August 19, 1959

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1. Express as a single term the sum of $5i$ and $-2\sqrt{-1}$. 1_____

2. Write an equation of the line whose y-intercept is 9 and whose slope is the same as that of the line whose equation is $y = 3x - 4$. 2_____

3. If y varies directly as x and if $y = \frac{1}{2}$ when $x = 6$, find the value of x when $y = 3$. 3_____

4. Simplify completely: $\dfrac{\dfrac{y}{x} - \dfrac{x}{y}}{\dfrac{1}{x} + \dfrac{1}{y}}$ 4_____

5. Write an equation which expresses the relationship between x and y shown in the following table:

x	-1	0	1	4
y	-6	-4	-2	4

5_____

6. Factor $3x^2 + 2x - 8$. 6_____

7. Solve the formula $S = \pi K(R + r)$ for R. 7_____

8. Write the first *two* terms in the expansion of $(x + 3y)^5$. 8_____

9. Solve the following set of equations for x:

$$x = y - 5$$
$$3x - y = -9$$

9_____

10. Solve the equation: $2 + \sqrt{4x + 7} = 5$ 10_____

11. A tower stands on level ground. At a point on the ground 30 feet from the base of the tower, the angle of elevation of the top of the tower is 74°. Find, to the *nearest foot*, the height of the tower. 11_____

12. Find the logarithm of 0.6504. 12_____

13. Find n if $\log n = 0.4950$. 13_____

14. The first term of an arithmetic progression is 6 and the twenty-fifth term is 22. Find the common difference. 14_____

15. Find a geometric mean between 6 and 7. 15_____

16. Find the sum of the roots of the equation $9x^2 - 5x + 2 = 0$. 16_____

17. Given: $y = -x^2 + 10x - 16$. The maximum value of y occurs when $x = m$. Find the value of m. 17_____

18. Point P is the intersection of the graph of $4x^2 + 9y^2 = 36$ and the positive portion of the *y-axis*. Write the coordinates of P. 18_____

Directions (19-25): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b*, *c* or *d*.

19. $\dfrac{1}{2-\sqrt{11}}$ is equivalent to (a) $\dfrac{2+\sqrt{11}}{9}$ (b) $\dfrac{2+\sqrt{11}}{7}$ (c) $-\dfrac{2+\sqrt{11}}{7}$ (d) $-\dfrac{2+\sqrt{11}}{9}$ 19_____

20. The value of $3x^0 + (3x)^{-\frac{1}{3}}$ when $x = 3$ is (a) $1\frac{1}{3}$ (b) $3\frac{1}{3}$ (c) 6 (d) 4 20_____

21. The roots of the equation $2x^2 - 8x + 7 = 0$ are (a) rational and unequal (b) irrational and equal (c) irrational and unequal (d) imaginary 21_____

22. If $z = \dfrac{x^3}{y^2}$, then $\log z$ is equal to (a) $\dfrac{3 \log x}{2 \log y}$ (b) $\dfrac{3}{2} \log \dfrac{x}{y}$ (c) $\dfrac{3}{2}(\log x - \log y)$ (d) $3 \log x - 2 \log y$ 22_____

23. $(x^3)^{-2}$ is equal to (a) x (b) x^{-5} (c) x^{-6} (d) x^{-9} 23_____

24. The number 0.000036 may be written as (a) 3.6×10^{-5} (b) 3.6×10^{-4} (c) 3.6×10^{4} (d) 3.6×10^{5} 24_____

25. The graph of the equation $3x^2 + 3y^2 = 25$ is a (a) circle (b) pair of straight lines (c) parabola (d) hyperbola 25_____

Part II

Answer three questions from this part. Show all work.

26. Solve the following set of equations, group your answers and check: [7, 1, 2]

$$x^2 + xy = -2x$$
$$x - y = 5$$

27. Find to the *nearest tenth* the roots of the equation $2x^2 - 5x = 4$. [10]

28. The base edge of a regular pentagonal prism expressed in terms of its volume V and its altitude h is given by the formula $s = \sqrt{\dfrac{V \tan 36^\circ}{1.25\,h}}$.

If $V = 775$ and $h = 11.0$, using logarithms find s to the *nearest tenth*. [10]

29. *a* Draw the graph of $y = -x^2 + 3$ for values of x from $x = -3$ to $x = 3$, inclusive. [4]

b On the same set of axes used in part *a*, draw the graph of $xy = 8$. [4]

c From the graphs made in answer to parts *a* and *b*, estimate to *tenths* a common solution for the two equations. [2]

The following questions, *30 and *31, are based upon optional topics in the syllabus, and *one* of them may be substituted for any *one* question in *either* part II or part III. Therefore *one*, but *not both*, of these questions may be included in in the total of 5 required questions from parts II and III.

*30. Solve the following system of equations for x, y and z and check your results: [8, 2]

$$\begin{aligned} x + 2y + z &= -2 \\ 4y - z &= 1 \\ 3x - 2y - 2z &= 13 \end{aligned}$$

*31. Find the three roots of the equation $2x^3 - 9x^2 - 11x + 30 = 0$. [10]

Part III

Answer two questions from this part. Show all work unless otherwise directed. Only algebraic solutions will be accepted in 34-35.

32. *a* Express the sum of $\dfrac{x}{x+1}$ and $\dfrac{3}{1-x}$ as a single fraction. [3]

b Find the value of the fraction obtained in answer to *a* when $x = -2$. [2]

c Find the values of x for which the fraction obtained in answer to *a* will be equal to $\dfrac{9}{4}$. [5]

33. Write the equation or equations that would be used to solve the following problems. In *each* case state what the letter or letters represent. [*Solution of the equations is not required.*]

a How many ounces of water must be added to 24 ounces of a 10-percent solution of disinfectant to reduce it to an 8-percent solution? [5]

b A man buys a set of tools costing $50. He makes a down payment of $5 and arranges to pay the remainder of his bill by paying $1 at the end of the first week, $1.50 at the end of the second week and by increasing his payments by $.50 each successive week. How many weeks will it take him to pay off the debt? [5]

34. The units digit of a two-digit number is three more than the tens digit. One-half of the number formed by reversing the digits is ten less than the original number. Find the original number. [5, 5]

35. A man drove 280 miles to a conference and arrived an hour late. Had he increased his average speed five miles an hour, he would have arrived at the time for which the conference had been scheduled. What was his average speed? [5, 5]

June, 1959

1. $7i$
2. $\dfrac{5(\sqrt{3}+1)}{2}$
3. $1\frac{1}{2}$ or -4
4. $1\frac{1}{3}$
5. $32x^5 - 80x^4y$
6. 1.5973
7. 0.06066
8. $y = -2x + 6$
9. $1\frac{1}{2}$
10. -4
11. 3
12. hyperbola
13. $\frac{3}{4}$
14. $\dfrac{t-a}{n-1}$
15. 0
16. -4
17. $\frac{1}{2}$
18. 7/25
19. a
20. c
21. b
22. c
23. d
24. d
25. a
26. $-2, -5; \frac{2}{3}, 3$ and $-3.3, -3.4$, or -3.5
27. $-.5, -2.5$ (c) -6
28. 5.06
29. (b) 1.3, 1.4, or 1.5
30. 9
31. $5, -3, -1$
32. (a) $\dfrac{t}{u} = \dfrac{3}{2}$, $10t + u + 10u + t = 165$ (b) $\dfrac{200}{r+20} + \dfrac{225}{r-30} = \dfrac{5}{2}$
33. \$45 for strawberries, \$22 for peaches
34. 16
35. (a) 36, (b) 54

August, 1959

1. $3i$
2. $y = 3x + 9$
3. 36
4. $y - x$
5. $y = 2x - 4$
6. $(3x-4)(x+2)$
7. $\dfrac{S - \pi Kr}{\pi K}$
8. $x^5 + 15x^4y$
9. -2
10. $\frac{1}{2}$
11. 105
12. 9.6132—10
13. 3.126
14. $\frac{2}{3}$
15. $\sqrt{42}$ or $-\sqrt{42}$
16. 5/9
17. 5
18. 0, 2
19. c
20. b
21. c
22. d
23. c
24. a
25. a
26. $0, -5; 1\frac{1}{2}, -3\frac{1}{2}$
27. 3.1, —0.6
28. 6.4
29. (a) $-2.5, -3.2$
30. $2, -\frac{1}{2}, -3$
31. $1\frac{1}{2}, 5, -2$
32. (a) $\dfrac{x^2 - 4x - 3}{x^2 - 1}$, (b) 3, (c) $-1/5, -3$
33. (a) $.08(24 + x) = 2.4$ (b) $45 = \dfrac{n}{2}[2 + (n-1).50]$
34. 47
35. 35 mph

January, 1960

1. $\dfrac{3(4+\sqrt{3})}{13}$
2. $-1, 3$
3. $(6x+5)(x-2)$
4. $\dfrac{(2-x)x}{9+x}$
5. 27
6. $36, 54$
7. -87
8. 649.3
9. $9.3516 - 10$
10. $\dfrac{p-q}{q-p}$
11. $1\frac{1}{4}$
12. 12
13. $y = 2x - 5$
14. $y = 3x + 1$
15. $3i\sqrt{3}$
16. -6
17. $84x^5$
18. -2
19. 4
20. 3
21. 1
22. 1
23. 3
24. 2
25. 2
26. $2, -1; -1\frac{1}{4}, 2\frac{1}{4}$
27. (c) 0, 4; 4, 0
28. $-0.6, -3.4$
29. 82.7
30. (a) 6, (b) 1.8
31. $2, -\frac{1}{2}, -1\frac{1}{2}$
32. (a) $\dfrac{15}{x-10} + \dfrac{20}{x} = 1$
(b) $\dfrac{10t+u}{t+u} = 4$
$10u + t - 36 = 10t + u$
33. 6, 12, 18
34. 10 in., 20 in.
35. 2.6 of 33% solution and 27.4 oz. of 10% solution