The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS A

Tuesday, January 23, 2001 — 1:15 to 4:15 p.m., only

Print Your Name: Steve Watson

Print Your School's Name: IHS @ PH

Print your name and the name of your school in the boxes above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. Any work done on this sheet of scrap graph paper will *not* be scored. All work should be written in pen, except graphs and drawings, which should be done in pencil.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers for the questions in Parts II, III, and IV directly in this booklet. Clearly indicate the necessary steps you take, including appropriate formula substitutions, diagrams, graphs, charts, etc.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

Notice...

A minimum of a scientific calculator, a straightedge (ruler), and a compass must be available for your use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Record your answers in the spaces provided on the separate answer sheet. [40]

Use this space for computations.

1 There are 461 students and 20 teachers taking buses on a trip to a museum. Each bus can seat a maximum of 52. What is the *least* number of buses needed for the trip?

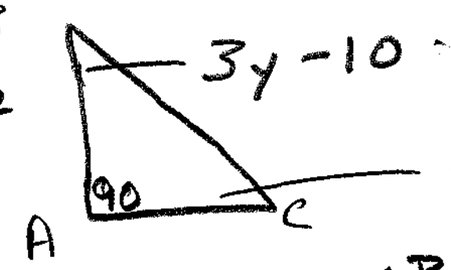
(1) 8
(2) 9
(3) 10
(4) 11

461 + 20 = 481 total

$\frac{481}{52} = 9.25$ ← Must round up to get a ride for everybody.

2 In right triangle ABC, $m\angle C = 3y - 10$, $m\angle B = y + 40$, and $m\angle A = 90$. What type of right triangle is triangle ABC?

(1) scalene
(2) isosceles
(3) equilateral not possible
(4) obtuse not possible



4y + 30 = 90
4y = 60
y = 15
∠B = 35°
∠C = 55°

3 If $x > 0$, the expression $(\sqrt{x})(\sqrt{2x})$ is equivalent to

(1) $\sqrt{2x}$
(2) $2x$
(3) $x^2\sqrt{2}$
(4) $x\sqrt{2}$

$\sqrt{x}\sqrt{x}\sqrt{2}$
$x\sqrt{2}$

4 Three times as many robins as cardinals visited a bird feeder. If a total of 20 robins and cardinals visited the feeder, how many were robins?

(1) 5
(2) 10
(3) 15
(4) 20

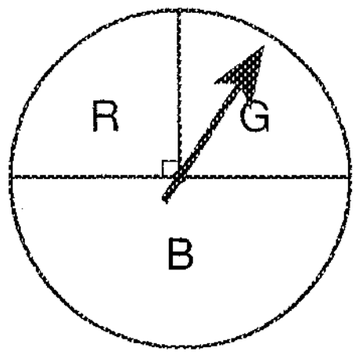
3C = R
C + R = 20
C + 3C = 20
4C = 20
C = 5
R = 15

5 One of the factors of $4x^2 - 9$ is

(1) $(x + 3)$
(2) $(2x + 3)$
(3) $(4x - 3)$
(4) $(x - 3)$

$4x^2 - 9 = (2x+3)(2x-3)$

6 At a school fair, the spinner represented in the accompanying diagram is spun twice.



What is the probability that it will land in section G the first time and then in section B the second time?

(1) $\frac{1}{2}$
(2) $\frac{1}{4}$
(3) $\frac{1}{8}$
(4) $\frac{1}{16}$

$P(G+B) = P(G) \cdot P(B)$
$P_{(G+B)} = \frac{1}{4} \cdot \frac{1}{2}$
$= \frac{1}{8}$

7 If a and b are integers, which equation is always true?

(1) $\frac{a}{b} = \frac{b}{a}$
(3) $a - b = b - a$
(2) $a + 2b = b + 2a$
(4) $a + b = b + a$

Let $a = 1$
Let $b = 2$
(1) $\frac{1}{2} \neq \frac{2}{1}$

Use this space for computations.

(2) $1 + 2(2) \neq 2 + 2(1)$
(3) $1 - 2 \neq 2 - 1$
(4) $+2 = 2 + 1$ ✓

8 The sum of $3x^2 + 4x - 2$ and $x^2 - 5x + 3$ is

(1) $4x^2 + x - 1$
(2) $4x^2 - x + 1$
(3) $4x^2 + x + 1$
(4) $4x^2 - x - 1$

$3x^2 + 4x - 2$
$x^2 - 5x + 3$
$4x^2 - x + 1$

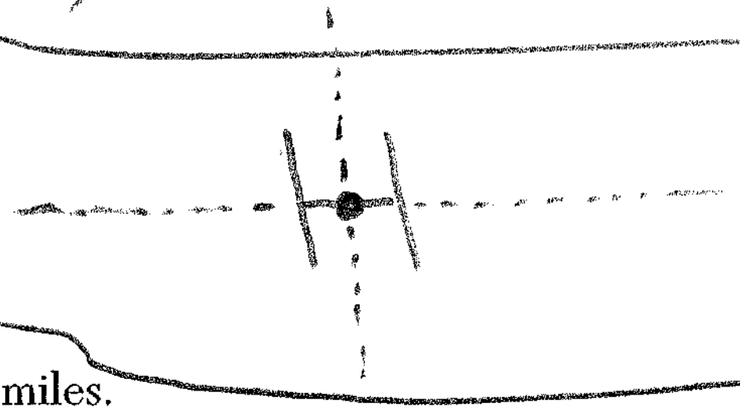
9 If $x \neq 0$, the expression $\frac{x^2 + 2x}{x}$ is equivalent to

(1) $x + 2$
(2) 2
(3) $3x$
(4) 4

$\frac{x^2 + 2x}{x} = \frac{x(x+2)}{x} = \frac{x+2}{1} = x + 2$

10 Helen is using a capital H in an art design. The H has

(1) only one line of symmetry
(2) only two points of symmetry
(3) two lines of symmetry and only one point of symmetry
(4) two lines of symmetry and two points of symmetry



11 The distance from Earth to the Sun is approximately 93 million miles. A scientist would write that number as

(1) 9.3×10^6
(2) 9.3×10^7
(3) 93×10^7
(4) 93×10^{10}

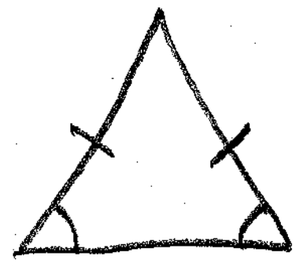
9.3 000 000.
7 6 5 4 3 2 1
9.3×10^7

12 Given the statement: "If two sides of a triangle are congruent, then the angles opposite these sides are congruent."

Given the converse of the statement: "If two angles of a triangle are congruent, then the sides opposite these angles are congruent."

What is true about this statement and its converse?

(1) Both the statement and its converse are true.
(2) Neither the statement nor its converse is true.
(3) The statement is true but its converse is false.
(4) The statement is false but its converse is true.



13 Which equation could represent the relationship between the x and y values shown in the accompanying table?

x	y
0	2
1	3
2	6
3	11
4	18

No
$y = x + 2$
$2 = 0 + 2$
$3 = 1 + 2$
$6 \neq 2 + 2$

Yes
$y = x^2 + 2$
$2 = (0)^2 + 2$
$3 = (1)^2 + 2$
$6 = (2)^2 + 2$
$11 = (3)^2 + 2$
$18 = (4)^2 + 2$

No
$y = x^2$
$2 \neq 0^2$

No
$y = 2^x$
$2 \neq 2^0$

(1) $y = x + 2$
(2) $y = x^2 + 2$
(3) $y = x^2$
(4) $y = 2^x$

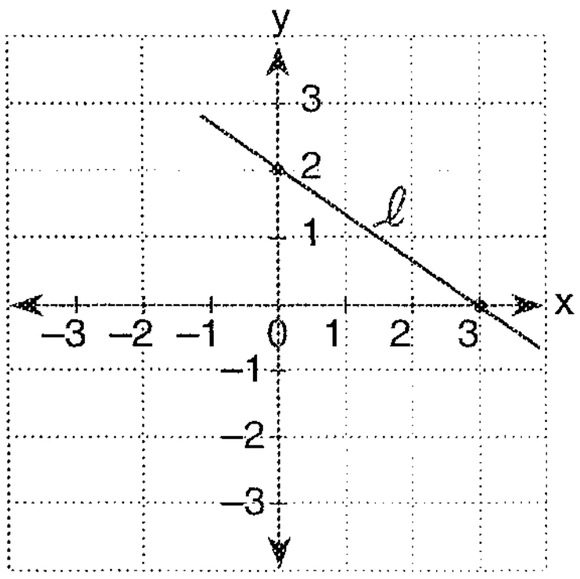
14 A locker combination system uses three digits from 0 to 9. How many different three-digit combinations with no digit repeated are possible?

(1) 30
(2) 504
(3) 720
(4) 1,000

Use this space for computations.

1rst 2nd 3rd
10 × 9 × 8 = 720

15 What is the slope of line ℓ in the accompanying diagram?



(1) $-\frac{3}{2}$
(2) $-\frac{2}{3}$
(3) $\frac{2}{3}$
(4) $\frac{3}{2}$

slope = m = $\frac{\text{rise}}{\text{run}}$
Read 2 points from left to right.
Goes: down 2 | over 3
rise = -2 | runs +3
Slope is negative
$\frac{\text{rise}}{\text{run}} = \frac{-2}{3}$

16 If $bx - 2 = K$, then x equals

(1) $\frac{K}{b} + 2$
(2) $\frac{K-2}{b}$
(3) $\frac{2-K}{b}$
(4) $\frac{K+2}{b}$

$bx - 2 = k$
$+2 \quad +2$
$bx = k+2$
$x = \frac{k+2}{b}$

17 In a molecule of water, there are two atoms of hydrogen and one atom of oxygen. How many atoms of hydrogen are in 28 molecules of water?

(1) 14
(2) 29
(3) 42
(4) 56

$\frac{\text{Hydrogen}}{\text{Oxygen}} = \frac{2}{1} = \frac{H}{28}$ (One Molecule, 28 Molecules)
$2(28) = 1H$
$56 = H$

18 From January 3 to January 7, Buffalo recorded the following daily high temperatures: 5°, 7°, 6°, 5°, and 7°. Which statement about the temperatures is true?

(1) mean = median
(2) mean = mode
(3) median = mode
(4) mean < median

Jan 3	5
Jan 4	7
Jan 5	6
Jan 6	5
Jan 7	7

mean = $\bar{x} = \frac{5+7+6+5+7}{5} = \frac{30}{5} = 6$
median = 5, 5, 6, 7, 7
modes = 5 and 7

19 In which of the accompanying figures are segments XY and YZ perpendicular?

3/4/5 Right Δ

$a^2+b^2=c^2$
$6^2+8^2=10^2$
$36+64=100$
$100=100$
∴ Right Δ

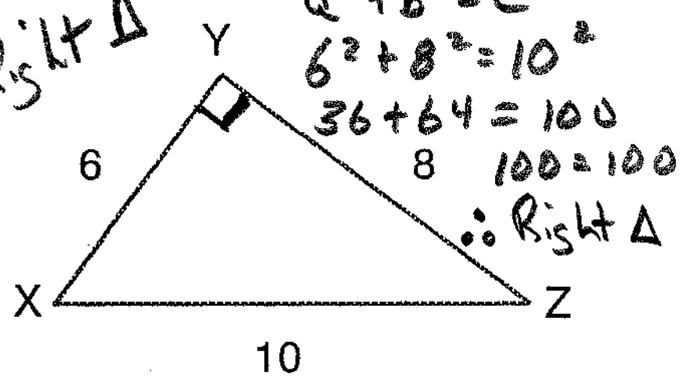


Figure 1

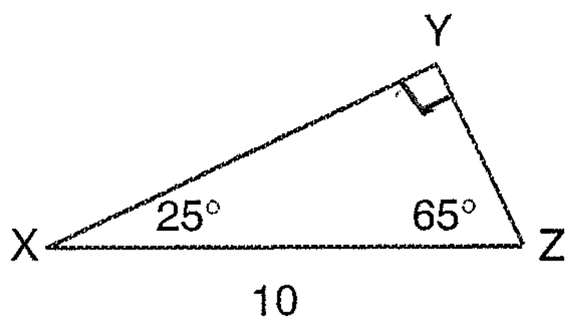


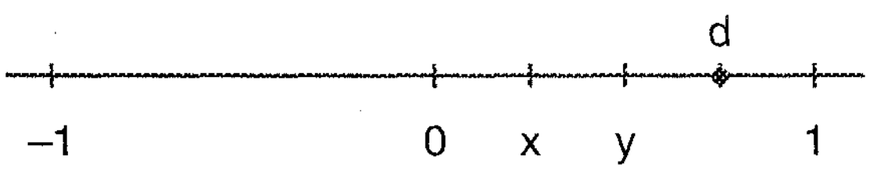
Figure 2

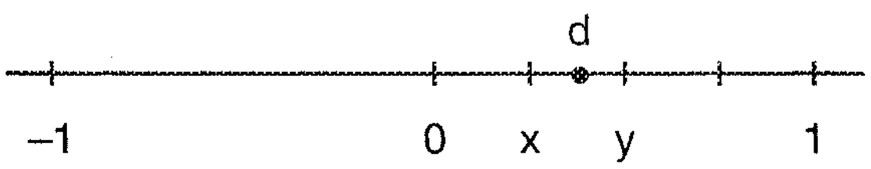
(1) figure 1, only
(2) figure 2, only
(3) both figure 1 and figure 2
(4) neither figure 1 nor figure 2

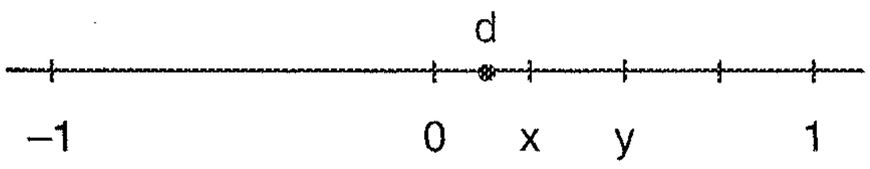
Use this space for computations.

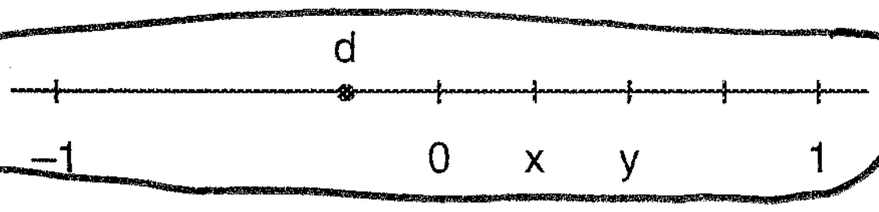
$\angle Y = 180° - (X+Z)$
$\angle Y = 180° - (25°+65°)$
$\angle Y = 180° - (90°)$
$\angle Y = 90°$

20 Let x and y be numbers such that $0 < x < y < 1$, and let $d = x - y$. Which graph could represent the location of d on the number line?

(1) 

(2) 

(3) 

(4) 

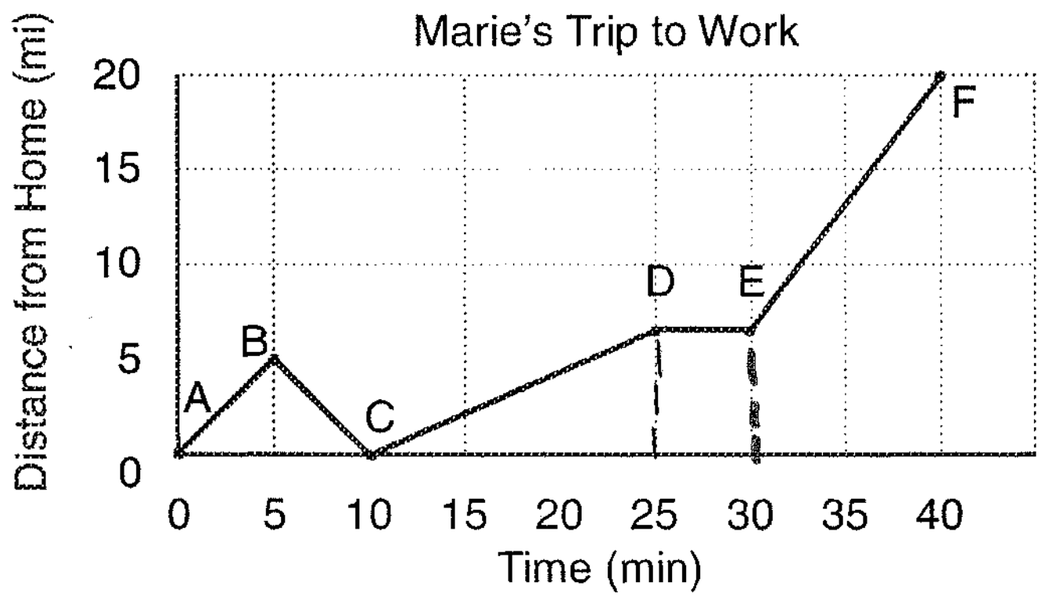
Let $x = \frac{1}{4}$
Let $y = \frac{1}{2}$

$0 < x < y < 1$
$0 < \frac{1}{4} < \frac{1}{2} < 1$
$d = x - y$
$d = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$
∴ d must be negative

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [10]

21 The accompanying graph shows Marie's distance from home (A) to work (F) at various times during her drive.



Wow! Between E and F, Mary drove 12 miles in 10 minutes. This is 1.2 miles per minute or 72 miles per hour. Mary was speeding any where in N.Y. Shame! Shame! Shame!

a Marie left her briefcase at home and had to return to get it. State which point represents when she turned back around to go home and explain how you arrived at that conclusion.

Point B is where Mary turned around. She was at home at points A and C. Point B was the furthest away from home in the area of the graph between points A and C.

b Marie also had to wait at the railroad tracks for a train to pass. How long did she wait?

Between points D and E, Mary's distance from home did not change. This is where she was stopped and waiting. She waited for 5 minutes

Price + Tax = 22

22 Sue bought a picnic table on sale for 50% off the original price. The store charged her 10% tax and her final cost was $22.00. What was the original price of the picnic table?

$$\left(\text{Original Price} \times \frac{1}{2}\right) + \frac{1}{10}\left(\text{Original Price} \times \frac{1}{2}\right) = 22$$

50% 10%

$$\frac{1}{2}x + \frac{1}{10}\left(\frac{1}{2}x\right) = 22$$

$$\frac{1}{2}x + \frac{1}{20}x = 22$$

$$\frac{1x}{2} + \frac{1x}{20} = 22$$

$$\frac{20x + 2x}{40} = 22$$

$$\frac{22x}{40} = 22$$

$$22x = 880$$

$x = \$40$ Answer

Check

$20.00 Price

$2.00 Tax

$22.00 Total ✓

23 A cardboard box has length $x - 2$, width $x + 1$, and height $2x$.

a Write an expression, in terms of x, to represent the volume of the box.

$V = \ell w h$

$V = (x-2)(x+1)\,2x$

$V = (x^2 + x - 2x - 2)\,2x$

$V = (x^2 - x - 2)\,2x$

$V = 2x^3 - 2x^2 - 4x$

b If $x = 8$ centimeters, what is the number of cubic centimeters in the volume of the box?

$V = 2x^3 - 2x^2 - 4x$

$x = 8$

$V = 2(8)^3 - 2(8)^2 - 4(8)$

$V = 2(512) - 2(64) - 32$

$V = 1024 - 128 - 32$

$V = 864\ cm^3$

Check

$\ell = x - 2 = 8 - 2 = 6$

$w = x + 1 = 8 + 1 = 9$

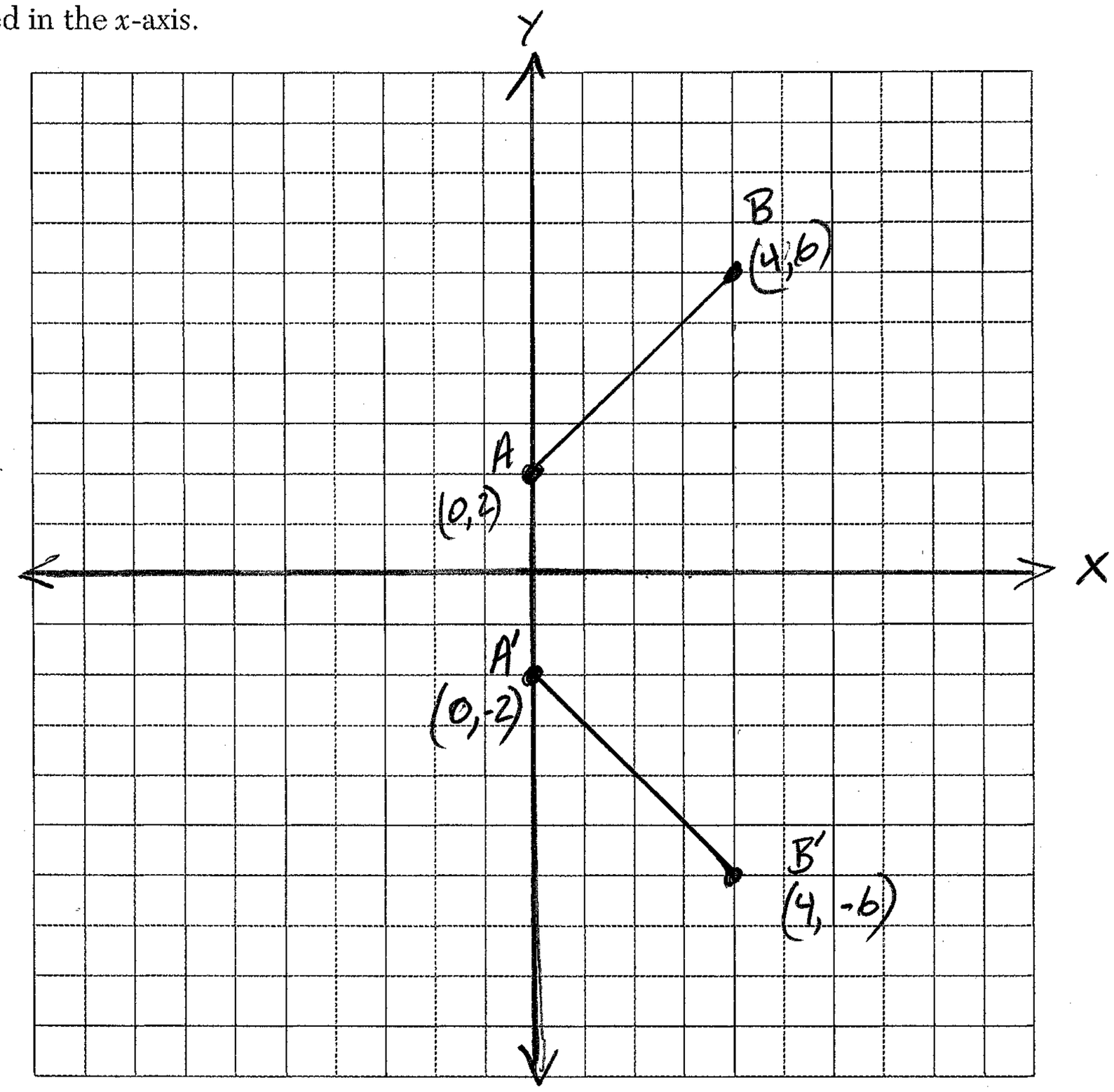
$h = 2x = 2(8) = 16$

$V = \ell w h$

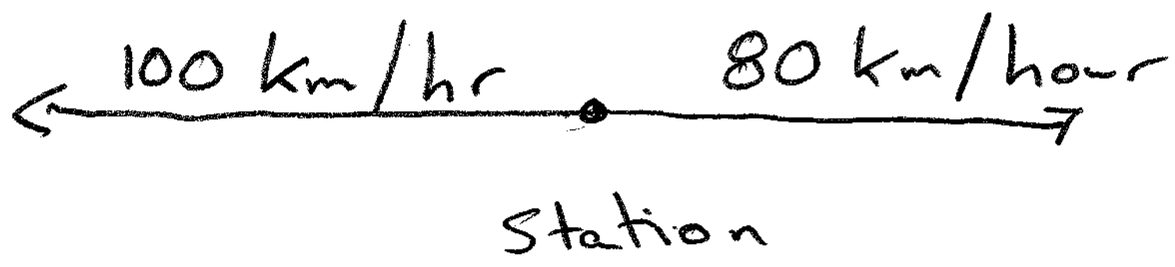
$V = 6 \cdot 9 \cdot 16$

$V = 864$ ✓

24 The coordinates of the endpoints of $\overline{AB}$ are $A(0,2)$ and $B(4,6)$. Graph and state the coordinates of A' and B', the images of A and B after $\overline{AB}$ is reflected in the x-axis.



25 Two trains leave the same station at the same time and travel in opposite directions. One train travels at 80 kilometers per hour and the other at 100 kilometers per hour. In how many hours will they be 900 kilometers apart?



At the end of 1 hour, the trains are 180 km. apart

At the end of 2 hours, the trains are 360 km apart

3 hours = 3(180) = 540

4 hours = 4(180) = 720

Answer | 5 hours | = 5(180) = 900

Check $\frac{900}{180} = 5$

Part III

Answer all questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [15]

26 Sal has a small bag of candy containing three green candies and two red candies. While waiting for the bus, he ate two candies out of the bag, one after another, without looking. What is the probability that both candies were the same color?

$P_{(A+B)} = P_{(A)} \cdot P_{(B)}$

Both Green

$P_{(A)} = P_{(\text{green 1st pick})} = \frac{3 \text{ greens}}{5 \text{ total}} = \frac{3}{5}$

$P_{(B)} = P_{(\text{green 2nd pick})} = \frac{2 \text{ greens}}{4 \text{ total}} = \frac{1}{2}$

$P_{(A+B)} = \left(\frac{3}{5}\right)\left(\frac{1}{2}\right) = \frac{3}{10}$ ← This is the probability that both are green.

Both Red

$P_{(A)} = P_{(\text{red 1st pick})} = \frac{2 \text{ reds}}{5 \text{ total}} = \frac{2}{5}$

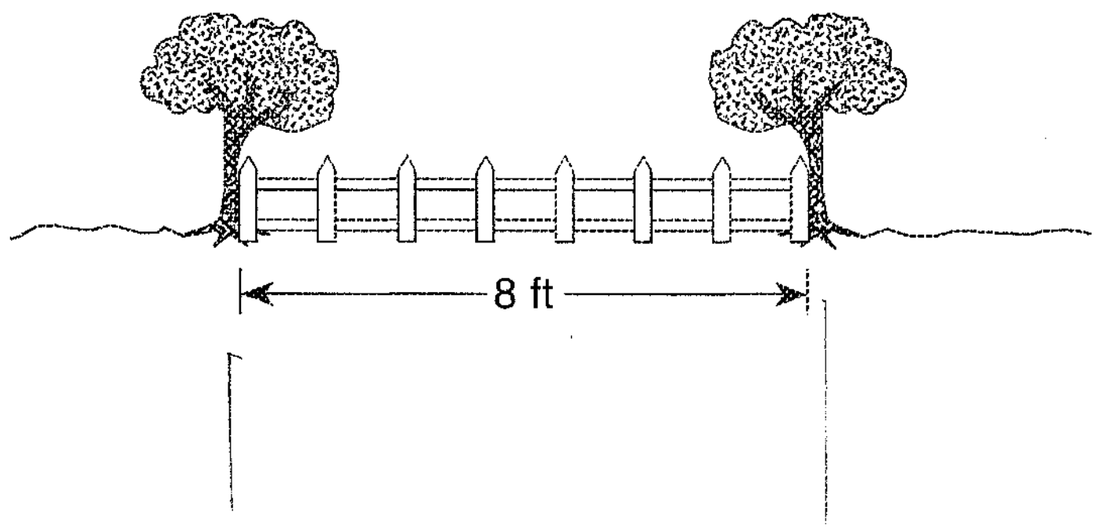
$P_{(B)} = P_{(\text{red 2nd pick})} = \frac{1 \text{ red}}{4 \text{ total}} = \frac{1}{4}$

$P_{(A+B)} = \left(\frac{2}{5}\right)\left(\frac{1}{4}\right) = \frac{2}{20} = \frac{1}{10}$ This is the probability that both are red

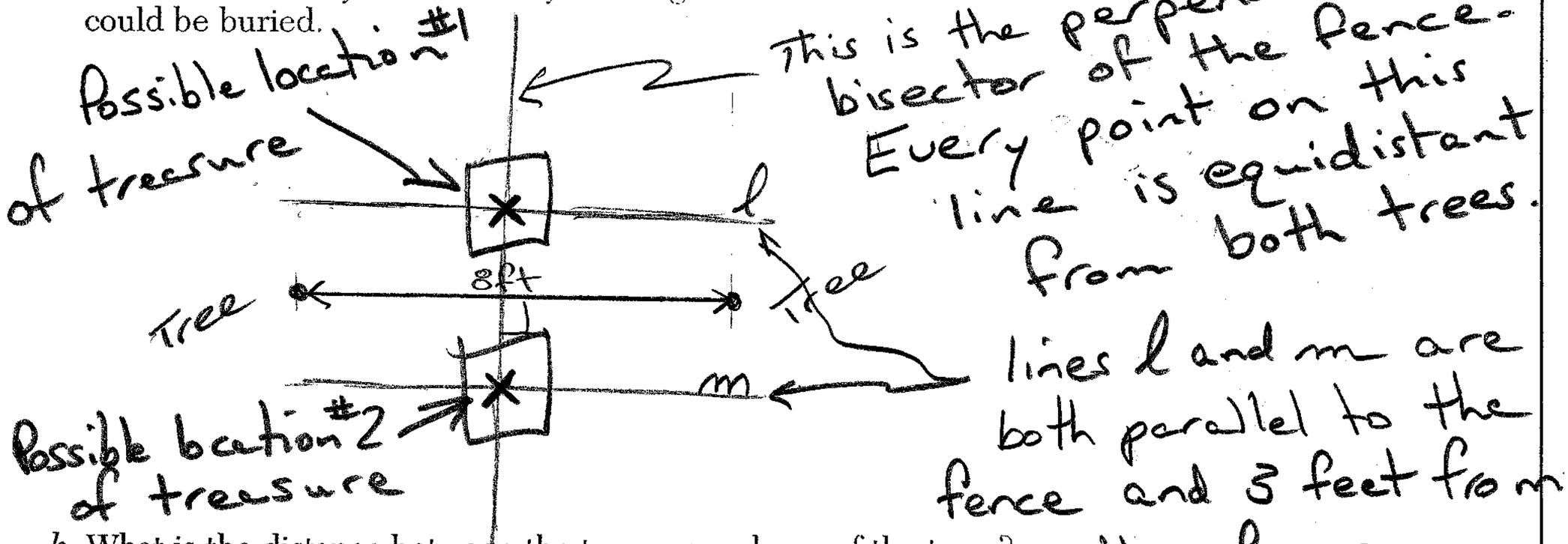
You have 2 chances of getting both the same color.

Add $\frac{3}{10} + \frac{1}{10}$. Total probability = $\boxed{\frac{4}{10} \text{ or } \frac{2}{5}}$

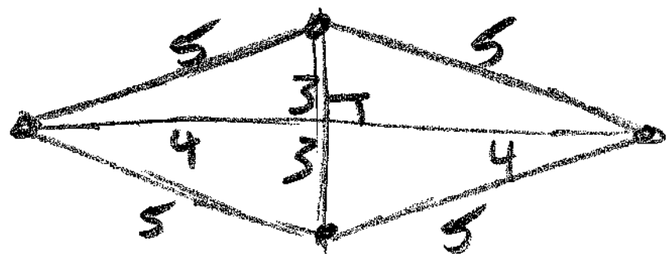
27 Steve has a treasure map, represented in the accompanying diagram, that shows two trees 8 feet apart and a straight fence connecting them. The map states that treasure is buried 3 feet from the fence and equidistant from the two trees.



a Sketch a diagram to show all the places where the treasure could be buried. Clearly indicate in your diagram where the treasure could be buried.



b What is the distance between the treasure and one of the trees?



$a^2 + b^2 = c^2$

$3^2 + 4^2 = c^2$

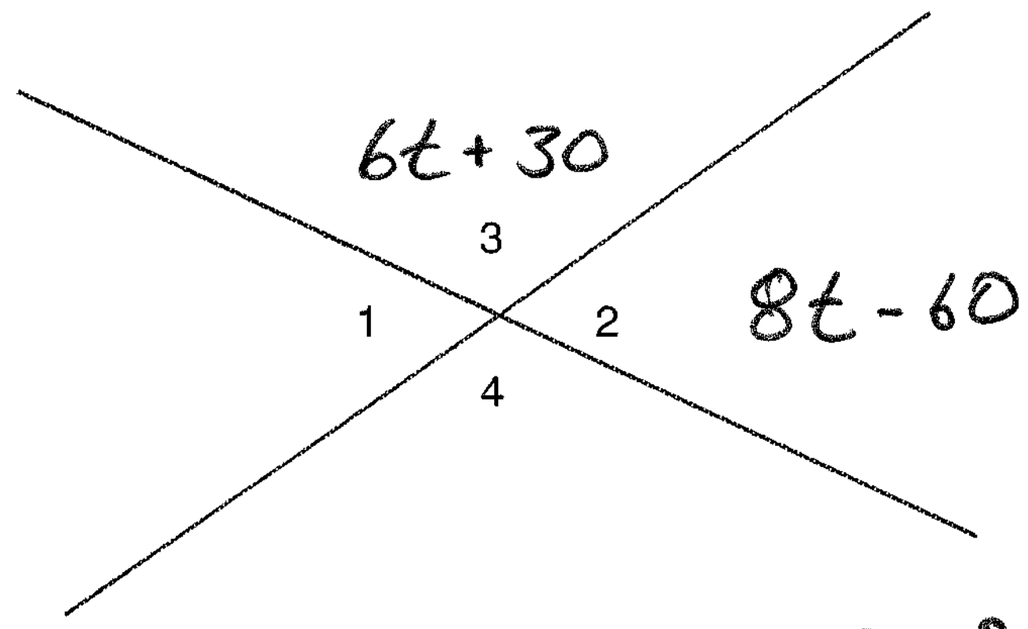
$9 + 16 = c^2$

$25 = c^2$

$5 = c$

The distance is 5 feet

28 In the accompanying figure, two lines intersect, $m\angle 3 = 6t + 30$, and $m\angle 2 = 8t - 60$. Find the number of degrees in $m\angle 4$.



$\angle 3 + \angle 2 = 180°$

$6t + 30 + 8t - 60 = 180$

$14t - 30 = 180$

$14t = 210$

$t = 15$

$m\angle 4 = m\angle 3$

$m\angle 3 = 6t + 30$

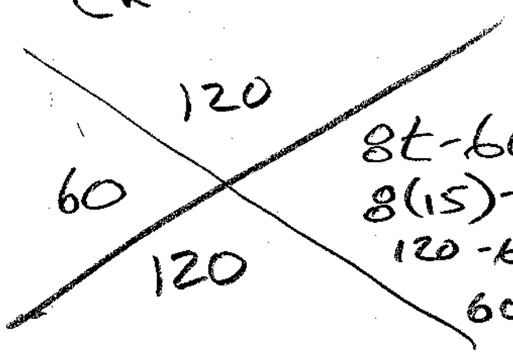
$m\angle 3 = 6(15) + 30$

$m\angle 3 = 90 + 30$

$m\angle 3 = 120$

$\therefore m\angle 4 = \boxed{120°}$

Check



$120 + 60 = 180$ ✓

$120 + 120 + 60 + 60 = 360$ ✓

29 Mark says, "The number I see is odd." Jan says, "That same number is prime." The teacher says, "Mark is correct or Jan is correct." Some integers would make the teacher's statement true while other integers would make it false. Give and explain *one* example of when the teacher's statement is true. Give and explain *one* example of when the teacher's statement is false.

Example of teacher's statement being true

9 → Mark's statement is true. 9 is odd

→ Jan's statement is false. 9 is not prime

Example of teacher's statement being false

4 → Mark's statement is false. 4 is not odd.

→ Jan's statement is false. 4 is not prime.

30 Juan has a cellular phone that costs \$12.95 per month plus 25¢ per minute for each call. Tiffany has a cellular phone that costs \$14.95 per month plus 15¢ per minute for each call. For what number of minutes do the two plans cost the same?

Monthly Costs ← total monthly cost

Juan $C = 12.95 + .25m$ ← minutes

Tiffany $C = 14.95 + .15m$

$12.95 + .25m = 14.95 + .15m$

$-.15m \qquad -.15m$

$12.95 + .10m = 14.95$

$-12.95 \qquad -12.95$

$.10m = 2.00$

$m = \frac{2.00}{.1}$

$m = \boxed{20}$

Check

Juan's Costs for 20 minutes: $12.95 + .25(20)$, $12.95 + 5$, \$17.95 total

Tiffany's Costs for 20 minutes: $14.95 + .15(20)$, $14.95 + 3.00$, \$17.95 total ✓

Part IV

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [20]

31 Solve algebraically for x: $\frac{1}{x} = \frac{x+1}{6}$

$$\frac{1}{X} = \frac{X+1}{6}$$

$$1(6) = X(X+1)$$

$$6 = X^2 + X$$

$$0 = X^2 + X - 6$$

$$0 = (X+3)(X-2)$$

$X+3=0$ $\qquad$ $X-2=0$

$X = -3$ and $X = 2$

Check

$X = -3$

$$\frac{1}{-3} = \frac{-3+1}{6}$$

$$-\frac{1}{3} = \frac{-2}{6}$$

$$-\frac{1}{3} = -\frac{1}{3}$$

✓

$X = 2$

$$\frac{1}{2} = \frac{2+1}{6}$$

$$\frac{1}{2} = \frac{3}{6}$$

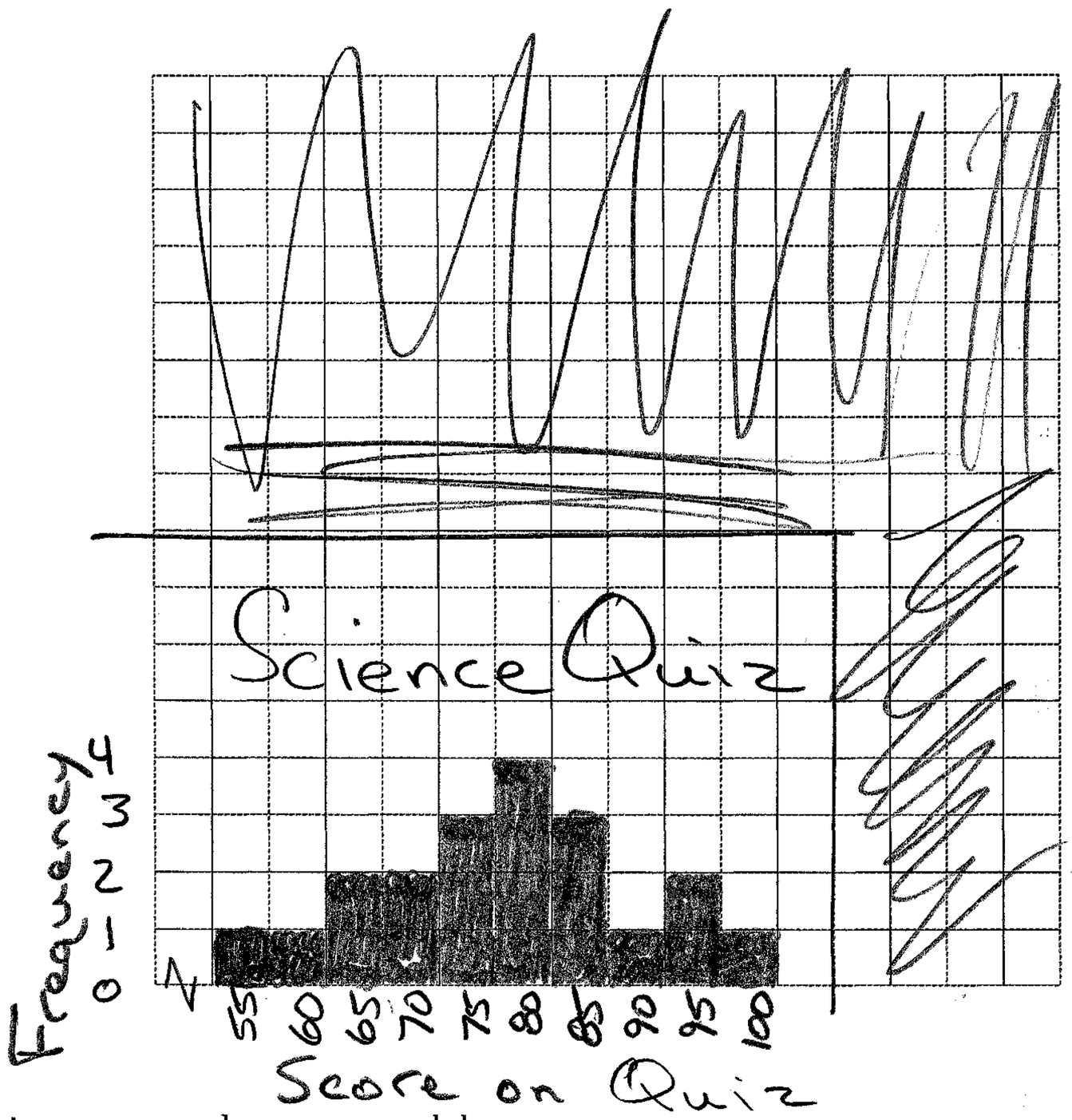
$$\frac{1}{2} = \frac{1}{2}$$

✓

32 On a science quiz, 20 students received the following scores: 100, 95, 95, 90, 85, 85, 85, 80, 80, 80, 80, 75, 75, 75, 70, 70, 65, 65, 60, 55.

Construct a statistical graph, such as a histogram or a stem-and-leaf plot, to display this data. [*Be sure to title the graph and label all axes or parts used.*]

If your type of plot requires a grid, show your work here.

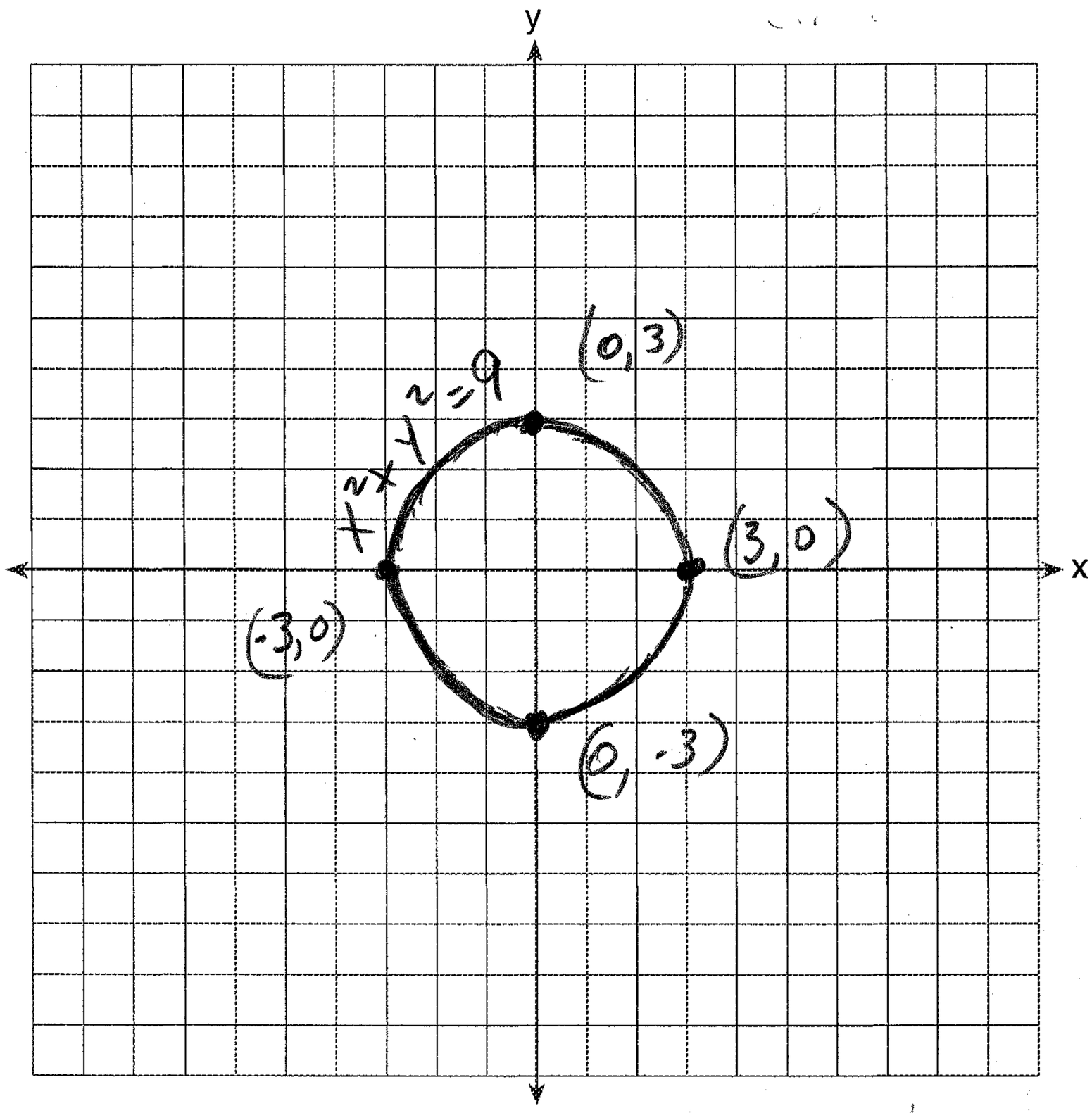


If no grid is necessary, show your work here.

33 John uses the equation $x^2 + y^2 = 9$ to represent the shape of a garden on graph paper.

$x^2+y^2=r^2$ is a circle.

a Graph $x^2 + y^2 = 9$ on the accompanying grid.



x	$x^2+y^2=9$	y
0	$y^2=9$	± 3
± 3	$x^2=9$	0

(0,3)
(0,-3)
(3,0)
(-3,0)

b What is the area of the garden to the *nearest square unit*?

$A = \pi r^2$

$A = \pi (3)^2$

$A = 9\pi$ units^2

34 There were 100 more balcony tickets than main-floor tickets sold for a concert. The balcony tickets sold for \$4 and the main-floor tickets sold for \$12. The total amount of sales for both types of tickets was \$3,056.

a Write an equation or a system of equations that describes the given situation. Define the variables.

Let m equal the # of main floor tickets sold
Let b equal the # of balcony tickets sold.

$b = m + 100$ ⟸ # sold

$4b + 12m = 3056$ ⟸ income

b Find the number of balcony tickets that were sold.

$b = m + 100$

$4(b) + 12m = 3056$

$4(m+100) + 12m = 3056$

$4m + 400 + 12m = 3056$

$16m + 400 = 3056$

$-400 \quad -400$

$16m = 2656$

$m = 166$

$b = m + 100$

$b = 166 + 100$

$b = 266$ ←

There were 266 balcony tickets sold.

Check

$4b + 12m = 3056$

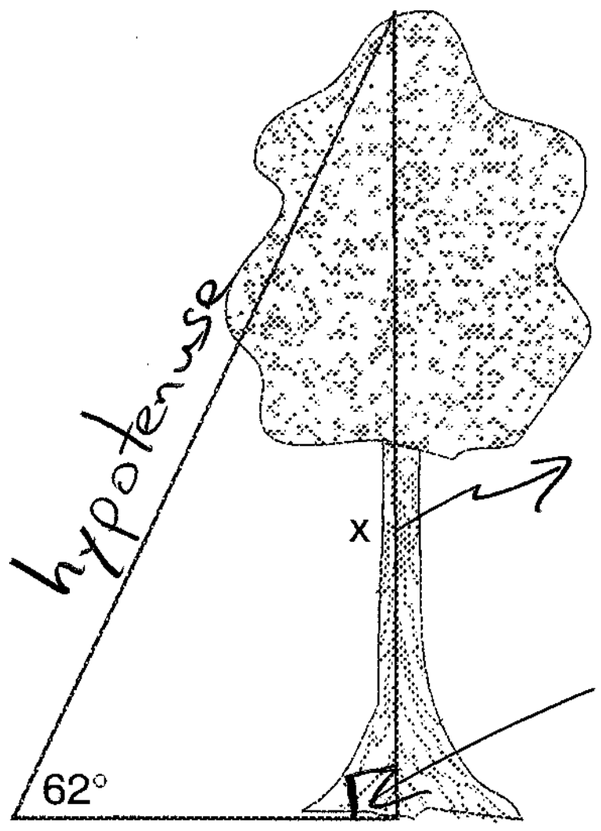
$4(266) + 12(166) = 3056$

$1064 + 1992 = 3056$

$3056 = 3056$ ✓

round at end

35 Find, to the *nearest tenth of a foot* the height of the tree represented in the accompanying diagram.



hypotenuse

opposite side

right angle

adjacent side

(a)

SOH-CAH-TOA

$\sin = \frac{opp}{hyp}$

$\cos = \frac{adj}{hyp}$

$\tan = \frac{opp}{adj.}$

(b)

We have an angle and its adjacent and opposite sides, so we use the tangent function

(c)

$\tan 62° = \frac{x}{15}$

Set Calculator to Degree Mode

$\tan 62° = 1.880726465$

$\frac{1.880726465}{1} = \frac{x}{15}$

$x = 15(1.880726465)$

$x = 28.2\cancel{089698}$

28.2 feet

Tear Here

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

MATHEMATICS A

Tuesday, January 23, 2001 — 1:15 to 4:15 p.m., only

ANSWER SHEET

Pupil Steve Watson Sex: ☐ Male ☐ Female Grade .....

Teacher ..... School IHS @ PH

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer all 20 questions in this part.

1 3	6 3	11 2	16 4
2 1	7 4	12 1	17 4
3 4	8 2	13 2	18 1
4 3	9 1	14 3	19 3
5 2	10 3	15 2	20 4

Your answers for Parts II, III, and IV should be written in the test booklet.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

Signature

Tear Here