

(4) whole numbers

Handwritten: $-\frac{1}{2} = -\frac{1}{2}$ $\quad \frac{?}{} = 1$

$\frac{2}{3} = \frac{2}{3}$ $\quad\quad \frac{4}{5} = \frac{4}{5}$

18 When Kimberly bought her new car, she found that there were 72 different ways her car could be equipped. Her choices included four choices of engine and three choices of transmission. If her only other choice was color, how many choices of color did she have?

(1) 6

(2) 12

(3) 60

(4) 65

Handwritten:

Engine Choices · Trans Choices · Color Choices = Total Choices

$4 \cdot 3 \cdot x = 72$

$12x = 72$

$x = 6$

19 Which is an irrational number?

(1) $\sqrt{9} = 3$

(2) 3.14

(3) $\sqrt{3}$

(4) $\frac{3}{4}$

**20** Which statement is logically equivalent to "If the team has a good pitcher, then the team has a good season"?

(1) If the team does not have a good season, then the team does not have a good pitcher.

(2) If the team does not have a good pitcher, then the team does not have a good season.

(3) If the team has a good season, then the team has a good pitcher.

(4) The team has a good pitcher and the team does not have a good season.

Handwritten: Given If 1, then 2

If not 2, then 1 ← Contrapositive

If not ② the team has a good season

Then not ② the team has a good pitcher

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.   [10]

21 Seth is thinking of a number between 20 and 30. The number is prime and not more than 2 away from a perfect square. What is the number?

21   22   23   24   25   26   27   28   29

↑ prime (1 × 23)   ↑ perfect square (5 × 5)   ↑ not prime (3 × 9)
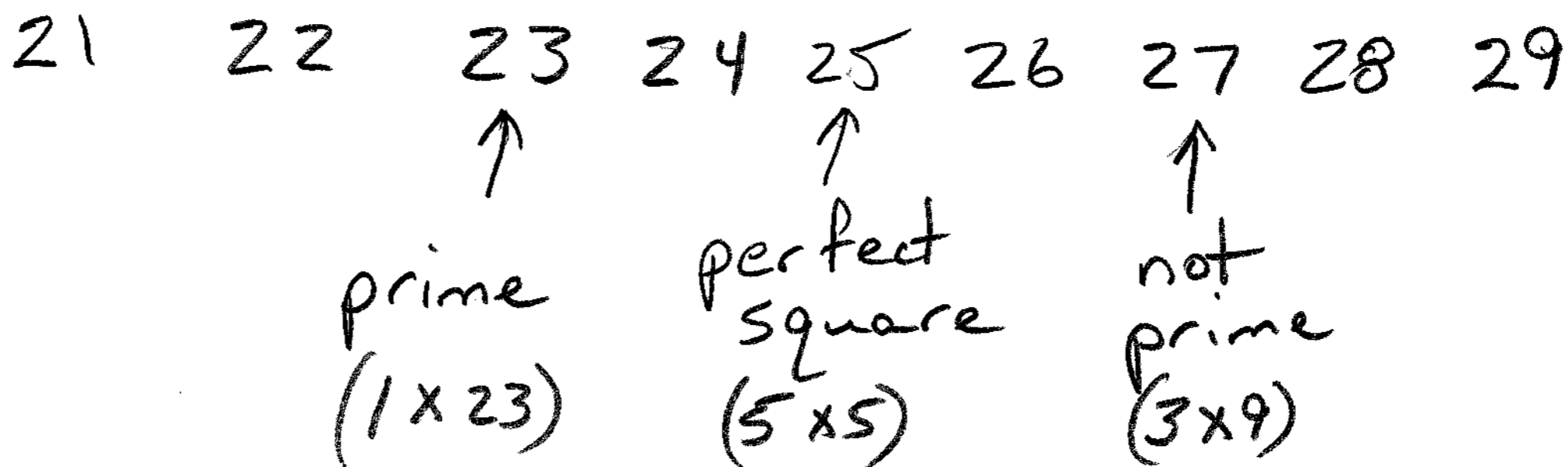
The number is $\boxed{23}$

22 A 12-foot tree casts a 16-foot shadow. How many feet tall is a nearby tree that casts a 20-foot shadow at the same time?

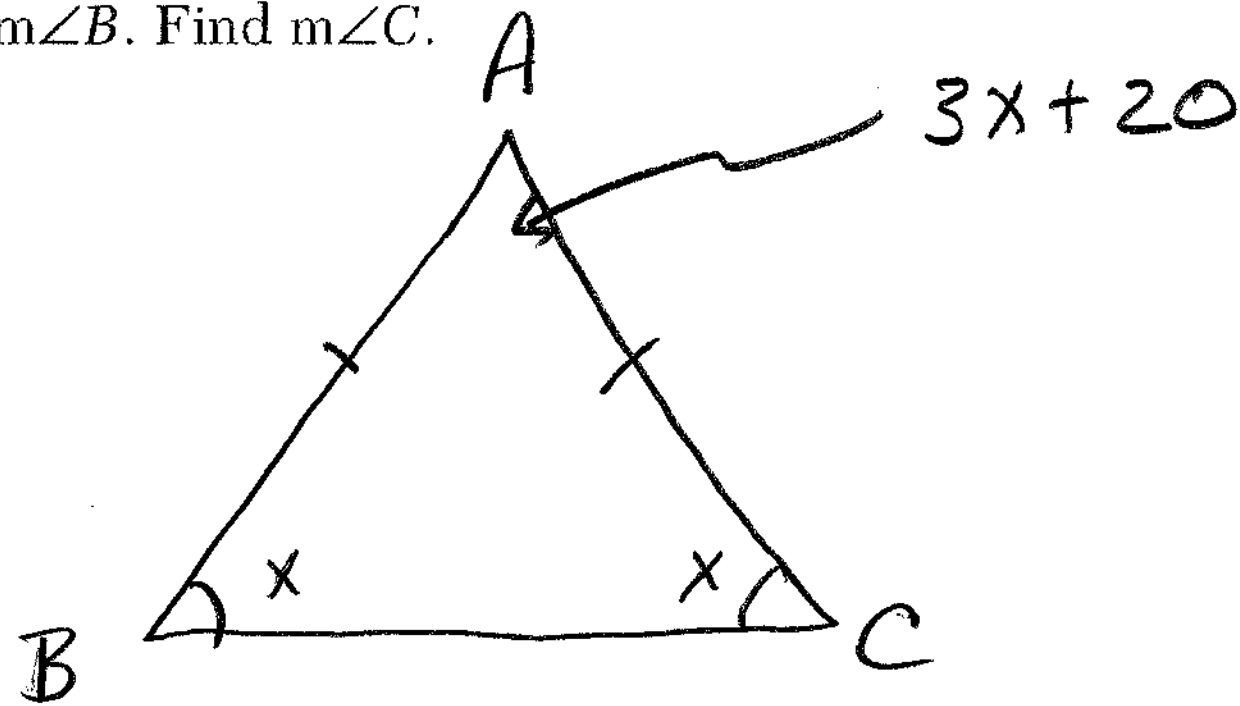12   Shadow
16

X   Shadow
20
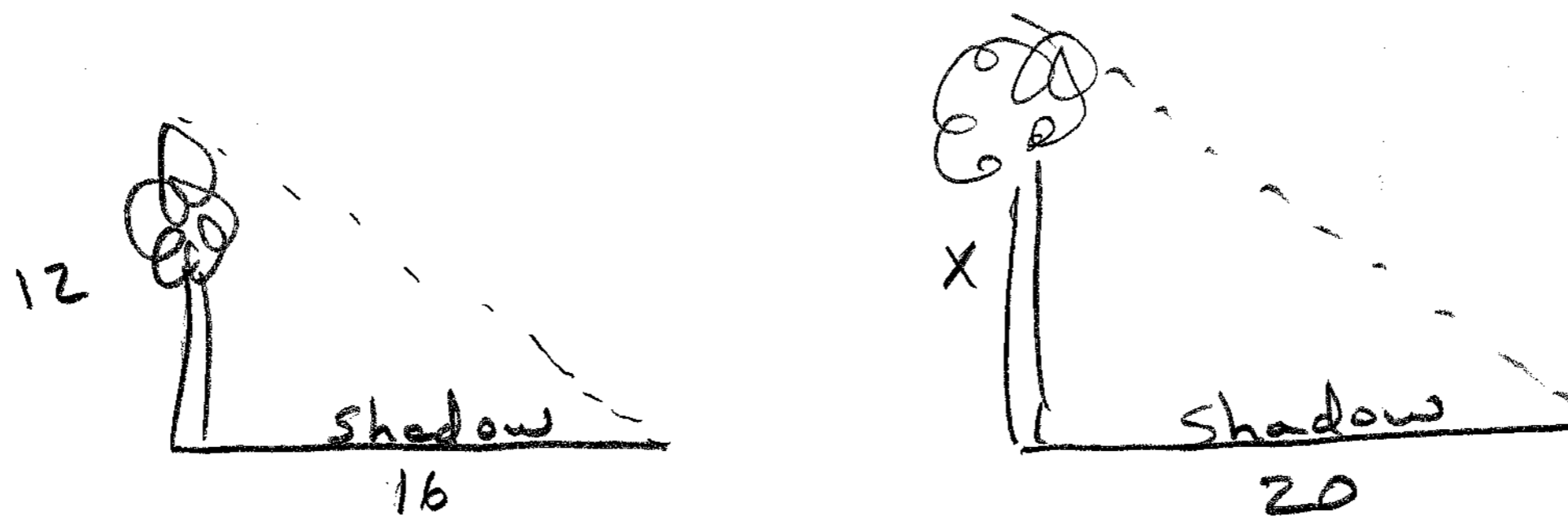
$$\frac{12}{16} = \frac{X}{20}$$

$$240 = 16X$$

$$15 = X$$

The tree is 15 feet tall

23 Vertex angle A of isosceles triangle ABC measures $20°$ more than three times m$\angle B$. Find m$\angle C$.

A

$3X + 20$

B x x C

$$3X + 20 + X + X = 180$$
$$5X + 20 \qquad\qquad = 180$$
$$5X \qquad\qquad\qquad = 160$$
$$X \qquad\qquad\qquad = 32$$

$$\boxed{m\angle C = 32}$$

24 Ashanti and Maria went to the store to buy snacks for their back-to-school party. They bought bags of chips, pretzels, and nachos. They bought three times as many bags of pretzels as bags of chips, and two fewer bags of nachos than bags of pretzels. If x represents the number of bags of chips they bought, express, in terms of x, how many bags of snacks they bought in all.

$$\text{chips} = X$$
$$\text{pretzels} = 3X$$
$$\text{nachos} = 3X - 2$$
$$\overline{\text{Total} \qquad 7X - 2}$$

They bought $\boxed{7X - 2}$ bags of chips

25 Construct a triangle with sides of lengths *a*, *b*, and *c*, as shown below. Be sure the longest side of your triangle lies on $\overline{PQ}$ and that point *P* is one of the triangle's vertices. [*Show all arcs necessary for a valid construction.*]

a ———————|

b ———————|

c ———————| longest side

ℓ ℓ

P ————— c ————— Q
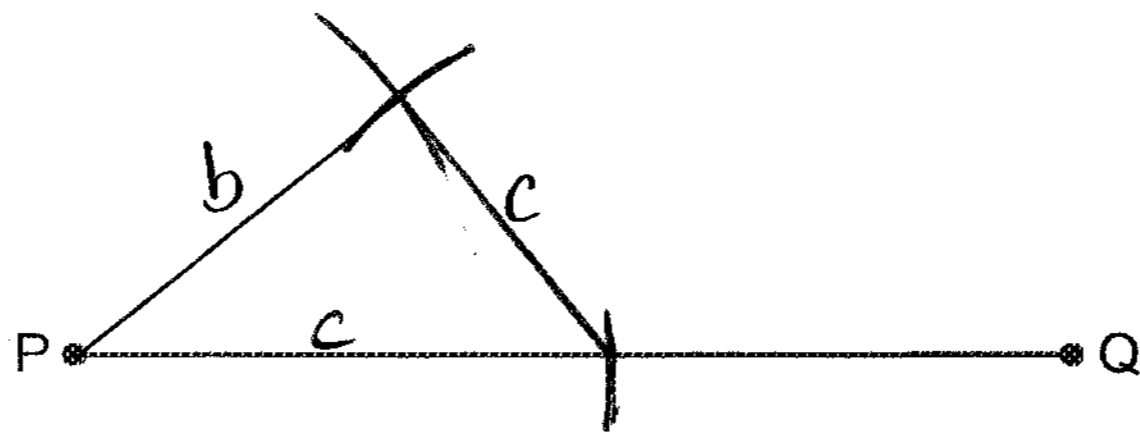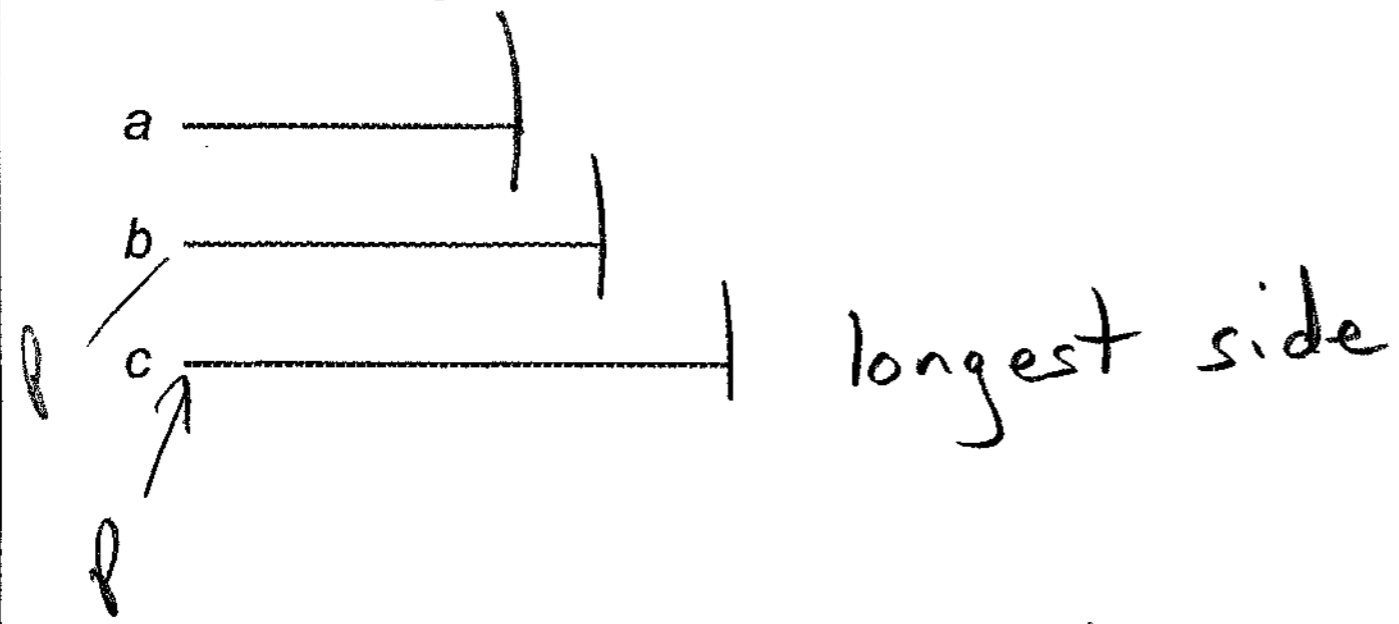
Part III

Answer all questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [15]

26 Jerry and Jean Jogger start at the same time from point A shown on the accompanying set of axes. Jerry jogs at a rate of 5 miles per hour traveling from point A to point R to point S and then to point C. Jean jogs directly from point A to point C on $\overline{AC}$ at the rate of 3 miles per hour. Which jogger reaches point C first? Explain or show your reasoning.
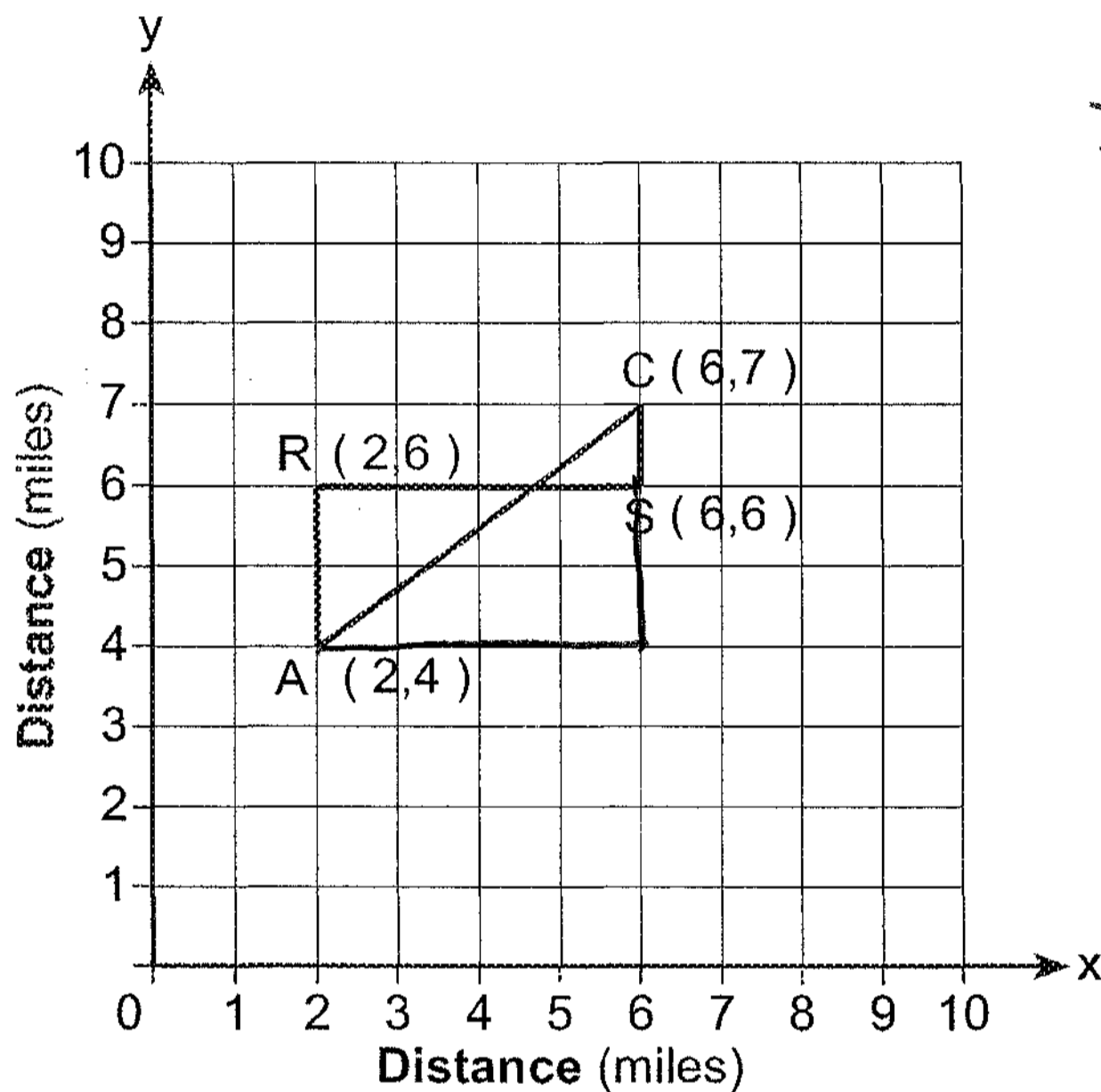
Jerry's Distance = 7
$\overline{AR} = 2$
$\overline{RS} = 4$
$\overline{SC} = 1$
————
7

Jean's Distance = 5
$a^2 + b^2 = c^2$
$4^2 + 3^2 = c^2$
$16 + 9 = c^2$
$25 = c^2$
$5 = c$

Jerry's Time = $\dfrac{7 \text{ miles}}{5 \text{ miles per hour}} = \dfrac{7}{5} = 1\frac{2}{5}$ hours

Jean's Time = $\dfrac{5 \text{ miles}}{3 \text{ miles per hour}} = \dfrac{5}{3} = 1\frac{2}{3}$ hours

Jerry reached point C first because $1\frac{2}{5}$ hours is less than $1\frac{2}{3}$ hours

27 In the accompanying diagram, a rectangular container with the dimensions 10 inches by 15 inches by 20 inches is to be filled with water, using a cylindrical cup whose radius is 2 inches and whose height is 5 inches. What is the maximum number of full cups of water that can be placed into the container without the water overflowing the container?

Radius (2 in)

5 in
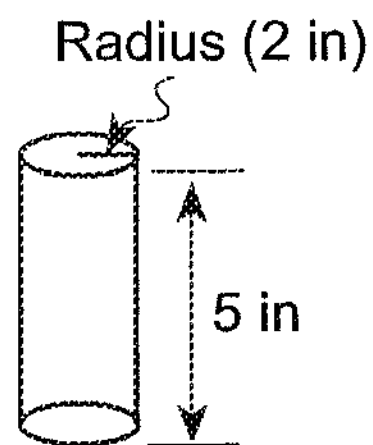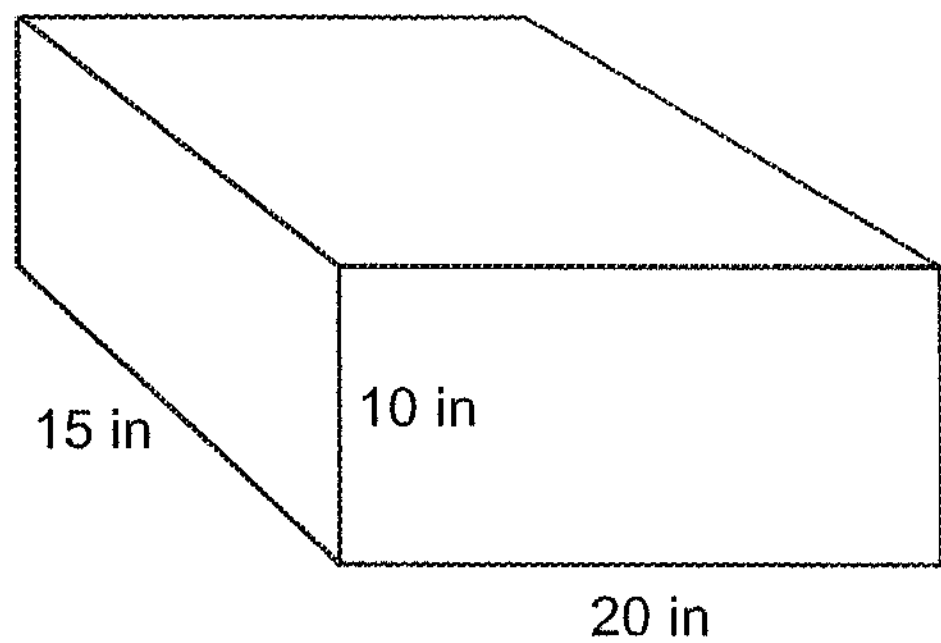
$$a = \pi r^2$$
$$a = \pi (2)^2$$
$$a = 4\pi$$

$$V = ah$$
$$V = 4\pi (5)$$
$$V = 20\pi \ in^3$$

$$V = lwh$$
$$V = (15)(20)(10)$$
$$V = 3000 \ in^3$$

$$\frac{3000}{20\pi} = 47.74648293$$

$\boxed{47}$ full cups can be placed in the container

28 A total of 600 tickets were sold for a concert. Twice as many tickets were sold in advance than were sold at the door. If the tickets sold in advance cost $25 each and the tickets sold at the door cost $32 each, how much money was collected for the concert?
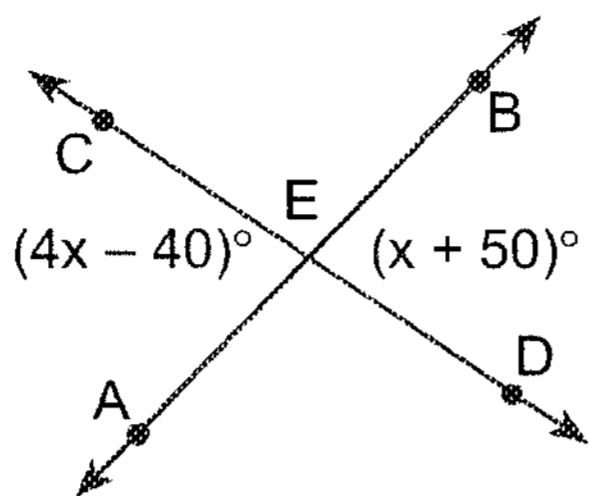
Advance Door
$$2X + X = 600$$
$$3X = 600$$
$$X = 200$$

400 tickets were sold in advance (400)(25)
200 tickets were sold at the door (200)(32)

$$(400)(25) + (200)(32) = Total$$
$$10,000 + 6,400 = \boxed{\$16,400} \ was \ collected$$

29 In the accompanying diagram, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at E. If $m\angle AEC = 4x - 40$ and $m\angle BED = x + 50$, find the number of degrees in $\angle AEC$.

C $(4x - 40)°$ E B $(x + 50)°$ A D

$$4X - 40 = X + 50$$
$$\underline{-X \qquad\qquad -X}$$
$$3X - 40 = \qquad 50$$
$$\underline{ +40 \qquad\qquad +40}$$
$$3X \qquad = \qquad 90$$
$$X \qquad = \qquad 30$$

$$\angle AEC = 4X - 40$$
$$\angle AEC = 4(30) - 40$$
$$\angle AEC = 120 - 40$$
$$\angle AEC = \boxed{80} \text{ degrees}$$

30 The students in Woodland High School's meteorology class measured the noon temperature every schoolday for a (week) Their readings for the first 4 days were Monday, 56°; Tuesday, 72°; Wednesday, 67°; and Thursday, 61°. If the mean (average) temperature for the (5 days) was exactly 63°, what was the temperature on Friday?

A week is 5 days in this problem.

$$\overline{X} = 63°$$
$$\overline{X} = \frac{X_1 + X_2 + X_3 + X_4 + X_n}{n}$$

$$63° = \frac{\overset{Mon}{56} + \overset{Tue}{72} + \overset{Wed}{67} + \overset{Thu}{61} + \overset{Fri}{X_5}}{5}$$

$$63 = \frac{256 + X_5}{5}$$

$$315 = 256 + X_5$$
$$\underline{-256 \qquad -256}$$

The temperature on Friday was $\boxed{59°}$

$$59 = X_5$$

[11]

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [20]

31 A square dartboard is represented in the accompanying diagram. The entire dartboard is the first quadrant from $x = 0$ to 6 and from $y = 0$ to 6. A triangular region on the dartboard is enclosed by the graphs of the equations $y = 2$, $x = 6$, and $y = x$. Find the probability that a dart that randomly hits the dartboard will land in the triangular region formed by the three lines.
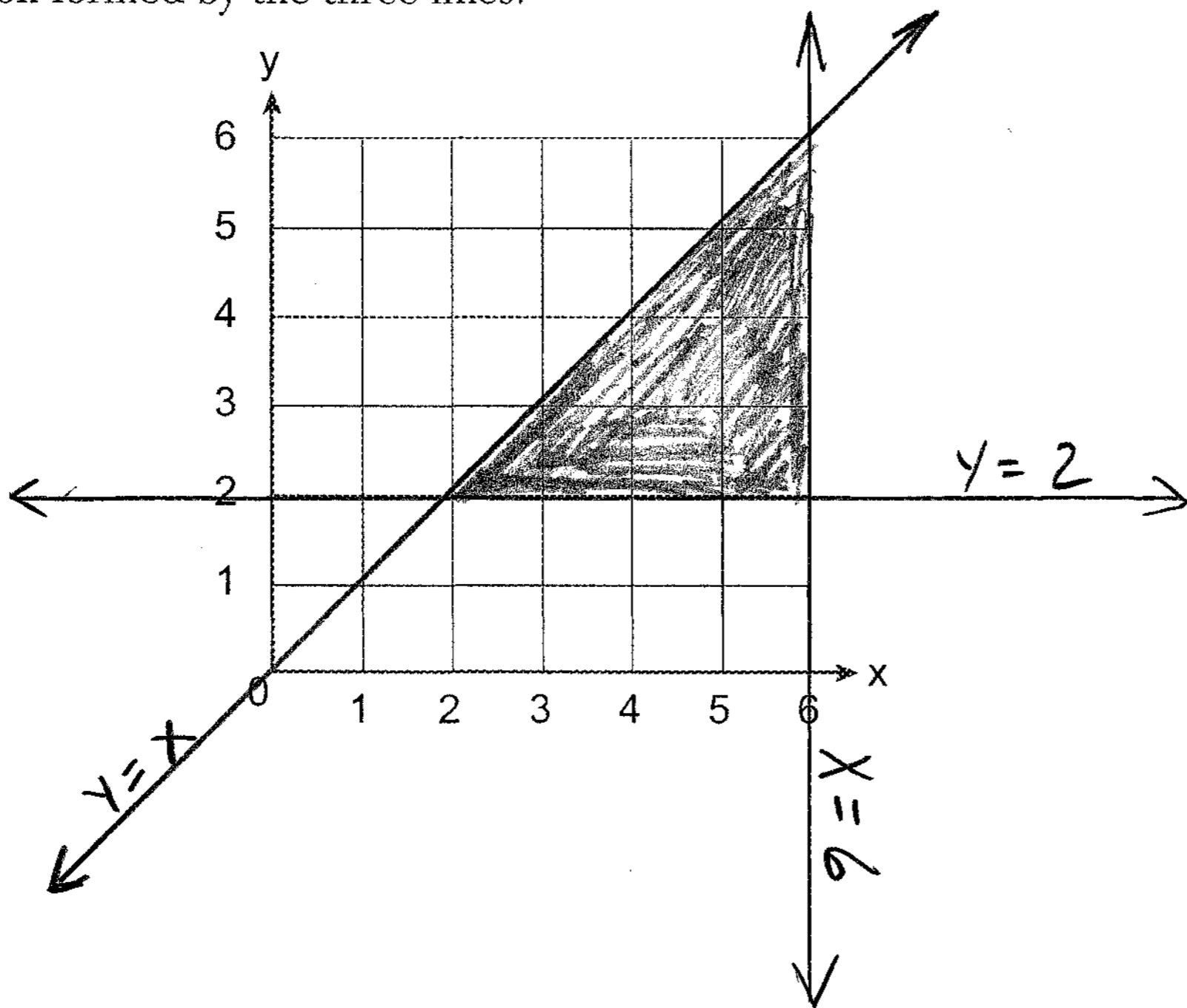
Area of Dartboard $= 6 \times 6 = 36$ units²

Area of Triangle $= \frac{1}{2}bh = \frac{1}{2}(4)(4) = \frac{1}{2}(16) = 8$ units²

$P(event) = \dfrac{desired\ outcome}{possible\ outcomes} = \boxed{\dfrac{8}{36}\ or\ \dfrac{2}{9}}$

32 When Tony received his weekly allowance, he decided to purchase candy bars for all his friends. Tony bought three Milk Chocolate bars and four Creamy Nougat bars, which cost a total of $4.25 without tax. Then he realized this candy would not be enough for all his friends, so he returned to the store and bought an additional six Milk Chocolate bars and four Creamy Nougat bars, which cost a total of $6.50 without tax. How much did *each* type of candy bar cost?

Answer

Creamy Nougats cost 50¢
Milk Chocolate costs 75¢

Check
6m + 4c = 6.50
4.50 + 2 = 6.50 ✓

Let M = # Milk Chocolate bars
Let C = # Creamy Nougat bars

1rst Trip to Store
3m + 4C = 4.25

2nd Trip to Store
6m + 4C = 6.50

subtract
$$6m + 4C = 6.50$$
$$3m + 4C = 4.25$$
$$3m \quad = 2.25$$
$$m \quad = .75$$

Solve for C
$$3m + 4c = 4.25$$
$$3(.75) + 4c = 4.25$$
$$2.25 + 4c = 4.25$$
$$-2.25 \qquad -2.25$$
$$4c = 2.00$$

C = .50

33 Javon's homework is to determine the dimensions of his rectangular backyard. He knows that the length is 10 feet more than the width, and the total area is 144 square feet. Write an equation that Javon could use to solve this problem. Then find the dimensions, in feet, of his backyard.

144 ft²   x
X + 10

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$X = \frac{-10 \pm \sqrt{10^2 - 4(1)(-144)}}{2(1)}$$

$$X = \frac{-10 \pm \sqrt{100 + 576}}{2}$$
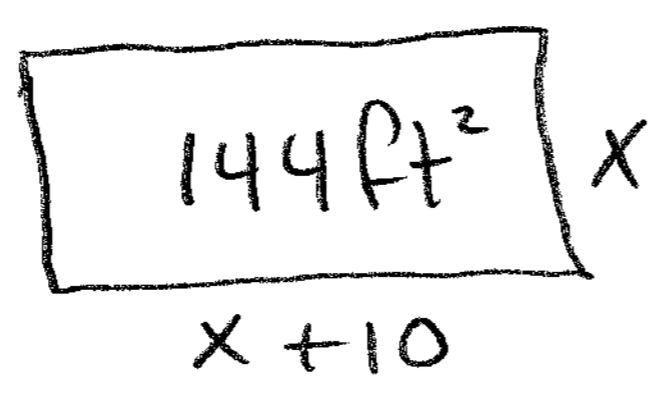
$$X = \frac{-10 \pm \sqrt{676}}{2}$$

$A = lw$

$A = (x+10)x$

$A = x^2 + 10x$

$144 = x^2 + 10x$

$0 = x^2 + 10x - 144$

$a = 1 \quad b = 10 \quad C = -144$

The dimensions are 8ft and 18ft. Answer

$$X = \frac{-10 \pm 26}{2}$$

$$X = \frac{16}{2} = 8 \qquad X = \frac{-36}{2} = -18$$
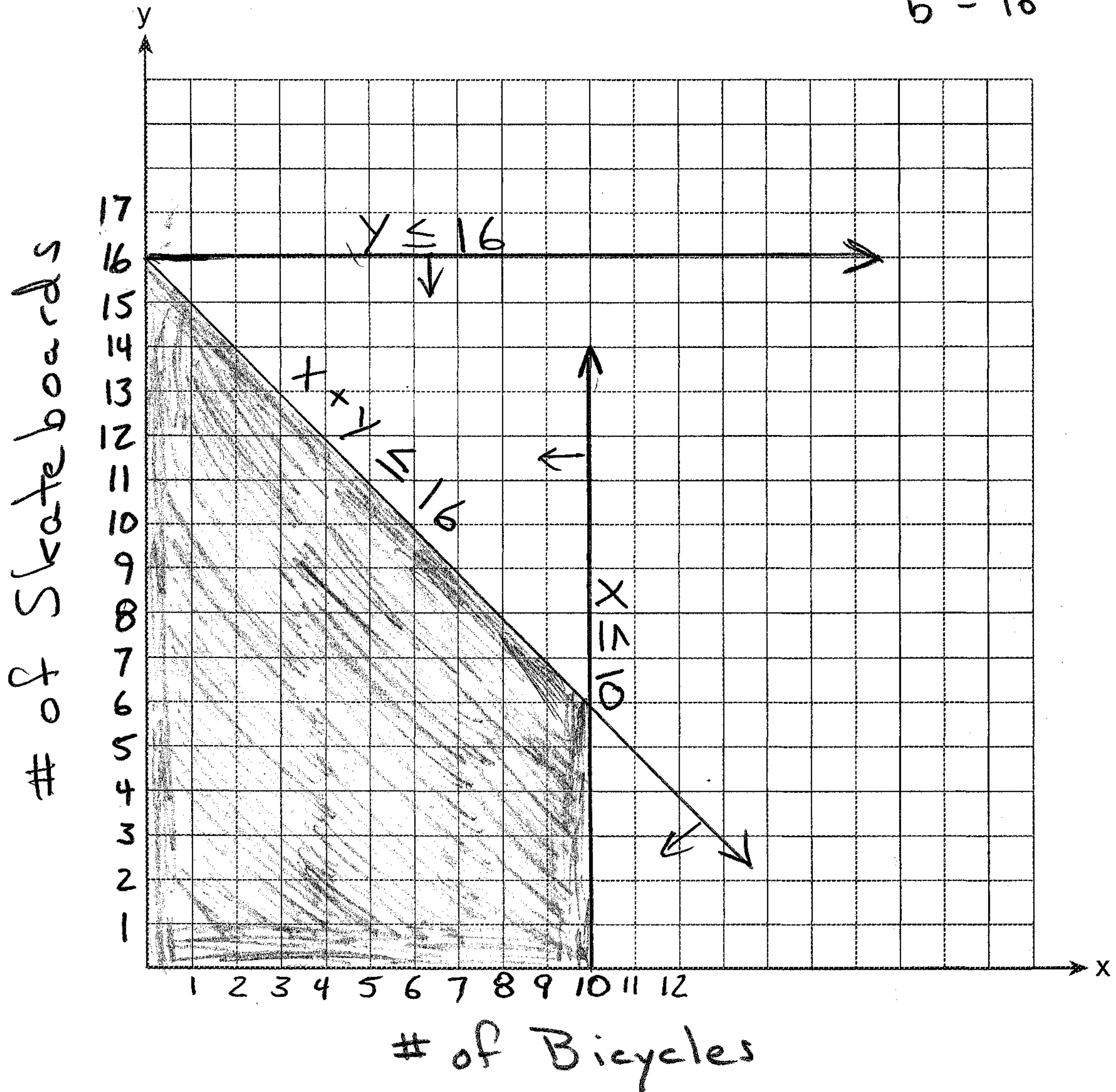
This is not a possible length

Check
8 × 18 = 144 ✓

34 A company manufactures bicycles and skateboards. The company's daily production of bicycles cannot exceed 10, and its daily production of skateboards must be less than or equal to 12. The combined number of bicycles and skateboards cannot be more than 16. If x is the number of bicycles and y is the number of skateboards, graph on the accompanying set of axes the region that contains the number of bicycles and skateboards the company can manufacture daily.
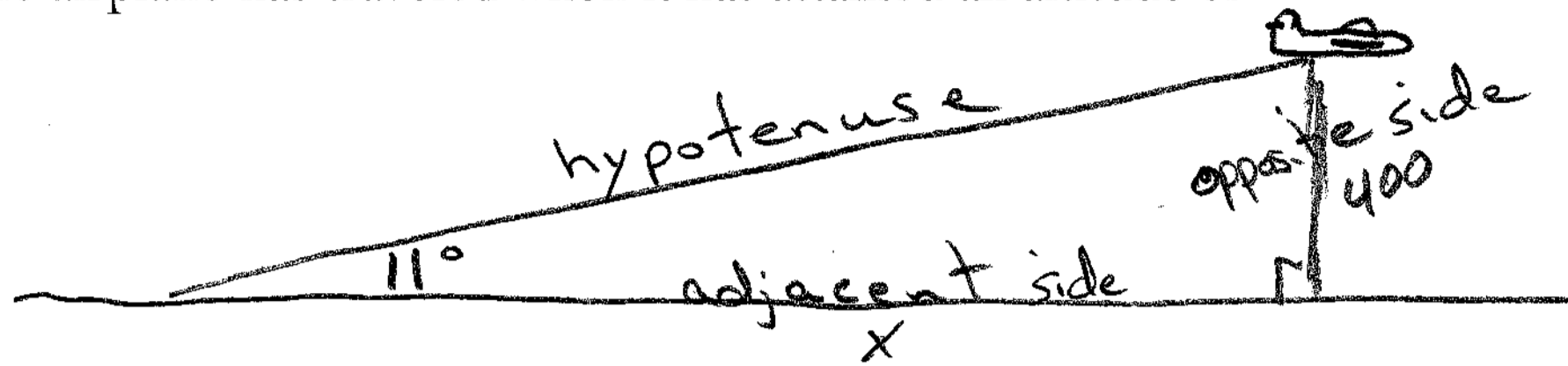
$$x + y \leq 16$$
$$y \leq -x + 16$$
$$m = -1$$
$$b = 16$$

y

# of Skateboards

17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

$Y \leq 16$

$x + y \leq 16$

$x \leq 10$

1 2 3 4 5 6 7 8 9 10 11 12   x

# of Bicycles

35 Draw and label a diagram of the path of an airplane climbing at an angle of 11° with the ground. Find, to the *nearest foot* the ground distance the airplane has traveled when it has attained an altitude of 400 feet.

hypotenuse

opposite side 400

11° adjacent side x

(a) SOH - CAH - TOA

$\sin = \frac{opp}{hyp}$

$\cos = \frac{adj}{hyp}$

$\ast \quad \tan = \frac{opp}{adj}$

We are working with an angle, its opposite side, and its adjacent side, so we use tangent

(b) $\tan = \frac{opp}{adj}$

$\tan 11° = \frac{400}{x}$

Set calculator to degree mode

$\tan 11° = .1943803091$

$.1943803091 = \frac{400}{x}$

$.1943803091 (x) = 400$

$x = 2057.821606$

Answer **2058 feet**

MATHEMATICS A

Tuesday, January 22, 2002 — 1:15 to 4:15 p.m., only

ANSWER SHEET

Student .... Sex: ☐ Male ☐ Female Grade ....

Teacher ....._Steve Watson_............ School ..._IHS @ PH_.......

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer all 20 questions in this part.

1 2	6 3	11 4	16 4
2 4	7 4	12 2	17 1
3 3	8 1	13 3	18 1
4 2	9 2	14 1	19 3
5 1	10 2	15 3	20 1

Your answers for Parts II, III, and IV should be written in the test booklet.

The declaration below should be signed when you have completed the examination.

Tear Here