The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

INTEGRATED ALGEBRA

Thursday, August 13, 2009 — 8:30 to 11:30 a.m., only

Student Name: _Steve Watson_

School Name: _IHS @ PH_

Print your name and the name of your school on the lines above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice…

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 30 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [60]

Use this space for computations.

1 If h represents a number, which equation is a correct translation of "Sixty more than 9 times a number is 375"?

(handwritten: +60 + 9x h =)

(1) $9h = 375$

(3) $9h - 60 = 375$

(2) $9h + 60 = 375$ *(circled)*

(4) $60h + 9 = 375$

2 Which expression is equivalent to $9x^2 - 16$?

(1) $(3x + 4)(3x - 4)$ *(circled)*

(3) $(3x + 8)(3x - 8)$

(2) $(3x - 4)(3x - 4)$

(4) $(3x - 8)(3x - 8)$

(handwritten: Difference of perfect squares)
$$a^2 - b^2 = (a+b)(a-b)$$
$$9x^2 - 16 = (3x+4)(3x-4)$$

3 Which expression represents $(3x^2y^4)(4xy^2)$ in simplest form?

(1) $12x^2y^8$

(3) $12x^3y^8$

(2) $12x^2y^6$

(4) $12x^3y^6$ *(circled)*

(handwritten:)
$$(3)(4)(x^2)(x)(y^4)(y^2)$$
$$(3 \times 4)(x^{2+1})(y^{4+2})$$
$$12 \quad x^3 \quad y^6$$

4 An online music club has a one-time registration fee of $13.95 and charges $0.49 to buy each song. If Emma has $50.00 to join the club and buy songs, what is the maximum number of songs she can buy?

(handwritten: Let x = a song)

(1) 73 *(circled)*

(3) 130

(2) 74

(4) 131

(handwritten:)
$$13.95 + 0.49x \leq 50.00$$
$$0.49x \leq 50 - 13.95$$
$$0.49x \leq 36.05$$
$$x \leq \frac{36.05}{.49}$$
$$x \leq 73.571428...$$
round down

(handwritten: Check 73 × 49¢ = $35.77 + registration 13.95 Total $49.72 Emma has 28¢ left, which is not enough for another song)

two (no vanilla) ↙

5 The local ice cream stand offers three flavors of soft-serve ice cream: vanilla, chocolate, and strawberry; two types of cone: sugar and wafer; and three toppings: sprinkles, nuts, and cookie crumbs. If Dawn does not order vanilla ice cream how many different choices can she make that have one flavor of ice cream, one type of cone, and one topping?
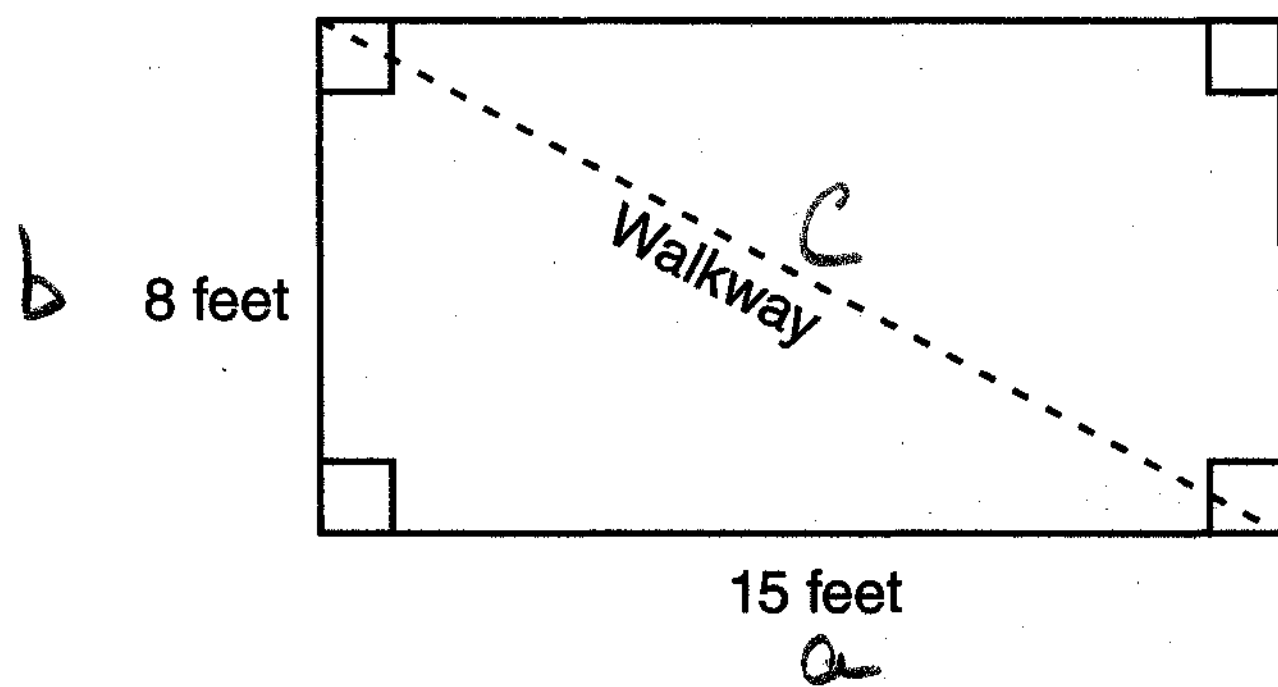
(1) 7   (3) 12

(2) 8   (4) 18

# of choices of

Flavors TypeCones Toppings   Total Choices

$$\boxed{2} \times \boxed{2} \times \boxed{3} = 12$$

6 Nancy's rectangular garden is represented in the diagram below.

b 8 feet   *Walkway C*

15 feet

a

If a diagonal walkway crosses her garden, what is its length, in feet?
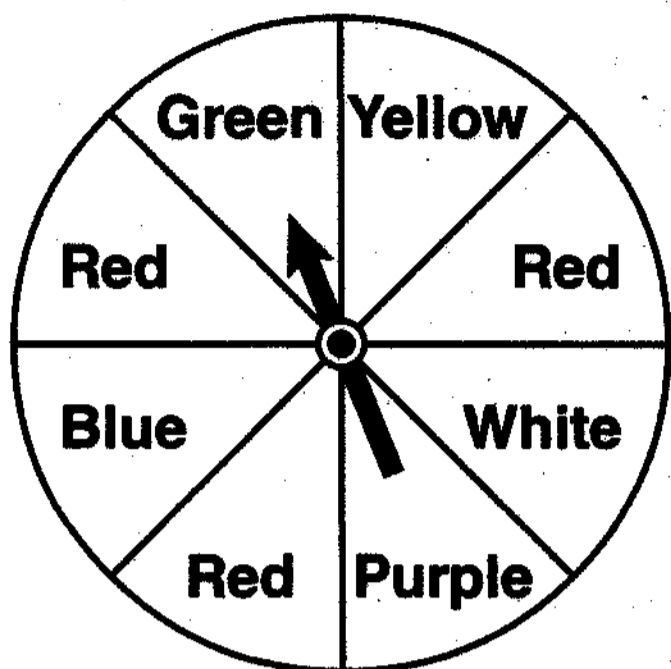
(1) 17   (3) $\sqrt{161}$

(2) 22   (4) $\sqrt{529}$

Pythagorean Theorem

$$a^2 + b^2 = c^2$$
$$15^2 + 8^2 = c^2$$
$$225 + 64 = c^2$$
$$289 = c^2$$
$$\sqrt{289} = \sqrt{c^2}$$
$$17 = c$$

7 The spinner below is divided into eight equal regions and is spun once. What is the probability of *not* getting red?

(1) $\dfrac{3}{5}$

(3) $\dfrac{5}{8}$

(2) $\dfrac{3}{8}$

(4) $\dfrac{7}{8}$

Yellow   1
Red   3
white   1
Purple   1
Blue   1
Green   1
          ___
Total   8

$P_{(red)} = \dfrac{3}{8}$

$P_{(not\,red)} = \dfrac{5}{8}$

8 Which relationship can best be described as causal?

(1) height and intelligence

(2) shoe size and running speed

(3) number of correct answers on a test and test score

(4) number of students in a class and number of students with brown hair

Correct answers cause the score to go up.

9 Solve for *x*: $\dfrac{3}{5}(x+2) = x - 4$

(1) 8

(3) 15

(2) 13

(4) 23

Check

$\dfrac{3}{5}(13 \times 2) = 13 - 4$

$\dfrac{3}{5}(15) = 9$

$\dfrac{45}{5} = 9$

$9 = 9$ ✓

$\dfrac{3}{5}(x+2) = x - 4$

$\dfrac{3}{5}(x) + \dfrac{3}{5}(2) = x - 4$

$\dfrac{3}{5}\left(\dfrac{x}{1}\right) + \dfrac{3}{5}\left(\dfrac{2}{1}\right) = x - 4$

$\dfrac{3x}{5} + \dfrac{6}{5} = x - 4$

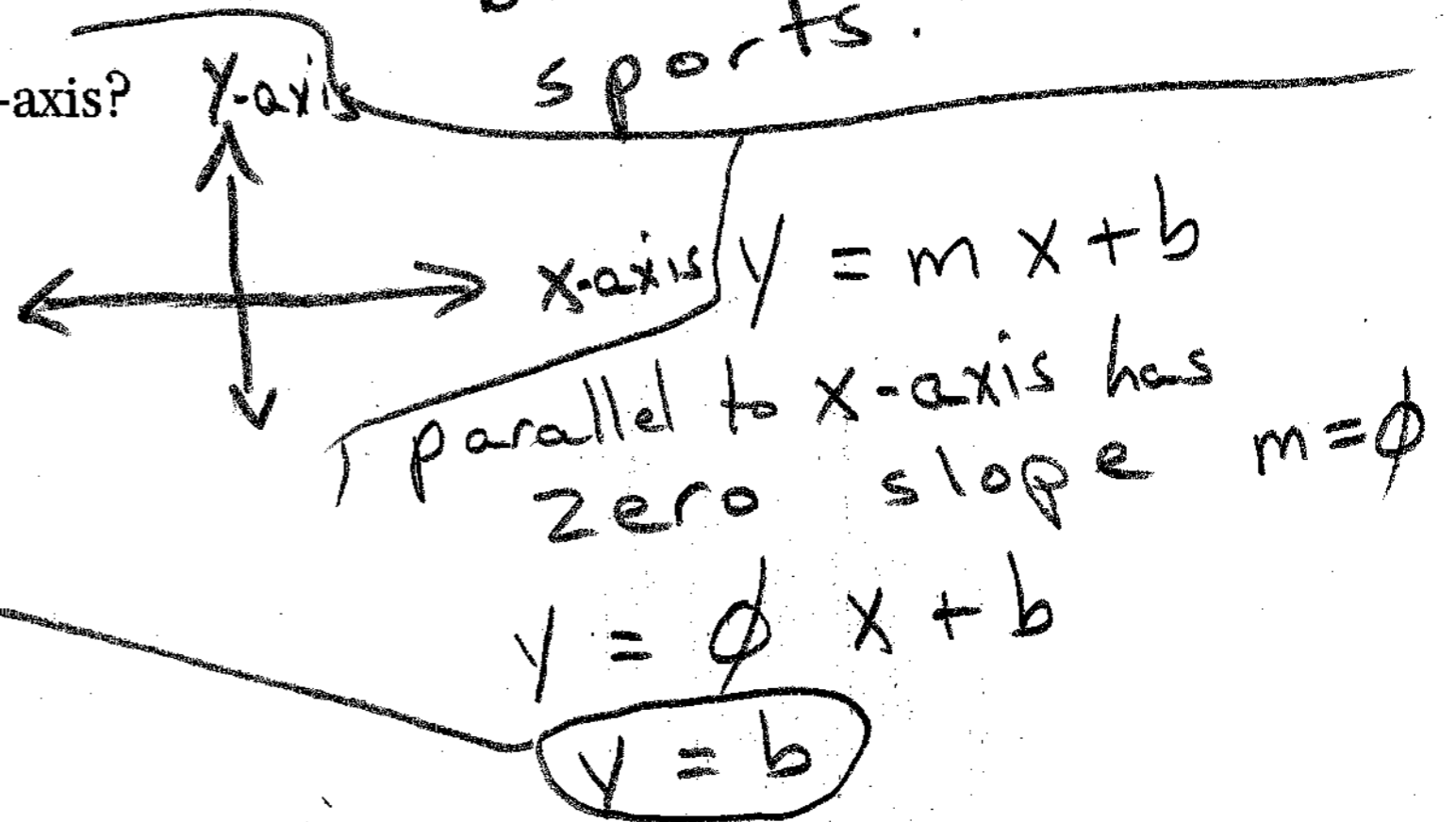M(5) $3x + 6 = 5(x-4)$

$3x + 6 = 5x - 20$

$26 = 2x$

$13 = x$

10 Erica is conducting a survey about the proposed increase in the sports budget in the Hometown School District. Which survey method would likely contain the *most bias*?

(1) Erica asks every third person entering the Hometown Grocery Store.

(2) Erica asks every third person leaving the Hometown Shopping Mall this weekend.

(3) Erica asks every fifth student entering Hometown High School on Monday morning.

(4) Erica asks every fifth person leaving Saturday's Hometown High School football game.

← These people are probably biased for school sports.

11 Which equation represents a line parallel to the *x*-axis?
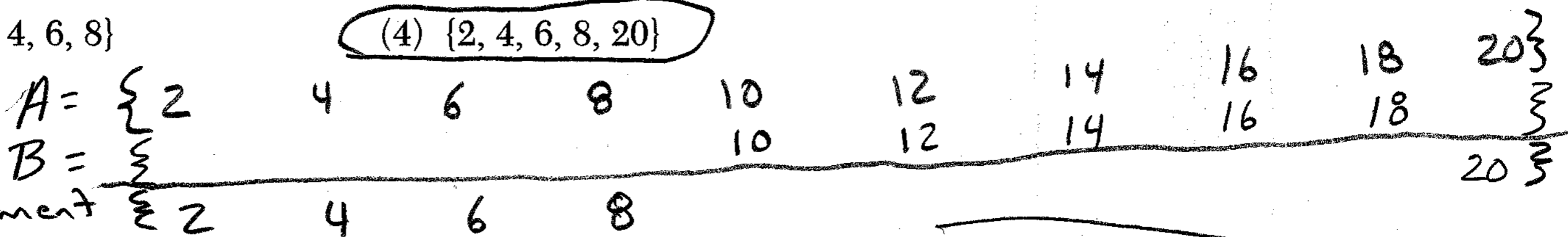
(1) $y = -5$

(2) $y = -5x$

(3) $x = 3$

(4) $x = 3y$

y-axis

x-axis $y = mx + b$

parallel to x-axis has zero slope m = 0

$y = 0 \cdot x + b$

$y = b$

12 Given:

$A = \{$All even integers from 2 to 20, inclusive$\}$

$B = \{10, 12, 14, 16, 18\}$

What is the complement of set *B* within the universe of set *A*?

(1) $\{4, 6, 8\}$

(2) $\{2, 4, 6, 8\}$

(3) $\{4, 6, 8, 20\}$

(4) $\{2, 4, 6, 8, 20\}$

$A = \{2 \quad 4 \quad 6 \quad 8 \quad 10 \quad 12 \quad 14 \quad 16 \quad 18 \quad 20\}$

$B = \{ \quad\quad\quad\quad\quad\quad 10 \quad 12 \quad 14 \quad 16 \quad 18 \quad 20\}$

Complement $\{2 \quad 4 \quad 6 \quad 8\}$

13 Which value of *x* is in the solution set of the inequality $-2(x - 5) < 4$?

(1) 0

(2) 2

(3) 3

(4) 5

$-2(x-5) < 4$

$-2x + 10 < 4$

$-2x \quad\quad < 4 - 10$

$-2x \quad\quad < -6$

$x \quad\quad > 3$ Note sign change

$D(-2) \quad x$

Check $5 > 3$ ✓

14 A tree casts a 25-foot shadow on a sunny day, as shown in the diagram below.

If the angle of elevation from the tip of the shadow to the top of the tree is 32°, what is the height of the tree to the *nearest tenth of a foot*?

(1) 13.2

(2) 15.6

(3) 21.2

(4) 40.0

Handwritten work:
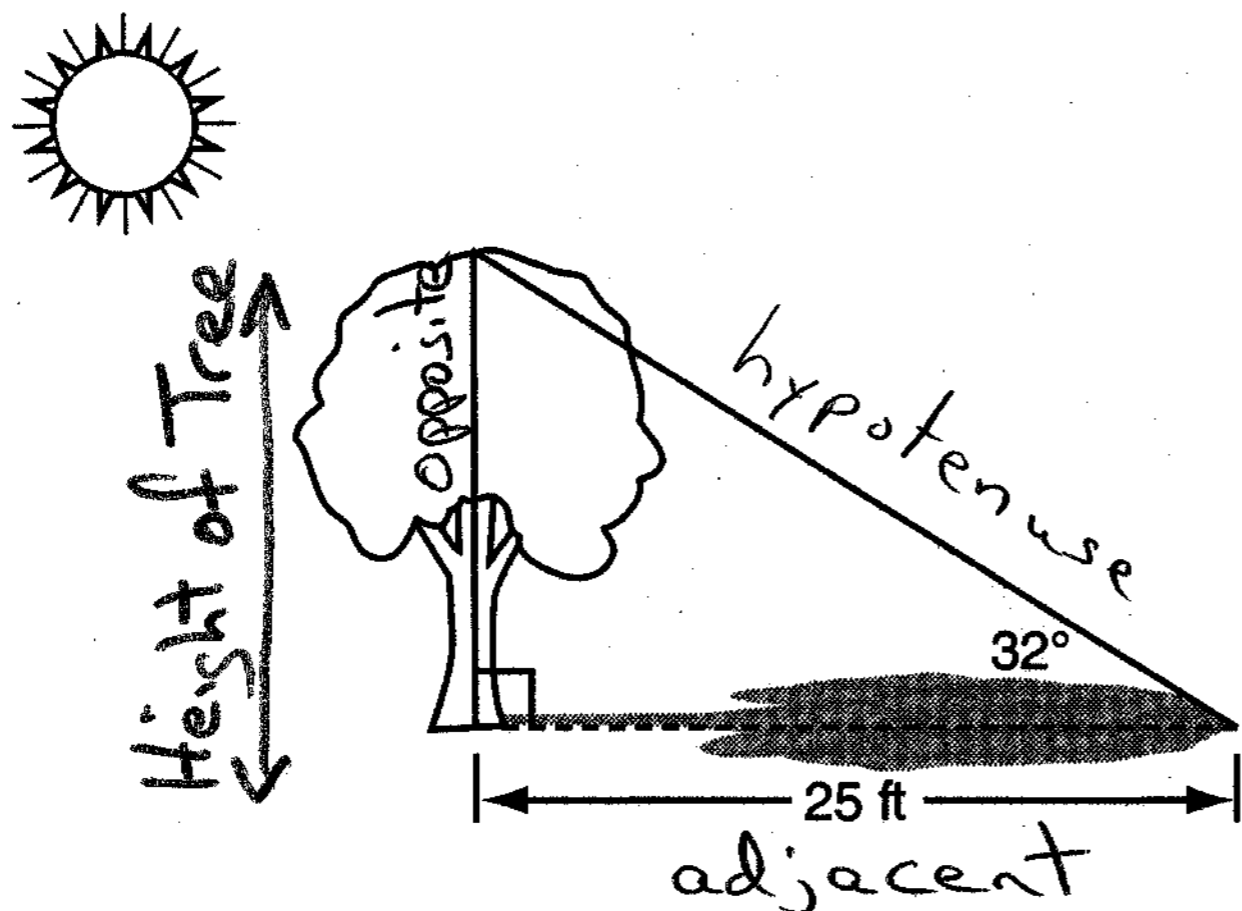
SOH - CAH - TOA

$\tan = \dfrac{opposite}{adjacent}$

$\tan 32° = \dfrac{opposite}{25}$

$25(\tan 32°) = opposite$

Remember to set calculator mode to degrees

$25(\tan 32°) = 15.6217338$

---

15 What is the slope of the line that passes through the points (−5,4) and (15,−4)?

(1) $-\dfrac{2}{5}$

(2) 0

(3) $-\dfrac{5}{2}$

(4) undefined

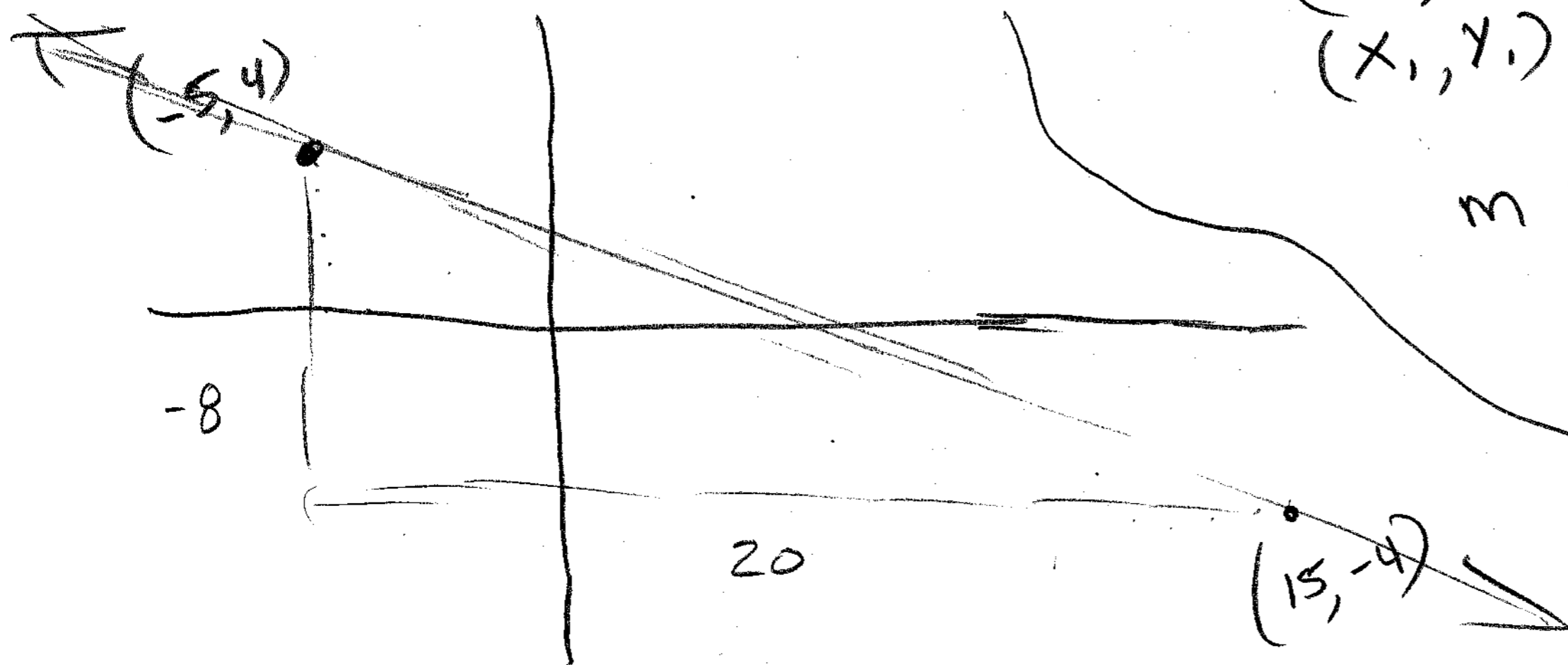*Handwritten work:*

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$\underset{(x_1, y_1)}{(-5, 4)} \quad \underset{(x_2, y_2)}{(15, -4)}$

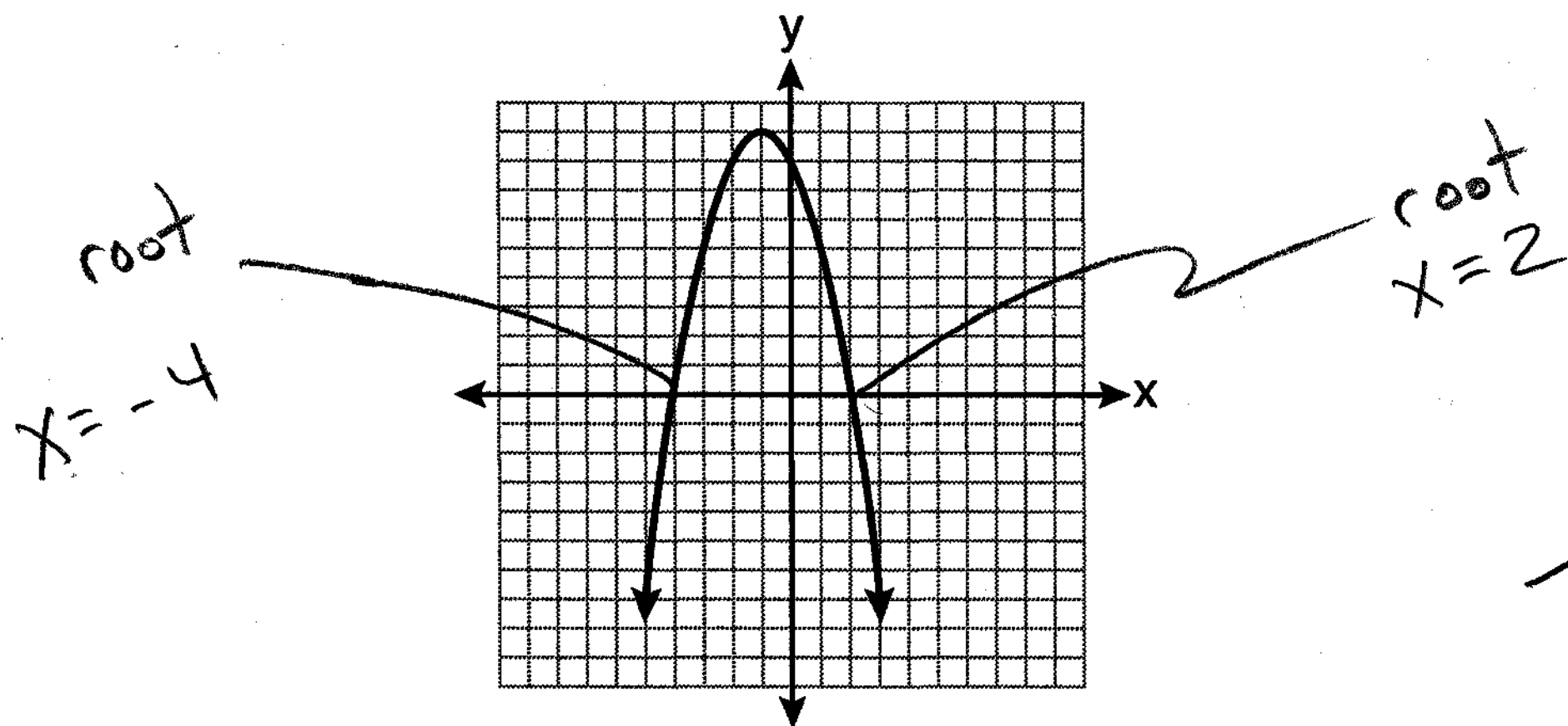$m = \dfrac{-4 - (4)}{15 - (-5)}$

$m = \dfrac{-8}{20}$

$m = \dfrac{-2}{5}$

$slope = \dfrac{rise}{run} = \dfrac{-8}{20} = \dfrac{-4}{10} = \dfrac{-2}{5}$

16 The equation $y = -x^2 - 2x + 8$ is graphed on the set of axes below.

root
x = -4

root
x = 2

The roots are where the graph crosses the x-axis.

Based on this graph, what are the roots of the equation $-x^2 - 2x + 8 = 0$?

(1) 8 and 0

(3) 9 and −1

(2) 2 and −4

(4) 4 and −2

add

17 What is the sum of $\frac{3}{2x}$ and $\frac{4}{3x}$ expressed in simplest form?

(1) $\frac{12}{6x^2}$

(3) $\frac{7}{5x}$

(2) $\frac{17}{6x}$

(4) $\frac{17}{12x}$

$$\frac{3}{2x} + \frac{4}{3x}$$

$$\frac{3(3x) + 2x(4)}{(2x)(3x)}$$

$$\frac{9x + 8x}{6x^2}$$

$$\frac{17x}{6x^2}$$

$$\frac{17}{6x}$$

18 Which value of x makes the expression $\dfrac{x^2 - 9}{x^2 + 7x + 10}$ undefined?

(1) –5

(3) 3

(2) 2

(4) –3

check
$(-5)^2 + 7(-5) + 10 = 0$
$25 - 35 + 10 = 0$
$0 = 0 ✓$

undefined occurs when you divide by zero.

Let $x^2 + 7x + 10 = 0$
$(x + 5)(x + 2) = 0$
$x + 5 = 0$ or $x + 2 = 0$
$x = -5$ $\qquad x = -2$

19 Which relation is *not* a function?

(1) {(1,5), (2,6), (3,6), (4,7)}

(2) {(4,7), (2,1), (–3,6), (3,4)}

(3) {(–1,6), (1,3), (2,5), (1,7)}

(4) {(–1,2), (0,5), (5,0), (2,–1)}

A function has one and only one value of y for each value of x.

20 What is the value of the y-coordinate of the solution to the system of equations $x - 2y = 1$ and $x + 4y = 7$?

(1) 1

(3) 3

(2) –1

(4) 4

$\begin{aligned} x - 2y &= 1 \\ \text{Subtract} \rightarrow x + 4y &= 7 \end{aligned}$
$-6y = -6$
$y = 1$

21 The solution to the equation $x^2 - 6x = 0$ is

(1) 0, only

(3) 0 and 6

(2) 6, only

(4) $\pm\sqrt{6}$

$x^2 - 6x = 0$
$x(x - 6) = 0$
$x = 0$ or $x - 6 = 0$
$\qquad\qquad x = 6$

Check

for 0
$0^2 - 6(0) = 0$
$0 = 0 ✓$

for 6
$(6)^2 - 6(6) = 0$
$36 - 36 = 0$
$0 = 0 ✓$

22 When $5\sqrt{20}$ is written in simplest radical form, the result is $k\sqrt{5}$. What is the value of k?

(1) 20

(2) 10

(3) 7

(4) 4

(2) circled)

$5\sqrt{20}$

$5\sqrt{4}\sqrt{5}$

$5(2)\sqrt{5}$

$10\sqrt{5}$

23 What is the value of the expression $|-5x + 12|$ when $x = 5$?

(1) −37

(2) −13

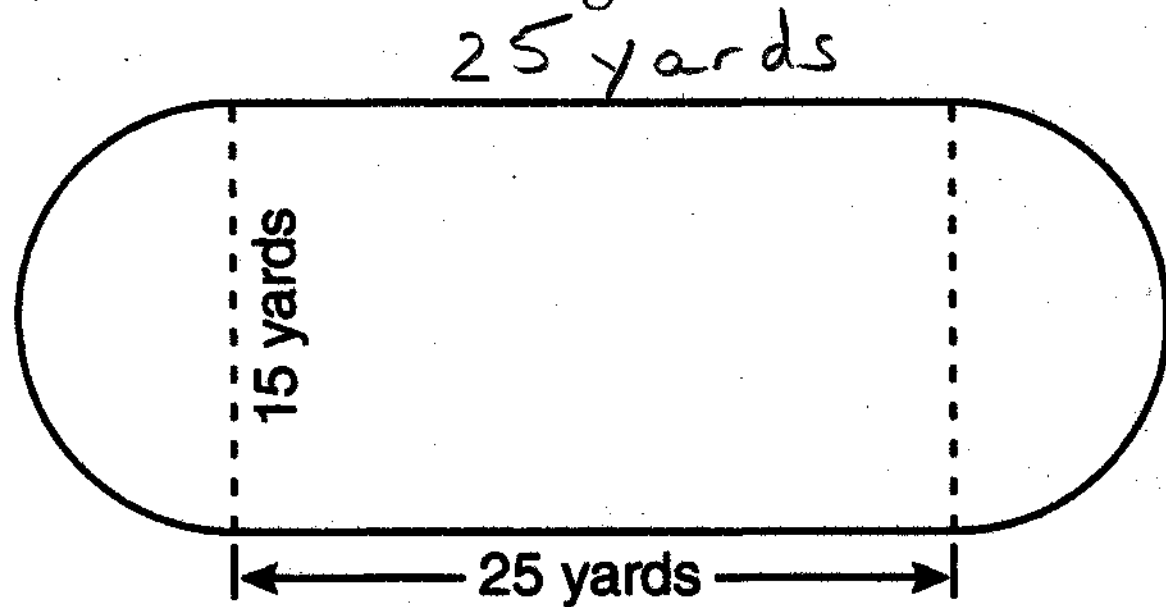(3) 13

(4) 37

(3) circled)

$|-5x+12|$

$|-5(5)+12|$

$|-25+12|$

$|-13|$

13

24 A playground in a local community consists of a rectangle and two semicircles, as shown in the diagram below.

25 yards

15 yards

25 yards

Which expression represents the amount of fencing, in yards, that would be needed to completely enclose the playground?

(1) $15\pi + 50$

(2) $15\pi + 80$

(3) $30\pi + 50$

(4) $30\pi + 80$

(1) circled)

We need enough fencing to go around two semi-circles (or one whole circle) plus two more lengths of 25 yards each for the sides.

The Circle Part
$C = \pi d$
$d = 15$
$C = 15\pi$

The Sides
$25 + 25 = 50$

Add them together
$15\pi + 50$

25 Which equation is represented by the graph below?
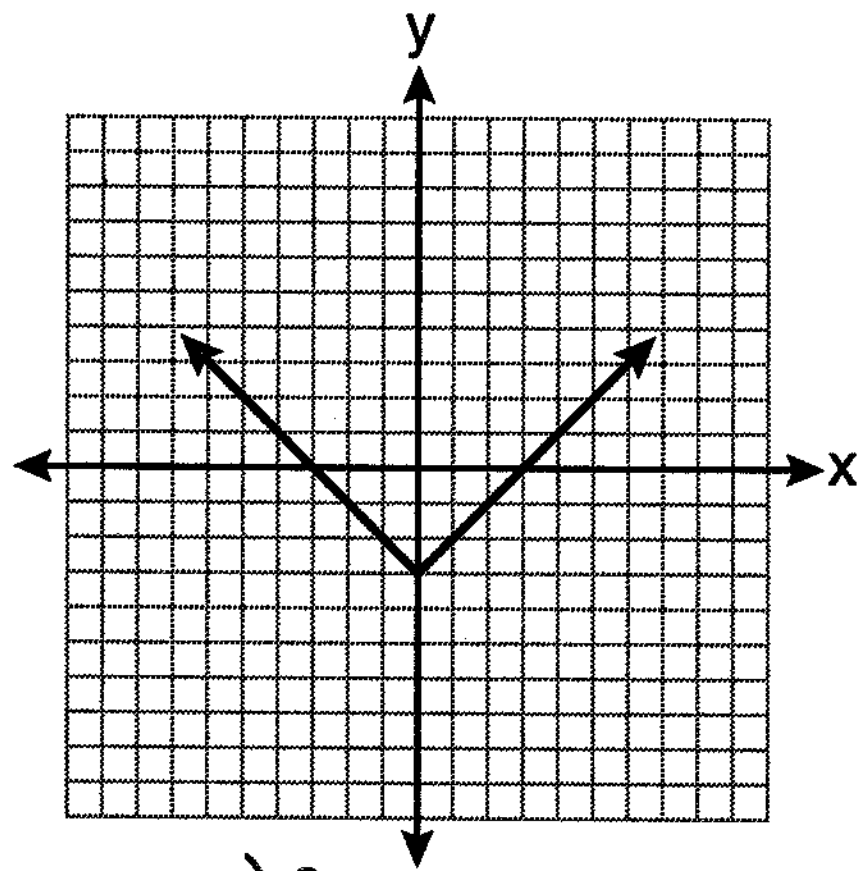
(1) $y = x^2 - 3$ ~~parabola~~

(2) $y = (x - 3)^2$

(3) $y = |x| - 3$

(4) $y = |x - 3|$

Handwritten table:

X	Y
-3	0
-2	-1
-1	-2
0	-3
1	-2
2	-1
3	0

Handwritten note: Use Graphing Calculator

26 Carrie bought new carpet for her living room. She calculated the area of the living room to be 174.2 square feet. The actual area was 149.6 square feet. What is the relative error of the area to the *nearest ten-thousandth*?

(1) 0.1412

(2) 0.1644

(3) 1.8588

(4) 2.1644

Handwritten:

$$\frac{Actual - Measured}{Actual}$$

$$\frac{174.2 - 149.6}{149.6}$$

$$\frac{24.6}{149.6} = .1644385027$$

27 What is an equation of the line that passes through the point (3, –1) and has a slope of 2?

(1) $y = 2x + 5$

(2) $y = 2x - 1$

(3) $y = 2x - 4$

(4) $y = 2x - 7$

Handwritten:

$y = mx + b$

$y = -1$
$m = 2$
$x = 3$
$b =$

$-1 = 2(3) + b$
$-1 = 6 + b$
$-7 = b$

$y = mx + b$

$y = 2x - 7$

Check
$y = 2x - 7$
$-1 = 2(3) - 7$
$-1 = 6 - 7$
$-1 = -1$ ✓

28 The ages of <u>three</u> brothers are <u>consecutive even integers.</u> Three times the age of the youngest brother exceeds the oldest brother's age by 48 years. What is the age of the *youngest* brother?

(1) 14   (3) 22

(2) 18   (4) 26 ⟵circled

Handwritten computations:

Let X = 1st bro's age
Let X+2 = 2nd brother
Let X+4 = 3rd brother

$$3(X) = (X+4) + 48$$
$$3X = X + 52$$
$$2X = 52$$
$$X = 26$$

Check
$$78 = 30 + 48$$
$$78 = 78 ✓$$

29 Cassandra bought an antique dresser for $500. If the value of her dresser increases 6% annually, what will be the value of Cassandra's dresser at the end of 3 years to the *nearest dollar*?

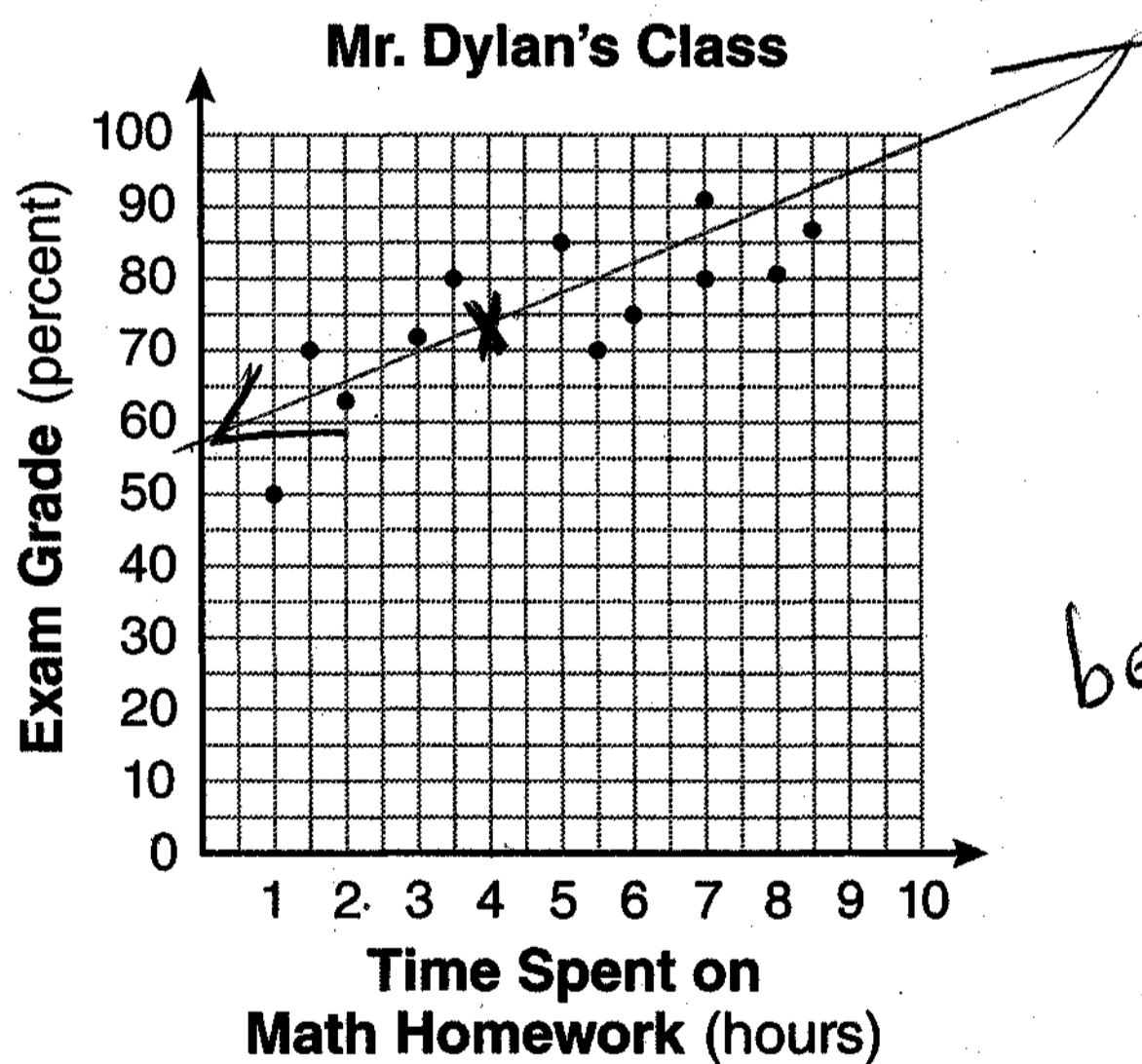(1) $415   (3) $596 ⟵circled

(2) $590   (4) $770

Handwritten computations:

$$Value = 500 \times (1.06) \times (1.06) \times (1.06)$$
$$Value = 500 (1.06)^3$$
$$Value = \$595.508$$
$$Value = \$596$$

30 The number of hours spent on math homework each week and the final exam grades for twelve students in Mr. Dylan's algebra class are plotted below.

Mr. Dylan's Class

Handwritten note:

When X = 4, y should be somewhere between 70 and 75.

Based on a line of best fit, which exam grade is the best prediction for a student who spends about <u>4 hours</u> on math homework each week?

(1) 62   (3) 82

(2) 72 ⟵circled   (4) 92

Answer all 3 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [6]

31 Chad complained to his friend that he had five equations to solve for homework. Are all of the homework problems equations? Justify your answer.
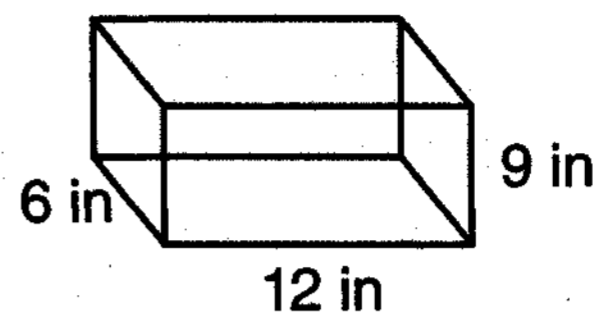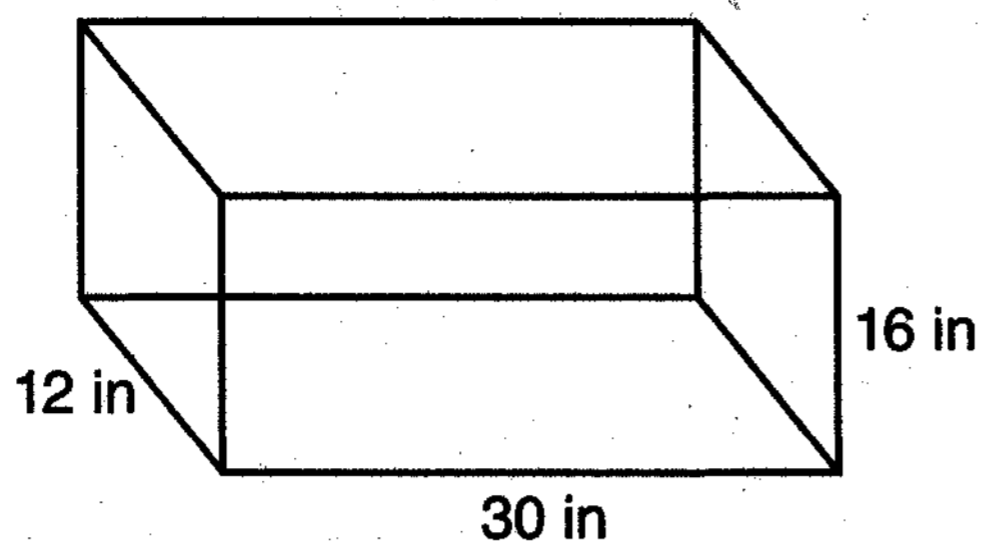
Math Homework

1. $3x^2 \cdot 2x^4$

2. $5 - 2x = 3x$

3. $3(2x + 7)$

4. $7x^2 + 2x - 3x^2 - 9$

5. $\dfrac{2}{3} = \dfrac{x + 2}{6}$

Name _Chad_

No

An equation is two mathematical expressions with an equal sign between them. Problems #1, 3, and 4 do not have equal signs and are not equations.

32 The diagram below represents Joe's two fish tanks.

Joe's larger tank is <u>completely filled</u> with water. He takes water from it to completely fill the small tank. Determine how many cubic inches of water will remain in the larger tank.

$$\text{Volume} = (\text{length})(\text{width})(\text{height})$$

<u>Big Tank</u>

$$V = (30)(12)(16)$$
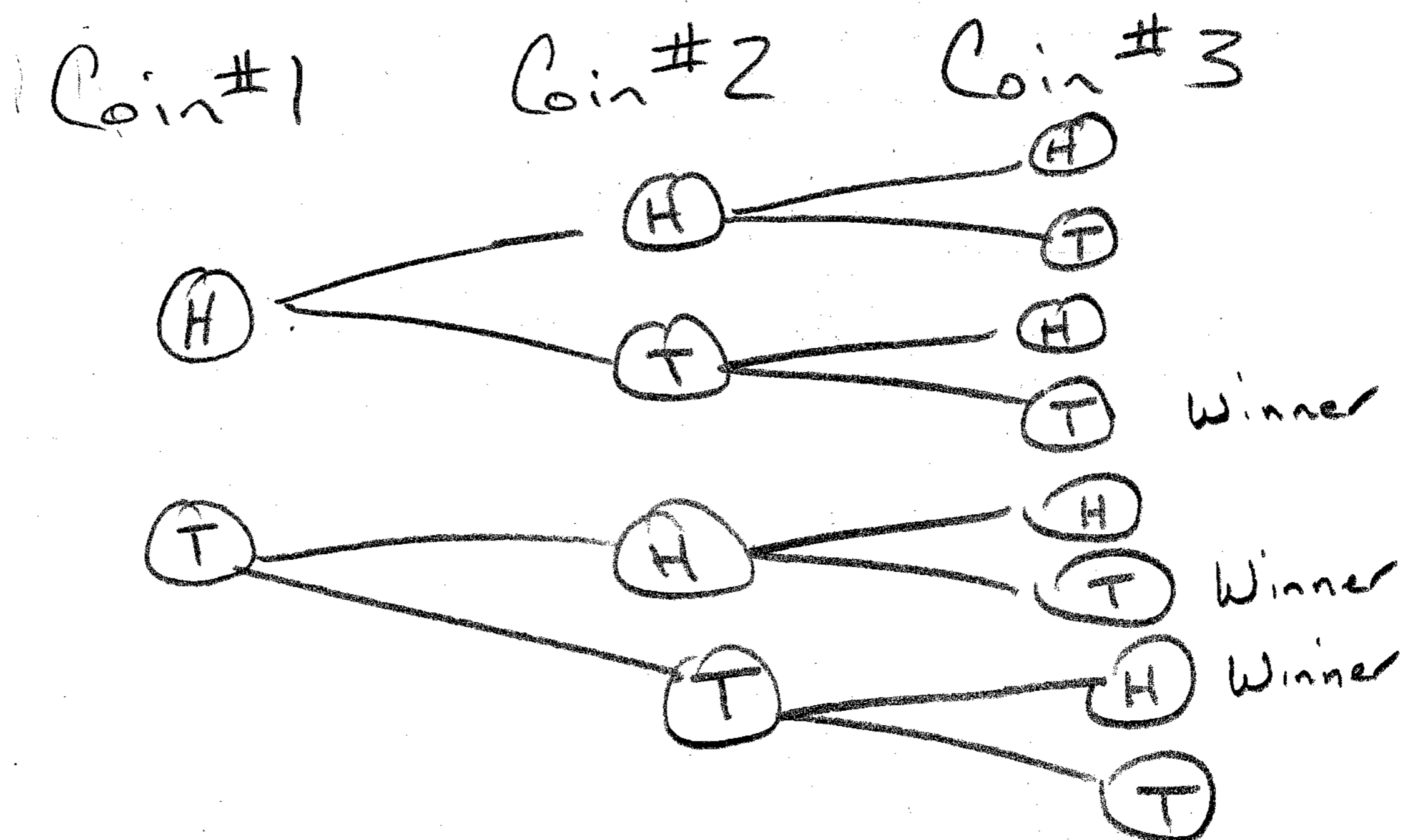
$$V = 5760$$

<u>Small Tank</u>

$$V = (12)(6)(9)$$

$$V = 648$$

Big Tank
− Small Tank

$$\begin{array}{r} 5760 \\ -\ 648 \\ \hline 5112 \end{array}$$

5,112 cubic inches of water
will remain in the big tank

33 Clayton has three fair coins. Find the probability that he gets two tails and one head when he flips the three coins.

Coin #1   Coin #2   Coin #3

H → H → H
H → H → T
H → T → H
H → T → T   Winner
T → H → H
T → H → T   Winner
T → T → H   Winner
T → T → T

There are 8 possible outcomes.
3 of the eight possible outcomes involve exactly 2 tails and 1 head.

The probability of getting 2 tails and 1 head is $\frac{3}{8}$

$$P_{(event)} = \frac{\#\ times\ event\ happens}{\#\ possible\ outcomes}$$

Answer all 3 questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.   [9]

34 Find algebraically the equation of the axis of symmetry and the coordinates of the vertex of the parabola whose equation is $y = -2x^2 - 8x + 3$.

axis of symmetry $\quad X = \dfrac{-b}{2a}$

standard form of a quadratic

$y = ax^2 + bx + c \quad$ or $\quad ax^2 + bx + c = 0$

$$y = -2x^2 - 8x + 3$$
$$\quad\quad\quad a \quad\quad\quad b \quad\quad\quad c$$

$$a = -2 \quad\quad b = -8 \quad\quad c = 3$$

$$X = \frac{-b}{2a} = \frac{-(-8)}{2(-2)} = \frac{8}{-4} = -2$$

The axis of symmetry is $\boxed{X = -2}$

To find the vertex, find the value of y at the axis of symmetry. State as a coordinate.

$$y = -2x^2 - 8x + 3$$
$$y = -2(-2)^2 - 8(-2) + 3$$
$$y = -2(4) + 16 + 3$$
$$y = -8 + 19$$
$$y = 11$$

$\boxed{\text{The vertex is at } (-2, 11)}$

Check using graphing calculator.

35 At the end of week one, a stock had increased in value from $5.75 a share to $7.50 a share. Find the <u>percent of increase</u> at the end of week one to the *nearest tenth of a percent*.

At the end of week two, the same stock had decreased in value from $7.50 to $5.75. Is the percent of decrease at the end of week two the same as the percent of increase at the end of week one? Justify your answer.

Week #1

$$\begin{array}{r} \$7.50 \\ -5.75 \\ \hline \$1.75 \end{array}$$

Up.

The stock went up $1.75

$$\frac{\text{Increase}}{\text{Starting Price}} \quad \frac{1.75}{5.75} = .3043478261$$

$$30.43478261 \%$$

$$30.4 \%$$

Week #2

$$\begin{array}{r} \$7.50 \\ -5.75 \\ \hline \$1.75 \end{array}$$

Down

The stock went down $1.75

$$\frac{\text{Decrease}}{\text{Starting Price}} \quad \frac{1.75}{7.50} = .2\overline{333}$$

$$23.\overline{33} \%$$

$$23.3 \%$$

The percent of decrease at the end of week two is <u>not</u> the same as the percent of increase at the end of week one. <u>Justification</u>: The starting prices in the two calculations are different, thus leading to different

results.

36 The chart below compares two runners.

Runner	Distance, in miles	Time, in hours
Greg	11	2
Dave	16	3

Based on the information in this chart, state which runner has the faster rate. Justify your answer.

$$\text{runner's rate} = \frac{\text{distance in miles}}{\text{time in hours}}$$

$$\text{Greg} = \frac{11}{2} = 5\tfrac{1}{2} \text{ miles per hour}$$

$$\text{Dave} = \frac{16}{3} = 5\tfrac{1}{3} \text{ miles per hour}$$

Justification

$5\tfrac{1}{2}$ is bigger than $5\tfrac{1}{3}$

Therefore, Greg has a faster rate than Dave.

Part IV

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

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37 Express in simplest form: $\dfrac{2x^2 - 8x - 42}{6x^2} \div \dfrac{x^2 - 9}{x^2 - 3x}$

$$\frac{2x^2 - 8x - 42}{6x^2} \div \frac{x^2 - 9}{x^2 - 3x}$$

$$\frac{2x^2 - 8x - 42}{6x^2} \times \frac{x^2 - 3x}{x^2 - 9}$$

$$\frac{(2)(x^2 - 4x - 21)}{(2)(3)(x^2)} \times \frac{(x)(x-3)}{(x+3)(x-3)}$$

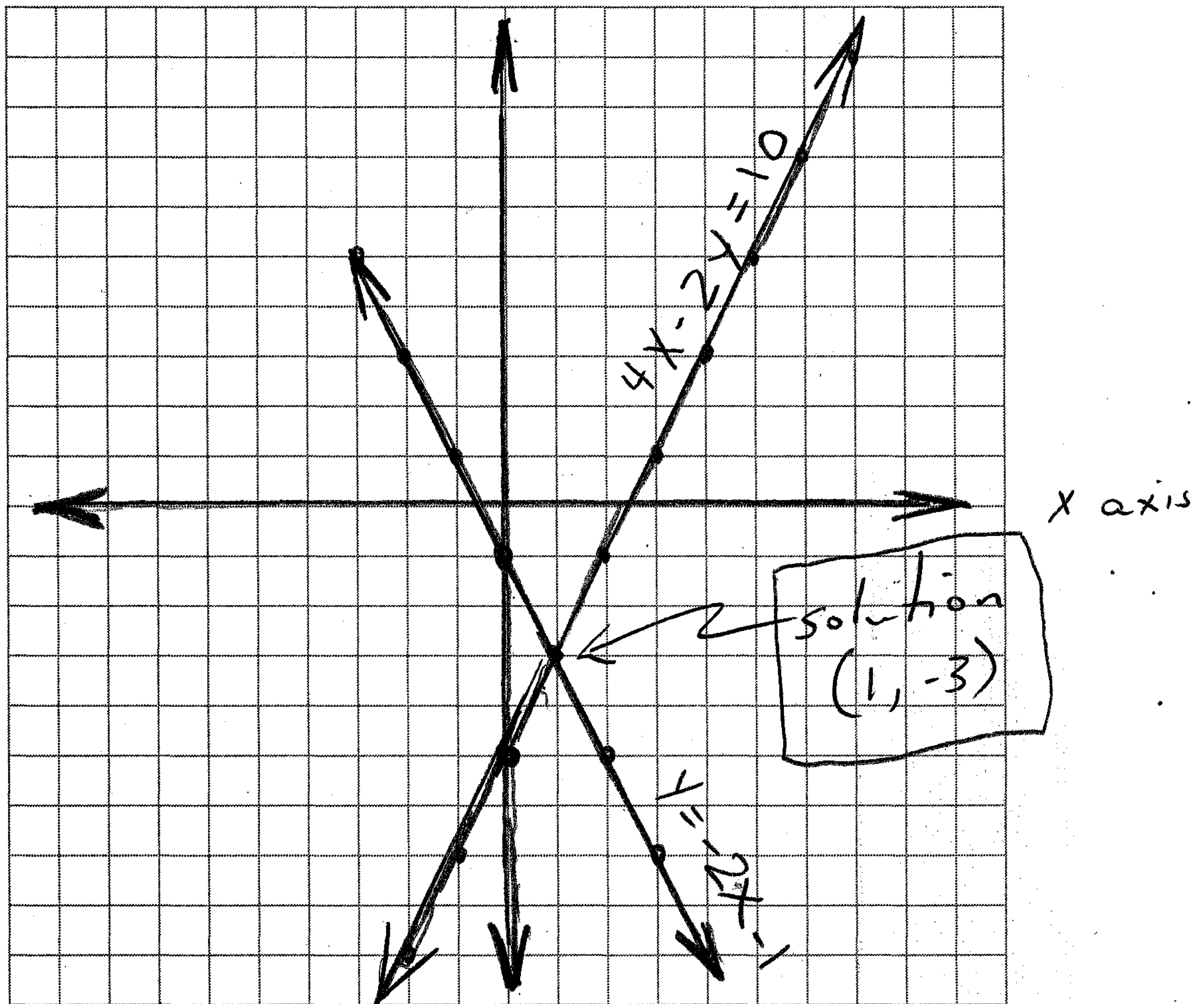$$\frac{(2)(x+3)(x-7)}{(2)(3)(x^2)} \times \frac{(x)(x-3)}{(x+3)(x-3)}$$

$$\frac{(x-7)(x)}{(3)(x)(x)}$$

$$\boxed{\frac{x-7}{3x}}$$

38 On the grid below, solve the system of equations graphically for x and y.

$$4X - 2y = 10$$
$$-2y = -4X + 10$$
$$y = 2X - 5$$

$$4x - 2y = 10$$
$$y = -2x - 1$$

$$y = -2X - 1$$

$$y = mx + b$$
$$y = 2X - 5$$
$$m = 2$$
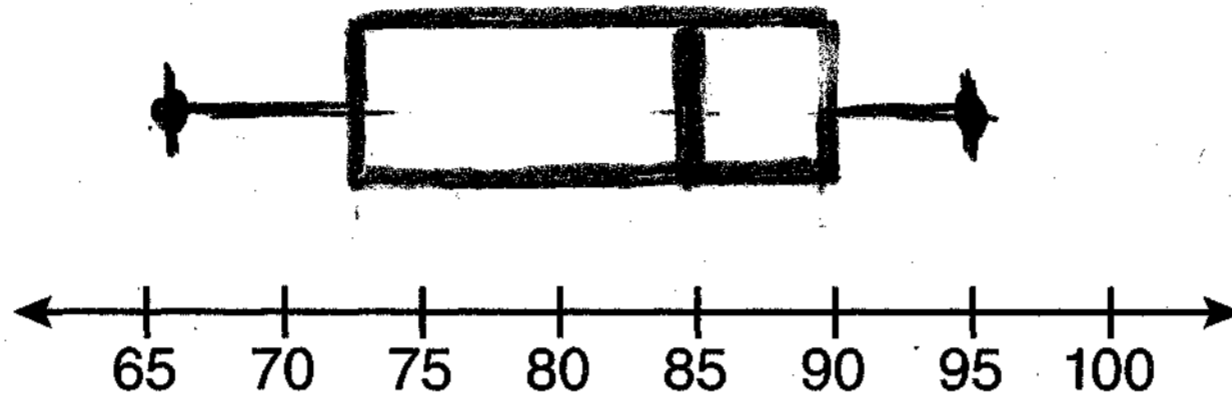$$b = -5$$

$$y = mx + b$$
$$y = -2X - 1$$
$$m = -2$$
$$b = -1$$

Check using graphing calculator

39 The test scores from Mrs. Gray's math class are shown below.

72, 73, 66, 71, 82, 85, 95, 85, 86, 89, 91, 92 $n = 12$ ✓

Construct a box-and-whisker plot to display these data.

$n = 12$ ✓

66, 71, 72 | 73, 82, 85 | 85, 86 89 | 91, 92, 95

Low		Median		High
66	72.5	85	90	95
	Q_1	Q_2	Q_3	