INTERMEDIATE ALGEBRA

Wednesday, June 18, 1947 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of elementary algebra.

Part II

Answer three questions from part II.

26 Find, correct to the *nearest tenth,* the roots of the equation $2x^2 - 5x + 1 = 0$ [10]

27 Solve the following system of equations, group your answers and check *one* set: [7, 2, 1]

$$x^2 + 3y^2 = 13$$
$$x - y = 3$$

28 a Draw the graph of the equation $y = x^2 - 7x + 10$, using integral values of x from 1 to 6 inclusive.   [6]

   b On the set of axes used in part a, draw the graph of $y = x - 6$ [2]

   c State whether the graphs of $y = x^2 - 7x + 10$ and $y = x - 6$ intersect in two points, are tangent or have no points in common.   [2]

29 The radius r of the circle circumscribed about a regular pentagon whose area is K is given by the formula $r = \sqrt{\dfrac{K}{5 \sin 54° \cos 54°}}$. Using logarithms, find r correct to the *nearest tenth of an inch* when $K = 108$ sq. in.   [10]

*30 a Write each member of the equation $4^{2x + 1} = 8^x$ as a power of 2. Find the value of x which satisfies the equation.   [3, 1]

   b Solve the equation $2^x = 12$. Express your answer correct to the *nearest tenth.*   [6]

* This question is based on one of the optional topics in the syllabus.

[1]

[OVER]

140

Answer one question from part III.

31 How many pounds of pure salt must be added to 30 pounds of a 2% solution of salt and water to increase it to a 10% solution? [7, 3]

32 Write the equations that may be used in solving *each* of the following problems. In each case state what the letter or letters represent. [Solution of the equations is not required.]

a The area of a rectangle is 50 sq. in. If the length and the width of the rectangle are each increased by 2 in., a new rectangle is formed whose area is 84 sq. in. Find the length of the given rectangle. [4]

b Three numbers whose sum is 30 are in arithmetic progression. The largest of these numbers is 8 greater than the square of the smallest. Find the numbers. [6]

Part IV

Answer one question from part IV.

33 Write the numbers (1) to (5) on your answer paper and opposite *each* write the letter a, b, c or d to indicate the correct answer to the corresponding question.

(1) Which of the following is false? (a) $x^5 \div x^2 = x^3$
(b) $(x^2)^3 = x^6$ (c) $x^3 \cdot y^3 = (xy)^3$ (d) $x^3 \cdot y^3 = (xy)^3$ [2]

(2) A root of the equation $x^3 - 6x^2 + 11x - 6 = 0$ is   (a) 0
(b) —1   (c) —2   (d) none of these answers   [2]

(3) The expression $\dfrac{\dfrac{x}{y} - \dfrac{y}{x}}{\dfrac{x}{y} + \dfrac{y}{x}}$ is equal to   (a) $\dfrac{x-y}{x+y}$

(b) —1   (c) $\dfrac{x^2 - y^2}{x^2 + y^2}$ (d) 0   [2]

(4) If the graph of the equation $y = ax^2 + bx + c$ does not intersect the x-axis and is not tangent to the x-axis, the equation $ax^2 + bx + c = 0$ has   (a) irrational roots
(b) no roots   (c) imaginary roots   (d) equal roots   [2]

(5) The graph of the equation $ax^2 + by = c$ in which neither a nor b is 0, is
(a) a straight line   (b) an ellipse   (c) a parabola   (d) a hyperbola   [2]

34 A man bought a number of articles for $120, paying the same price for each. He sold all but three of the articles at an advance of $5 on the cost of each, thereby making a monetary gain of $15 on the entire transaction.

The facts given above are represented graphically in the figure at the right. OA represents the number of articles bought, OC the number sold, AB the total amount paid for those bought, and CD the total amount received for those sold. Let OA be represented by x. Using the facts given above,
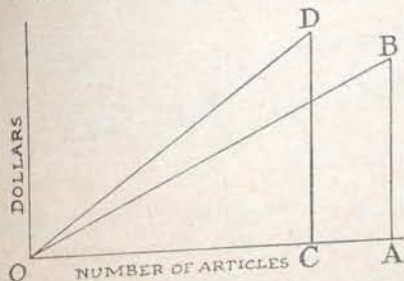
a Express OC in terms of x. [1]
b Express the slope of line OB in terms of x. [3]
c Express the slope of line OD in terms of x. [2]
d Write in terms of x the equation that would be used in finding x. [4]

[Suggestion: By how much does the slope of OD exceed the slope of OB?]
[2]

Fill in the following lines:

Name of school.................................Name of pupil...

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Solve for the positive value of V the equation $h = \dfrac{V^2}{2g}$

1.......................

2 During a certain year a man has an income of d dollars. He is allowed to deduct e dollars from his income, the remainder being subject to a tax of 3%. Write the expression for the amount of his tax.

2.......................

3 Using the formula $V = \frac{1}{3}\pi r^2 h$, find the value of V if $\pi = \frac{22}{7}$, $r = 3$ and $h = 42$

3.......................

4 Factor $y^2 - .09x^2$

4.......................

5 Find the sum of the fractions $\dfrac{3}{x-2}$ and $\dfrac{1}{2-x}$

5.......................

6 If x varies directly as y and $x = 8$ when $y = 3$, find y when $x = 32$

6.......................

7 For the following set of equations, find a solution in which the values of both x and y are positive:

$$x^2 + y^2 = 50$$
$$x^2 - y^2 = -48$$

7 x.......................

y.......................

8 Write the linear equation expressing the relationship between R and T as shown in the following table:

T	0	3	9	15
R	-4	5	23	41

8.......................

9 Solve the equation $\sqrt[3]{3x - 1} = 2$

9.......................

10 Simplify $2\sqrt{6} \times 3\sqrt{2}$

10.......................

11 Write the expression $\dfrac{1}{\sqrt{5} + \sqrt{2}}$ with rational denominator.

11.......................

12 Express in terms of i the sum of $\sqrt{-9}$ and $2\sqrt{-1}$

12.......................

13 Find the value of $2x^0$

13.......................

14 Find the value of $8^{\frac{1}{3}} \times 2^{-2}$

14.......................

15 Write the equation of the straight line which passes through the origin and whose slope is -1

15.......................

16 Find the sum of the roots of the equation $x^2 - 5x + k = 0$

16.......................

17 If the roots of a quadratic equation are real and equal, the discriminant of the equation is (a) greater than 0, (b) less than 0, (c) equal to 0. Which is correct, (a), (b) or (c)?

17.......................

18 Find the sum of the first 16 terms of the series 1, 3, 5, ...

18.......................

19 A geometric mean between two numbers is 8. If one of the numbers is 2, find the other number.

19.......................

[3]

20 Find the sum of the infinite geometric series 5, $\frac{4}{4}$, $\frac{4}{4}$, ...   20.....................

21 Find the first three terms in the expansion of $(x + a)^8$ 21.....................

22 Express $\log ab^3$ in terms of $\log a$ and $\log b$.   22.....................

23 Find the logarithm of 27.83   23.....................

24 Find the number whose logarithm is $8.5420 - 10$ 24.....................

25 Find, correct to the *nearest degree*, the smaller acute angle of the
right triangle whose legs are 14 and 20.   25.....................

[4]