The University of the State of New York

297TH HIGH SCHOOL EXAMINATION

INTERMEDIATE ALGEBRA

Wednesday, June 19, 1946 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of elementary algebra.

Part II

Answer three questions from part II.

26 Find, correct to the *nearest tenth*, the roots of the equation $3x^2 - 7x - 1 = 0$ [10]

27 *a* Write the name of the graph of *each* of the following equations: [1, 1]

(1) $x^2 - y^2 = 16$

(2) $x^2 + y^2 = 34$

b If the graphs of the two equations given in *a* were drawn on the same set of axes, in how many points would they intersect? [2]

c Find, *algebraically*, the coordinates of the points of intersection of the graphs of these equations. [6]

28 The velocity V, in feet per second, of a jet of water necessary to produce P foot-pounds of power per second is given by the formula $V = \sqrt[3]{\frac{64\,P}{62.2\,A}}$ where A is the area of a cross section of the jet in square feet. Using logarithms, find V correct to the *nearest tenth* if $P = 21.3$ foot-pounds per second and $A = .116$ square foot. [10]

29 A man wishes to pay off a debt of \$580 by making monthly payments in which each payment after the first is \$2 more than that of the preceding month. According to this plan, how long will it take him to pay the debt if the first payment is \$10 and no interest charge is made? [10]

*30 Find, correct to the *nearest year*, the time in which \$1500 will amount to \$2170 if interest is compounded annually at $2\frac{1}{2}\%$. [Use the formula $A = P(1+r)^n$] [10]

* This question is based on one of the optional topics in the syllabus.

Part III

Answer one question from part III.

31 Write the equations that may be used to solve the following problems. In each case state what the letter or letters represent. [Solution of the equations is not required.]

a Flying with the wind, an airplane can travel 1080 miles in 6 hours, but flying directly against the wind, it can travel only $\frac{2}{3}$ of this distance in the same time. Find the speed of the plane in still air. [5]

b How many ounces of pure disinfectant should be added to 30 ounces of a 4% solution of disinfectant to make a 10% solution? [5]

32 A man bought a certain number of articles of the same kind for \$320. He sold all but 10 of the articles at an advance of \$2 on the cost of each and received \$420 from the sales. Find the number of articles bought. [6, 4]

Part IV

Answer one question from part IV.

33 Each of the following statements may be correctly completed by *one or more* of the given choices. Write on your answer paper the numbers (1)–(5) and after *each* indicate the *correct* answer to the corresponding question by writing *one or more* of the letters *a, b, c, d.* [10]
[In each of the five parts of this question, one credit will be allowed for each correct choice made and one credit will be deducted for each incorrect choice. The minimum credit on each part will be 0.]

(1) The expression $\frac{x}{y} = 4$ states that (*a*) x varies directly as y,
(*b*) y varies directly as x, (*c*) x varies inversely as y,
(*d*) y varies inversely as x

(2) A root of the equation $x^3 - 6x^2 + 11x - 6 = 0$ is (*a*) 1, (*b*) -1, (*c*) -2, (*d*) 4

(3) The point whose coordinates are (3, 4) lies on the straight line whose equation is (*a*) $x = 3$, (*b*) $y = 3$, (*c*) $x = 4$, (*d*) $y = 4$

(4) The expression $\dfrac{1 + \frac{1}{x}}{1 - \frac{1}{x^2}}$ is equal to (*a*) $\dfrac{1}{1 - \frac{1}{x}}$

(*b*) $\dfrac{1}{x-1}$, (*c*) $\dfrac{x}{x-1}$, (*d*) $-\dfrac{x}{1-x}$

(5) If the first term of a geometric progression is 3 and the seventh term is 192, (*a*) the common ratio is ± 2, (*b*) the fourth term is 24 if the ratio is negative, (*c*) the sum of the first four terms is $+45$ if the ratio is positive, (*d*) the sum of the first four terms is -45 if the ratio is negative

34 The parabola $y = ax^2 + bx + c$ passes through the points whose coordinates are given in the following table:

x	-1	0	1	2	3	4	5	6	7
y	12	5	0	-3	-4	-3	0	5	12

a Draw the graph of the equation $y = ax^2 + bx + c$ from $x = -1$ to $x = 7$ inclusive. [4]

b From the graph made in answer to *a*, tell whether *each* of the following statements is *true* or *false*:

(1) The equation of the axis of symmetry is $x = 3$ [1]
(2) The roots of the equation $ax^2 + bx + c = -4$ are real and equal. [1]
(3) The roots of the equation $ax^2 + bx + c = 5$ are 0 and 6 [1]
(4) The roots of the equation $ax^2 + bx + c = -5$ are imaginary. [1]
(5) The coefficients a and c are positive numbers. [1]
(6) The coefficient b is a negative number. [1]

Fill in the following lines:

Name of school..... Name of pupil.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Factor $16x^2 - 25$ 1.....

2 Solve the formula $V = \pi r^2 h$ for the positive value of r. 2.....

3 Find a value of x and a value of y which satisfy the following set of equations: $y = x^2$, $y = 2x - 1$ 3.....

4 Solve the equation $\sqrt{x-3} - 2 = 0$ for x. 4.....

5 The length of a rectangular field is 2 rods more than three times its width. Its area is 208 square rods. If w represents the width of the field, write the equation which should be used to find w. 5.....

6 Express $x + \frac{3}{x} = \frac{11}{4}$ as a quadratic equation with integral coefficients. 6.....

7 A coil of wire 60 feet long weighs $1\frac{1}{2}$ pounds. How long is a coil of wire of the same kind which weighs $\frac{1}{4}$ pound? 7.....

8 Write the equation of the straight line which passes through the following points: $(0, 1)$, $(1, 3)$, $(2, 5)$, $(3, 7)$ 8.....

9 What is the slope of the graph of the equation $2x + y = 3$? 9.....

10 Find the value of the expression $3^2 \times 2^0 + 2^{-1}$ 10.....

11 Express the sum of $2\sqrt{-9}$ and $3\sqrt{-1}$ in terms of i. 11.....

12 Express $\frac{5}{\sqrt{5}-1}$ as an equivalent fraction with a rational denominator. 12.....

13 Find the first term of an arithmetic progression if the last term is 14, the number of terms is 37 and the common difference is $\frac{1}{4}$. 13.....

14 Find the sum of the infinite geometric progression in which the first term is 3 and the common ratio is $\frac{1}{2}$. 14.....

15 Find the second term in the expansion of $(x + 2y)^5$ 15.....

16 Find the logarithm of 30.57 16.....

17 If $\log x = 0.4285$, find x correct to the *nearest thousandth.* 17.....

18 Find, correct to the *nearest degree,* the smallest angle of the right triangle whose hypotenuse is 15 and one of whose legs is 5. 18.....

19 What is the name of the graph of the equation $x^2 + \frac{y^2}{4} = 1$? 19.....

20 Which, if any, of the following points *does not* lie on the graph of the equation $y = x^2 - 3x - 1$? $(1, -3)$; $(2, 3)$; $(-2, 9)$ 20.....

[OVER]

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Directions (questions 21–22) — If the blank in each statement is replaced by one of the words *always*, *sometimes* or *never*, the resulting statement will be true. Write on the line at the right the word that will correctly complete the statement.

21 In the expression $f = 1 - \frac{1}{r}$, if r is positive and increases in value, then f ... increases in value. 21.....

22 If a, b and c are unequal positive numbers, then $\frac{a}{b}$ is ... equal to $\frac{a+c}{b+c}$ 22.....

Directions (questions 23–25) — Indicate the correct answer to *each* question by writing on the line at the right the letter a, b, c or d

23 The result of dividing x^3 by x^{-2} is (*a*) x, (*b*) x^5, (*c*) $x^{\frac{1}{2}}$, (*d*) x^{-1}. 23.....

24 If the discriminant of the equation $ax^2 + bx + c = 0$, in which a, b and c are real numbers, is positive, the graph of the equation $y = ax^2 + bx + c$ (*a*) intersects the x axis, (*b*) is tangent to the x axis, (*c*) neither intersects nor is tangent to the x axis. 24.....

25 Given the equation $px^2 - qx + q = 0$ in which p is not equal to 0. The sum of the roots of this equation is equal to the product of its roots (*a*) for all values of p and q, (*b*) for only certain values of p and q, (*c*) for no values of p and q. 25.....