The University of the State of New York

313TH HIGH SCHOOL EXAMINATION

INTERMEDIATE ALGEBRA

Tuesday, August 21, 1951 — 12 m. to 3 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in intermediate algebra previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1951 or number and length in minutes of lessons taken in the summer of 1951 under a tutor licensed in the subject and supervised by the principal of the school you last attended.

The minimum time requirement is four or five recitations a week for half a school year after the completion of elementary algebra. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1951 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer three questions from part II.

26 Find to the *nearest tenth* the roots of the equation $2x^2 - 4x - 7 = 0$. [10]

27 *a* Draw the graph of the equation $x^2 + y^2 = 25$. [3]

b On the set of axes used in *a* draw the graph of the equation $y = x^2 - 3$ for values of x from -3 to $+3$ inclusive. [5]

c Estimate to the *nearest tenth* the coordinates of the points of intersection of the graphs drawn in answer to *a* and *b*. [2]

28 The radius r of a regular polygon of 12 sides is given by the formula $r = \sqrt{\frac{A}{12 \sin 15^\circ \cos 15^\circ}}$ in which A is the area of the polygon. Using logarithms, find r to the *nearest tenth* when $A = 825$. [10]

29 Given the equations $I = \frac{E}{R}$ and $I = \sqrt{\frac{P}{R}}$

a Express R in terms of E and P. [7]

b If $P = 2420$ and $E = 110$, find R. [3]

*30 In how many years will \$500 amount to \$773 if interest is compounded annually at 2%? [$A = P(1 + r)^n$] [10]

*31 Solve the following system of equations: [10]

$$x - 2y + 3z = 8$$
$$3x + y - 6z = 5$$
$$x - 3y - 27z = 0$$

* This question is based upon one of the optional topics in the syllabus.

Part III

Answer one question from part III.

32 Write the equations that would be used to solve *each* of the following problems. In each case state what the letter or letters represent. [Solution of the equations is not required.]

a The dimensions of a rectangle are in the ratio 2 : 5. If the smaller dimension is increased by 3 feet and the larger dimension is decreased by 2 feet, the area of the resulting rectangle is 198 square feet. Find the dimensions of the original rectangle. [5]

b To do a certain piece of work alone one machine requires 10 minutes longer than another. The two machines working together can do the work in 12 minutes. How long does it take the faster machine to do this work alone? [5]

33 Mr. Jones had a certain amount of money to invest. Part of this amount he invested in bonds paying 3% annually and the rest in a mortgage paying 4% annually. His total annual income from the two investments was $265. If he had invested the entire amount at $3\frac{1}{2}\%$, his annual income would have been $280. How much did he invest in the mortgage? [7, 3]

Part IV

Answer one question from part IV.

34 Each of the following statements may be completed correctly by *two* and only two of the four choices given. Write on your answer paper the letters *A* through *E* and after *each* letter indicate the correct choices by writing *two* of the numbers 1, 2, 3 and 4. [Unless otherwise stated, *a*, *b* and *c* are positive integers.] [10 credits — 1 credit for each correct answer.]

A The graph of $ax + by + c = 0$
(1) has a positive slope
(2) is perpendicular to the x-axis if b is zero
(3) intersects the y-axis at the point $(0, \frac{c}{b})$
(4) passes through the origin if c is zero

B The graph of $ax^2 + by^2 = c$
(1) is a circle if a equals b
(2) is a hyperbola if a is not equal to b
(3) intersects the x-axis and the y-axis
(4) is a parabola if b equals zero

C The graph of $ax^2 - by^2 = c$
(1) intersects the y-axis
(2) intersects the x-axis
(3) passes through the origin if c equals zero
(4) is an ellipse

D The graph of $y = ax^2 + bx + c$
(1) is symmetric to the *y-axis*
(2) is tangent to the x-axis if $b^2 = 4ac$
(3) intersects the x-axis if b^2 is less than $4ac$
(4) is a straight line if a equals zero

E The graph of $xy = a$
(1) is a hyperbola
(2) passes through the point $(1, a)$
(3) intersects the x-axis and the y-axis
(4) intersects the line whose equation is $y = -x$

35 *a* The first term of an arithmetic progression is 1, the common difference is 3 and the sum of the first n terms is 51.
(1) Write the equation that can be used to find n. [2]
(2) Using this equation, find the number of terms. [3]

b The first term of a geometric progression is 1, the common ratio is 3 and the sum of the first n terms is 121.
(1) Write the equation that can be used to find n. [2]
(2) Using this equation, find the number of terms. [3]

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Factor: $x^3 + 4x^2 + 4x$ 1.....

2 Perform the indicated operation: $a^n \div a$ 2.....

3 Express $\frac{2a}{a-b} + \frac{b}{b-a}$ as a single fraction in its lowest terms. 3.....

4 Simplify the complex fraction $\frac{\frac{r}{s}}{\frac{1}{r} + \frac{1}{s}}$ 4.....

5 Find the value of $x^{\frac{2}{3}} - 4x^0$ when $x = 8$. 5.....

6 Express $\frac{1}{\sqrt{5} - \sqrt{2}}$ with a rational denominator.

6.....

7 Express $\sqrt{75} - 6\sqrt{\frac{1}{3}}$ as a single term. 7.....

8 Solve the following formula for r: $V = \frac{4}{3}\pi r^3$ 8.....

9 Find the coordinates of the point in which the graph of the equation 9 x =.....

$y = x$ intersects the graph of $y = 2x - 3$. y =.....

10 A straight line has a slope of 3 and intersects the y-axis at the point (0,2). Write an equation of this line. 10.....

11 Write a quadratic equation whose roots are $+2$ and -2. 11.....

12 Find the product of the roots of the equation $2x^2 + 3x + 5 = 0$. 12.....

13 If y varies directly as x, and $y = 5$ when $x = 2$, find y when $x = 3$. 13.....

[OVER]

14 The first term of an arithmetic progression is 3 and the fifth term is 19. Find the common difference. 14.....

15 The first term of a geometric progression is $\frac{1}{2}$ and the fourth term is 32. Find the common ratio. 15.....

16 Find the sum of the infinite geometric series whose first term is 9 and whose common ratio is $\frac{1}{10}$. 16.....

17 The length of a rectangle is s and its perimeter is $2p$. Express its width in terms of s and p. 17.....

18 Find the logarithm of 34.64. 18.....

19 Find the number whose logarithm is $8.6752 - 10$. 19.....

20 In triangle ABC, angle $C = 90°$, angle $A = 82°$ and side $AC = 200$ feet. Find side BC to the *nearest foot*. 20.....

Directions (questions 21–25): Indicate the correct answer to *each* of the following by writing the letter a, b, c or d on the dotted line at the right.

21 If the discriminant of a quadratic equation is 8, the roots of the equation are (a) imaginary (b) rational and equal (c) rational and unequal (d) irrational and unequal 21.....

22 Which of the following numbers is imaginary? (a) $\sqrt[3]{-8}$ (b) $\sqrt{-8}$ (c) $-\sqrt{8}$ (d) none of these 22.....

23 The number 9.3×10^7 is equal to (a) 930,000,000 (b) 93,000,000 (c) 9,300,000 (d) none of these 23.....

24 If r ounces of water are added to 10 ounces of a solution which is 50% alcohol, the amount of alcohol in the resulting mixture divided by the total amount of that mixture is

(a) $\frac{5}{10+r}$ (b) $\frac{10}{10+r}$ (c) $\frac{5+r}{10+r}$ (d) none of these 24.....

25 Which of the following sets of equations when solved graphically will have *more than* two points of intersection?

(a) $x^2 + y^2 = 4$, $x - y = 1$ (b) $xy = 4$, $x + y = 4$ (c) $x - y = 1$, $x + y = 4$ (d) $x^2 + y^2 = 16$, $xy = 4$ 25.....