INTERMEDIATE ALGEBRA

Wednesday, January 21, 1953 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (a) name of school where you have studied, (b) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of elementary algebra.

Part II

Answer three questions from part II.

26 Two positive numbers are in the ratio $1:2$. If their product is added to their sum, the result is 6.

    a Write an equation from which the two numbers can be found. [2]

    b Solve the equation written in answer to a and find to the *nearest tenth* the smaller of the two numbers. [8]

27 a Draw the graph of $y = x^2 - 4x$ from $x = -1$ to $x = 5$ inclusive. [6]

    b On the set of axes used in a, draw the graph of $y = 2x - 6$. [2]

    c Estimate to the *nearest tenth* the coordinates of the points of intersection of the graphs made in answer to a and b. [2]

28 Using logarithms, find to the *nearest tenth* the value of $\dfrac{45.6 \sin 36°}{\sqrt[3]{.574}}$ [10]

29 Solve the following system of equations and check: [8, 2]
$$x^2 - 3y^2 = 13$$
$$x + 3y = 1$$

*30 Solve the equation $4x^3 - 4x^2 - 5x + 3 = 0$ [10]

*31 Solve the following system of equations: [10]
$$3x + 2y = 5$$
$$4x - 3z = 7$$
$$6y - 6z = -5$$

* This question is based upon one of the optional topics in the syllabus.

Part III
Answer one question from part III.

32 Write the equations that would be used in solving the following problems. In each case state what the letter or letters represent. [Solution of the equations is not required.]

    a The rate of a motor boat in still water is 12 miles per hour. If the rate of the current is 3 miles per hour, how far upstream can the boat travel before returning to its starting point if the time allowed for the entire trip is 4 hours? [5]

    b In an arithmetic progression the common difference is 2. If the sum of the first four terms of the progression is divided by the third term, the quotient is 3 and the remainder is 5. Find the first term. [5]

33 A man has a sum of money invested in bonds which yield interest at the rate of 4%. He borrows for one year an amount of money equal to $\frac{2}{3}$ the value of the bonds and pays interest at the rate of 5%. His net return for the year amounts to $27. How much money did he have invested in bonds? [5, 5]

Part IV
Answer one question from part IV.

34 *a* If $y = mx + b$ represents the line l through points A (2, 4) and B (7, 7), find the slope and the y-intercept of l. [4]

    b Write the equation of line l. [2]

    c Write the equation of the line l' which is parallel to l and has a y-intercept equal to —3. [2]

    d Find to the *nearest degree* the acute angle that l' makes with the x-axis. [2]

35 *a* A solution of water and alcohol is 90% alcohol. Express in terms of n the amount of water that must be added to n gallons of this solution to make it 80% alcohol. [5]

    b A solution of water and alcohol is 80% alcohol. Express in terms of n the amount of alcohol that must be added to n gallons of this solution to make it 90% alcohol. [5]

Fill in the following lines:

Name of pupil ....Name of school ....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Write the three factors of $2x^2 - 8$.

1................

2 Express $\dfrac{5}{3-\sqrt{2}}$ as an equivalent fraction with a rational denominator.

2................

3 Express in terms of i the sum of $\sqrt{-12}$ and $\sqrt{-3}$. [Write answer in simplest form.]

3................

4 Solve the following formula for a: $K = \frac{1}{2}h\,(a + b)$.

4................

5 Write the equation which expresses the relationship between x and y shown in the following table:

x	-2	0	2	4
y	-7	-1	5	11

5................

6 The tens digit of a two-digit number is represented by t and the units digit by u. Express in terms of t and u the number divided by the sum of the digits.

6................

7 Express $\dfrac{1 - \dfrac{1}{a}}{1 + \dfrac{1}{a}}$ as a fraction reduced to lowest terms.

7................

8 Find the value of $2x^{\frac{2}{3}} + x^{-1}$ when $x = 8$.

8................

9 If x varies directly as y and $x = 16$ when $y = 12$, find x when $y = 21$.

9................

10 If $x = \dfrac{12}{y}$ and y is positive, does x increase or does it decrease as y increases?

10................

11 If y yards of material can be bought for d dollars, how many yards can be bought for k dollars?

11................

12 A wire stretched from the top of the mast of a boat to point A on the deck makes an angle of $56°$ with the deck. If the distance from point A to the foot of the mast is 19 feet, find to the *nearest foot* the height of the mast.

12................

13 Find the logarithm of 45.76

13................

[3]   [OVER]

14 Find the number whose logarithm is 3.8380

14. . . . .

15 Express $\log \dfrac{x^3}{y}$ in terms of $\log x$ and $\log y$.

15. . . . .

16 One root of the equation $x^2 - 5x + c = 0$ is 3. Find the other root.

16. . . . .

17 The roots of the equation $ax^2 + bx + c = 0$ are real, rational and unequal. What integer between 13 and 20 may be the value of the discriminant of the equation?

17. . . . .

18 If the roots of the equation $ax^2 + bx + c = 0$ are real and equal, the graph of the equation $y = ax^2 + bx + c$ is tangent to the x-axis. Is this statement true or is it false?

18. . . . .

19 Solve for x: $5 + \sqrt{4x + 3} = 7$

19. . . . .

20 If the abscissa of every point on a line is 2, write the equation of the line.

20. . . . .

21 Find the sum of the first 20 terms of the progression $3, 6, 9, \ldots$

21. . . . .

22 Write the positive geometric mean between 6 and $\frac{2}{3}$.

22. . . . .

23 Find the common ratio of the infinite geometric progression whose first term is 3 and whose sum is 6.

23. . . . .

24 Find the third term of the expansion of $(x + y)^4$.

24. . . . .

25 The expression $a^2 - (a - x)(a + x) = x^2$ is an identity. Is this statement true or is it false?

25. . . . .