

The University of the State of New York  
306TH HIGH SCHOOL EXAMINATION

INTERMEDIATE ALGEBRA

Wednesday, June 22, 1949—9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of elementary algebra.

Part II

Answer three questions from part II.

26 Find to the *nearest tenth* the roots of the equation

$$3x^2 - 7x - 1 = 0 \quad [10]$$

27 Solve the following system of equations, group your answers and check *one* set:

$$\begin{aligned} xy + y + 5 &= 0 \\ 2x + y &= 1 \end{aligned} \quad [7, 2, 1]$$

28 a Draw the graph of  $y = x^2 - 4x$  from  $x = -1$  to  $x = 5$  inclusive. [6]

b On the set of axes used in a, draw the graph of  $y = x - 5$ . [2]

c From the graphs drawn in answer to a and b, estimate to the *nearest tenth* the solutions common to both equations. [2]

29 The side  $a$  opposite angle  $A$  of a triangle inscribed in a circle whose area is  $K$  is given by

$$\text{the formula } a = 2 \sqrt{\frac{K}{\pi}} \sin A$$

Using logarithms, find  $a$  to the *nearest tenth* of an inch when  $K = 153$  square inches and  $A = 71^\circ$ . (Use  $\pi = 3.14$ ) [10]

\*30 Solve the following system of equations:

$$\begin{aligned} 2x + 3y - z &= 10 \\ 6x - y - 2z &= -2 \\ y + z &= 1 \end{aligned} \quad [10]$$

\*31 Solve the equation  $2x^3 - 3x^2 - 18x - 8 = 0$  [10]

\* This question is based upon one of the optional topics in the syllabus.

## Part III

Answer one question from part III.

32 Write the equations that would be used in solving the following problems. In each case state what the letter or letters represent. [Solution of the equations is not required.]

a The sum of the digits of a 2-digit number is 10. Three times the number is 2 more than the number obtained by interchanging the digits. Find the original number. [5]

b How many ounces of water should be added to 18 ounces of alcohol 95% pure so that the resulting solution will be 72% pure alcohol? [5]

33 A boy wants to save \$18.75 so that he can buy his brother a bond as a graduation gift. He makes an arrangement with the school bank whereby he will pay 15 cents the first week and increase his payments by 5 cents per week each week thereafter until the \$18.75 is accumulated. How many payments must he make? [10]

## Part IV

Answer one question from part IV.

34 If the blank in each of the following statements is filled by one of the words *always*, *sometimes*, or *never*, the resulting statement will be true. Write the numbers (1) to (5) and opposite each write the word that will correctly complete the corresponding statement. [Consider only the cases where  $a$ ,  $b$  and  $c$  are real numbers.]

(1) The graph of the equation  $y = ax^2 + bx + c \dots$  intersects the  $x$ -axis if  $b^2 - 4ac$  is less than zero. [2]

(2) The sum of the roots of  $x^2 + bx + c = 0$  is  $\dots$  equal to their product if  $b = -c$ . [2]

(3) The graph of the equation  $ax^2 + by^2 = c$  is  $\dots$  a hyperbola if  $a$  is not equal to  $b$ . [2]

(4) The roots of  $ax^2 + c = 0$  are  $\dots$  imaginary if  $a$  and  $c$  have unlike signs. [2]

(5) When drawn on the same axes, the graphs of the equations  $x^2 + y^2 = a^2$  and  $y = a$  are  $\dots$  tangent if  $a$  is not equal to zero. [2]

35 A motorist can cover a distance of  $h$  miles if he travels at  $r$  miles an hour. If he were to increase his rate by  $x$  miles an hour, he would cover the same distance in 1 hour less time.

a Express in terms of  $h$  and  $r$  the time required for the trip under the original conditions. [2]

b Express in terms of  $h$ ,  $r$  and  $x$  the time required for the trip under the new conditions. [2]

c Write an equation that can be used to solve for  $x$  in terms of  $h$  and  $r$ . [2]

d Solve for  $x$  the equation written in answer to c. [3]



INTERMEDIATE ALGEBRA  
Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

- 1 Write the three factors of  $x^3 - 9x$  1.....
- 2 Express  $2\sqrt{-16}$  in terms of  $i$ . 2.....
- 3 Express  $\frac{1}{3 + \sqrt{2}}$  as an equivalent fraction with a rational denominator. 3.....
- 4 Solve the equation  $\sqrt{4n} - 1 = 5$  4.....
- 5 Solve for the positive value of  $r$ :  $V = \frac{1}{3} \pi r^2 h$  5.....
- 6 Write an equation of the straight line whose slope is 3 and whose  $y$ -intercept is  $-1$  6.....
- 7 Write an equation expressing the relationship between  $x$  and  $y$  shown by the following table:

$x$	0	1	3	4
$y$	$-1$	2	8	11

- 8 If the parcel post rate is  $p$  cents for the first pound and  $a$  cents for each additional pound, express in cents the cost of sending a package of  $n$  pounds where  $n$  is an integer greater than 1. 7.....
- 9 Find the sum of the roots of the equation  $2x^2 - 3x + 4 = 0$  8.....
- 10 Find *two* geometric means between 3 and 192. 9.....
- 11 Find the *31st* term of the progression 3, 7, 11, ... 10.....
- 12 Find the sum of the infinite geometric progression whose first term is 2 and whose ratio is  $\frac{2}{3}$ . 11.....
- 13 Find, to the *nearest degree*, the angle of elevation of the sun when an 8-foot vertical pole casts a shadow 10 feet long. 12.....
- 14 If  $\log N = 3.4090$ , find  $N$  to the *nearest integer*. 13.....
- 15 Find  $\log 7.106$  14.....
- 16 If  $\log a^3 = 1.3608$ , find  $\log a$ . 15.....
- 17 If  $y$  varies directly as  $x$  and  $y = 20$  when  $x = 12$ , find  $y$  when  $x = 15$ . 16.....
- 18 Simplify the complex fraction  $\frac{\frac{1}{a}}{1 - \frac{1}{a}}$  17.....
- 19 Find the value of  $8^{\frac{3}{2}} + 8^0$  18.....

20 If  $y = 1 - \frac{1}{x}$  and  $x$  is positive, does  $y$  increase or does it decrease as  $x$  increases? [Answer increase or decrease.] 20.....

21 How many terms are there in the expansion of  $(a + b)^{10}$ ? 21.....

Directions (questions 22-25) — Indicate the correct answer to each question by writing on the line at the right the letter  $a$ ,  $b$ , or  $c$ .

22 If 0.000065 is expressed in the form  $6.5 \times 10^n$ , the value of  $n$  is (a)  $-5$ , (b)  $-6$ , (c)  $-7$  22.....

23 The fraction  $\frac{y-x}{-z}$  is equal to (a)  $\frac{y+x}{z}$ , (b)  $\frac{x-y}{z}$ , (c)  $-\frac{x-y}{z}$  23.....

24 If the discriminant of a quadratic equation with real coefficients is greater than zero, the roots of the equation are always (a) real and rational, (b) real and unequal, (c) rational and unequal. 24.....

25  $x^m \cdot y^m$  equals (a)  $(xy)^m$ , (b)  $xy^m$ , (c)  $(xy)^{2m}$  25.....