The University of the State of New York

290TH HIGH SCHOOL EXAMINATION

INTERMEDIATE ALGEBRA

Wednesday, January 19, 1944 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in intermediate algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of elementary algebra.

Part II

Answer three questions from part II.

26 If a body is thrown downward with an initial velocity of V feet per second, the distance S in feet traversed in t seconds is given by the formula $S = Vt + \frac{1}{2}gt^2$. Find the value of t, correct to the *nearest tenth*, when $S = 12$, $V = 20$ and $g = 32$. [10]

27 Solve the following set of equations, group your answers and check *one* set:

$$2x^2 + 3y^2 = 23$$
$$3x^2 - y^2 = 7 \quad [7, 2, 1]$$

28 Solve *each* of the following by the use of logarithms:

a Find the value of $750 \times (1.03)^6$. [5]

b Beginning at point A in a level field, a tunnel, AB, 367 feet long was dug, making an angle of 19° with the horizontal. Find, correct to the *nearest foot*, how far from A a vertical shaft should be sunk in order to meet the tunnel at B. [5]

29 *a* Draw the graph of $x^2 - 2x - 3 = y$ [6]

b From the graph made in answer to *a*, read the roots of the following equations: (1) $x^2 - 2x - 3 = 0$, (2) $x^2 - 2x - 3 = -3$, (3) $x^2 - 2x - 3 = -4$ [1, 1, 1]

c Give *one* value of y for which the roots of $x^2 - 2x - 3 = y$ will be imaginary. [1]

*30 Solve for x, y and z:

$$2x + 3y = 2$$
$$8x - 4z = 3$$
$$3y - 8z = -1 \quad [10]$$

* This question is based on one of the optional topics in the syllabus.

Part III

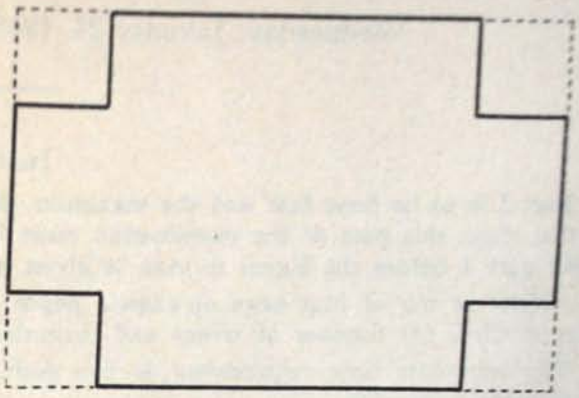
Answer one question from part III.

31 Write the equations that would be used in solving the following problems. In *each* case state what is represented by the letter or letters you use. [Solution of the equations is not required.]

a A certain grade of gun metal (a mixture of tin and copper) contains 16% tin. How much tin must be added to 410 pounds of this gun metal to make a mixture that is 18% tin? [5]

b A ship was loaded with 2000 tons of coal. If 50 tons more had been put on per hour, it would have taken 2 hours less time to load the ship. How long did it take to load the ship? [5]

32 The length of a rectangular sheet of tin is 4 inches greater than its width. From each corner a piece 2 inches square is cut out. The tin is then folded to form a box with open top. If the box contains 64 cubic inches, what were the dimensions of the sheet of tin? [10]



Part IV

Answer one question from part IV.

33 *a* Write .000059 as the product of 5.9 and a power of 10. [2]

b Write the equation of a parabola whose axis of symmetry is the y axis. [2]

c Express as a power of 2 the quotient of 4^x divided by 2^y. [2]

d If in the formula $r = \frac{K}{c}$, K is a constant and $r = 500$ when $c = 10$, find r when $c = 50$. [2]

e Give the value of a for which the graph of $x = a$ is tangent to the graph of $y^2 = x - 2$. [2]

34 *a* Derive the formula for the sum S of an arithmetic progression in terms of the first term a, the last term l and the number of terms n. [6]

b Using the formula derived in answer to part *a* of this question, derive the formula for S in terms of a, n and the common difference d. [1]

c By use of the formula derived in answer to *b*, show that the sum of the first n positive odd integers is n^2. [3]

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Write the factors of $ax^2 - 9a$ 1.....

2 Find the missing term in the progression 2, —6, 18, ..., 162 2.....

3 Find the *17th* term in the progression 3, 7, 11, ... 3.....

4 Write the value of $27^{-\frac{1}{3}}$ 4.....

5 Solve for h: $V = \frac{\pi r^2 h}{3}$ 5.....

6 Solve the equation: $\sqrt{x^2 + 4} = x + 1$ 6.....

7 What values of x satisfy the equation $2x^2 - 3x - 2 = 0$? 7.....

8 Write $\sqrt{-9x^2}$ in terms of the imaginary unit i. 8.....

9 Write $\frac{3}{\sqrt{5}}$ as an equivalent fraction with a rational denominator. 9.....

10 Write the *second* term of the expansion of $(2x - y)^4$ 10.....

11 The units digit of a two-digit number is a and the tens digit is b. Represent the number in terms of a and b. 11.....

12 On a certain night, to determine the ceiling over an airport, a ceiling light projector threw a spotlight vertically on the underside of a cloud. At a distance of 500 feet from the projector, the angle of elevation of the spot of light on the cloud was found to be 66°. What was the ceiling (height of the cloud)? 12.....

13 If y varies directly as x and if $y = 2$ when $x = 5$, what is the value of y when $x = 15$? 13.....

14 Find, correct to the *nearest integer*, the number whose logarithm is 3.6487 14.....

15 If $10^x = 78.36$, find x correct to *four decimal places*. 15.....

Directions (questions 16–21) — Indicate the correct answer to *each* question by writing on the line at the right the letter a, b, c or d.

16 The expression $\frac{2}{\sqrt{3} - 1}$ equals (*a*) $\frac{2\sqrt{3} + 1}{2}$, (*b*) $\sqrt{3} + 1$, (*c*) $\frac{\sqrt{3} + 1}{2}$, (*d*) $\sqrt{3}$ 16.....

17 The sum of $\frac{1}{a - b}$ and $\frac{1}{b - a}$ equals (*a*) 0, (*b*) 1, (*c*) $\frac{2}{a - b}$, (*d*) $\frac{2}{b - a}$ 17.....

18 The sum of the roots of the equation $3x^2 - 5x + 4 = 0$ is
(*a*) $\frac{5}{3}$, (*b*) $-\frac{5}{3}$, (*c*) $\frac{4}{3}$, (*d*) 5 18

19 The expression $(a^2)^{-3}$ equals (*a*) a^{-6}, (*b*) a^{-5}, (*c*) a^{-1}, (*d*) a^{-9} 19

20 The slope of the graph of $2x + 3y = 5$ is
(*a*) -2, (*b*) $\frac{2}{3}$, (*c*) $-\frac{2}{3}$, (*d*) $\frac{1}{3}$ 20

21 The roots of the equation $2x^2 - 3x + 3 = 0$ are (*a*) real, equal and rational, (*b*) real, unequal and irrational, (*c*) real, unequal and rational, (*d*) imaginary 21

Directions (questions 22–25) — State whether each of the statements is true or false.

22 The product of two irrational numbers is always irrational. 22

23 The graphs of the two equations $2x - 3y = 4$ and $6x = 9y + 15$ are parallel. 23

24 The mantissa of a logarithm is always positive. 24

25 The graph of $ax^2 + by^2 = c$ is always a circle. 25