INTERMEDIATE ALGEBRA

Friday, June 15, 1956—1:15 to 4:15 p.m., only

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1. **Express** $x^3 - a^2x$ as the product of three factors. 1.....

2. **Find** the positive root of $2x^2 - x - 6 = 0$. 2.....

3. **Solve** the following pair of equations for x:
$$5x - 2y = a$$
$$3x - 2y = b$$ 3.....

4. Express $\dfrac{2}{3 - \sqrt{2}}$ as an equivalent fraction with a rational denominator. 4.....

5. Express as a single term the sum $3\sqrt{-4}$ and $3i$. 5.....

6. Find the value of x that satisfies the equation $2\sqrt{x - 1} = 1$. 6.....

7. Find the value of $3x^0 + x^{-\frac{3}{2}}$ when $x = 4$. 7.....

8. Find the slope of the line that passes through the points $(0, 0)$ and $(2, 5)$. 8.....

9. Find the logarithm of 312.3 9.....

10. Find the number whose logarithm is $9.8174 - 10$. 10.....

11. The legs of a right triangle are 4 and 10. Find, to the *nearest degree*, the smaller acute angle of the triangle. 11.....

12. Express $\log \dfrac{a}{\sqrt{b}}$ in terms of $\log a$ and $\log b$. 12.....

13. If x varies inversely as y and $x = 8$ when $y = 9$, find y when $x = 12$. 13.....

14. Reduce the fraction $\dfrac{2x - 2y}{3y - 3x}$ to lowest terms. 14.....

15. Simplify the complex fraction: $\dfrac{1 + \dfrac{1}{x}}{1 - \dfrac{1}{x^2}}$ 15.....

16. Solve for b in terms of A, h and c: $A = \frac{1}{2}h(b + c)$. 16.....

17. The first term of an arithmetic progression is 9 and the seventh term is 57. Find the common difference. 17.....

18. Find two numbers that, when inserted between 3 and 192, form with these numbers a geometric progression of four terms. 18.....

19. Find the sum of the infinite geometric progression 18, 12, 8, ... 19.....

20. The sum of the roots of the equation $x^2 - kx + 7 = 0$ is 3. Find the value of k. 20.....

21. Find the product of the roots of the equation $2x^2 - 3x - 5 = 0$. 21.....

22. The parabola whose equation is $y = ax^2 + x$ passes through the point $(1,3)$. Find the value of a. 22.....

23. Write the third term in the expansion of $(x + y)^5$. 23.....

Directions (24-25): Indicate the correct completion for *each* of the following by writing the letter *a*, *b* or *c* on the line at the right.

24. The value of the discriminant of the equation $3x^2 - 5x - 4 = 0$ is (*a*)—23 (*b*) $\sqrt{73}$ (*c*) 73 24.....

25. The graph of the equation $y^2 = x^2 + 9$ is (*a*) a circle (*b*) an ellipse (*c*) a hyperbola 25.....

Part II

Answer three questions from this part. Show all work.

26. Find to the *nearest tenth* the roots of the equation $2x^2 + 7x + 4 = 0$. [10]

27. Solve the following system of equations and check: [8, 2]
$$x^2 - xy + y^2 = 63$$
$$y - x = 3$$

28. *a.* Draw the graph of the equation of $y = x^2 - 4x + 5$ from $x = 0$ to $x = 4$ inclusive. [6]
b. On the same set of axes used in part *a*, draw the graph of the equation $x - y = -1$. [2]
c. From the graphs made in answer to parts *a* and *b*, find the common solutions of the two equations. [2]

29. Using logarithms, find to the *nearest thousandth* the value of: [10]
$$\sqrt[3]{\frac{.75 \times 4.95}{(\tan 72°)^2}}$$

*The following questions, *30 and *31, are based upon optional topics in the syllabus, and one of them may be substituted for any one question in either part II or part III. Therefore one, but not both, of these questions may be included in the total of 5 required questions from parts II and III.*

*30. In how many years (n) will \$480 amount to \$725 if money earns interest at the rate of 3% compounded annually? [Use $A = P(1 + r)^n$. Find n to the *nearest year*.] [10]

*31. Solve the following system of equations for x, y and z and check: [8, 2]
$$3x - 2y + z = 4$$
$$2x + 4y - 3z = 9$$
$$-x + 8y - 2z = 4$$

Part III

Answer two questions from this part. Show all work unless otherwise directed.

32. In an arithmetic progression the third term is 16 and the tenth term is 65. Using the formulas for finding the nth term and the sum of n terms of an arithmetic progression,

a. find the first term [6]

b. find the sum of the first 21 terms [4]

33. Write the equations that would be used in solving the following problems. In *each* case state what the letter or letters represent. [*Solution of the equations is not required.*]

a. A chemist has an 18% solution and a 45% solution of a disinfectant. How many ounces of *each* should be used to make 12 ounces of a 36% solution? [5]

b. When a certain two-digit number is divided by the sum of its digits, the quotient is 7. If the digits are reversed, the resulting number is 18 less than the original number. Find the original number. [5]

34. Two men, A and B, travelled 96 miles and 100 miles respectively. A's average rate was 8 miles an hour less than B's, and A's trip took one-half hour more than B's. Find the average rate of each. [10]

35. If the blank in each of the following statements is replaced by one of the words *always, sometimes* or *never,* the resulting statement will be true. Write the numbers 1-5 on your answer paper and opposite *each* number write the word that will correctly complete the corresponding statement. [Consider only the cases where a, b and c are real numbers and a is *not* equal to zero.] [10]

(1) The graph of the equation $y = ax^2 + bx + c$ . . . passes through the origin when $c = 0$.

(2) The graph of the equation $y = ax^2 + bx + c$ is . . . tangent to the x-axis when $b^2 = 4ac$.

(3) The graph of the equation $y = ax^2 + bx + c$ is . . . symmetric to the y-axis when b is *not* equal to zero.

(4) The roots of the equation $ax^2 + bx + c = 0$ are . . . real if a, b and c have the same sign.

(5) Both roots of the equation $ax^2 + bx + c = 0$ . . . have the same sign if a and c have opposite signs.