The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

INTEGRATED ALGEBRA

Wednesday, June 12, 2013 — 1:15 to 4:15 p.m., only

Student Name: Steve Watson

School Name: www.JMAP.org

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

INTEGRATED ALGEBRA

Part I

Answer all 30 questions in this part. Each correct answer will receive 2 credits. Record your answers on your separate answer sheet. [60]

Use this space for computations.

1 Which expression represents "5 less than twice x"?

(1) $2x - 5$

(2) $5 - 2x$

(3) $2(5 - x)$

(4) $2(x - 5)$

2 Gabriella has 20 quarters, 15 dimes, 7 nickels, and 8 pennies in a jar. After taking 6 quarters out of the jar, what will be the probability of Gabriella randomly selecting a quarter from the coins left in the jar?

(1) $\frac{14}{44}$

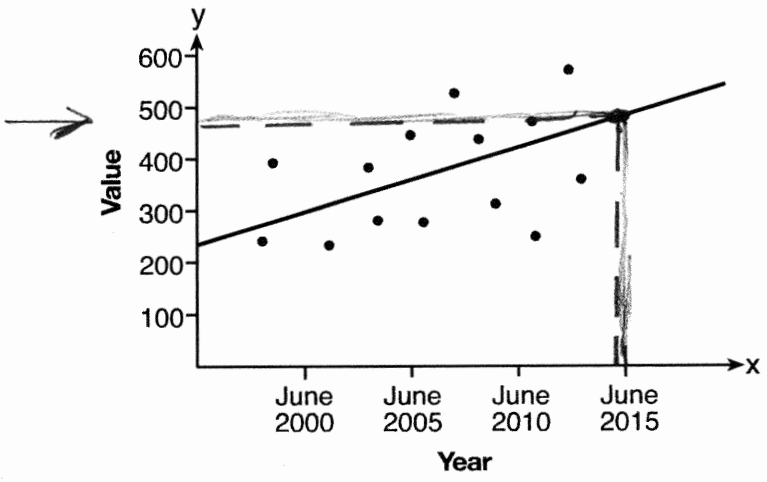
(2) $\frac{30}{44}$

(3) $\frac{14}{50}$

(4) $\frac{20}{50}$

	Start	After 6Q removed
Quarters	20	14
Dimes	15	15
Nickles	7	7
Pennies	8	8
Total	50	44

3 Based on the line of best fit drawn below, which value could be expected for the data in June 2015?



(1) 230

(2) 310

(3) 480

(4) 540

Use this space for computations.

4 If the point $(5,k)$ lies on the line represented by the equation $2x + y = 9$, the value of k is

(1) 1
(3) −1
(2) 2
(4) −2

$2x + y = 9$
$y = -2x + 9$ ← Input into graphing calculator and find $f(5)$.
$y = -2(5) + 9$
$y = -10 + 9$
$y = -1$

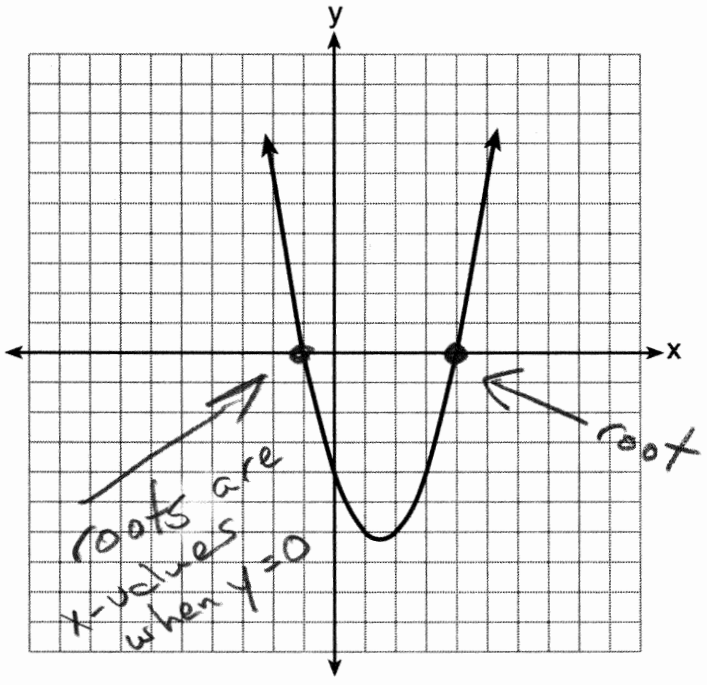
5 A soda container holds $5\frac{1}{2}$ gallons of soda. How many ounces of soda does this container hold?

1 quart = 32 ounces 1 gallon = 4 quarts

(1) 44
(3) 640
(2) 176
(4) 704

32 ounces / ~~quart~~
× 4 ~~quarts~~ / gallon
128 ounces / gallon
× 5.5 gallons
704 ounces

6 The roots of a quadratic equation can be found using the graph below.



What are the roots of this equation?

(1) −4, only
(3) −1 and 4
(2) −4 and −1
(4) −4, −1, and 4

Use this space for computations.

7 If the area of a rectangle is represented by $x^2 + 8x + 15$ and its length is represented by $x + 5$, which expression represents the width of the rectangle?

● $x + 3$
(2) $x - 3$
(3) $x^2 + 6x + 5$
(4) $x^2 + 7x + 10$

$x^2+8x+15$?
$x+5$
$x^2+8x+15$
$(x+5)(\ ?\ )$
$(x+5)(x+3)$

8 Which set of data describes a situation that would be classified as qualitative? → not numerical

● the colors of the birds at the city zoo — colors are not numbers
(2) the shoe size of the zookeepers at the city zoo — shoe sizes are numbers
(3) the heights of the giraffes at the city zoo — heights are numbers
(4) the weights of the monkeys at the city zoo — weights are numbers

$6! = (6)(5)(4)(3)(2)(1) = 720$

$(5)(4)(3)(2)(1)(3)(2)(1)$

$(4)(3)(2)(1)$

$= (5)(3)(2)(1) = 30$

9 The value of the expression $6! + \frac{5!(3!)}{4!} - 10$ is

(1) 50
● 740
(2) 102
(4) 750

$720 + 30 - 10 = 740$

10 Which interval notation represents $-3 \leq x \leq 3$?

● $[-3, 3]$
(2) $(-3, 3]$
(3) $[-3, 3)$
(4) $(-3, 3)$

< and > are curves
≤ and ≥ are square corners

11 The solutions of $x^2 = 16x - 28$ are

(1) -2 and -14
● 2 and 14
(3) -4 and -7
(4) 4 and 7

$x^2 = 16x - 28$
$x^2 - 16x + 28 = 0$
$(x-2)(x-14) = 0$
$x-2=0 \mid x-14=0$
$x=2 \mid x=14$

28
1 28
2 14
4 7

Use this space for computations.

12 If the expression $(2y^a)^4$ is equivalent to $16y^8$, what is the value of a?

(1) 12 (3) 32

(2) 2 (4) 4

$(2y^a)^4 = 16y^{4a}$

$16y^{4a} = 16y^8$

$4a = 8$

$a = 2$

13 Which table shows bivariate data?

Only 1 variable

Age (yr)	Frequency
14	12
15	21
16	14
17	19
18	15

(1)

Two numerical variables

Time Spent Studying (hr)	Test Grade (%)
1	65
2	72
3	83
4	85
5	92

Only 1 numerical variable

Type of Car	Average Gas Mileage (mpg)
van	25
SUV	23
luxury	26
compact	28
pickup	22

(2)

Only 1 numerical variable

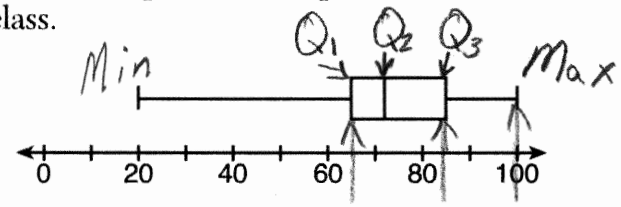
Day	Temperature (degrees F)
Monday	63
Tuesday	58
Wednesday	72
Thursday	74
Friday	78

(4)

Note: It can be argued that bivariate data does not have to be numerical, in which case options (2) and (4) could also be correct.

Use this space for computations.

14 The box-and-whisker plot below represents the results of test scores in a math class.



What do the scores 65, 85, and 100 represent?

(1) Q_1, median, Q_3

(2) Q_1, Q_3, maximum

(3) median, Q_1, maximum

(4) minimum, median, maximum

15 The expression $\frac{x-3}{x+2}$ is undefined when the value of x is

(1) -2, only

(2) -2 and 3

(3) 3, only

(4) -3 and 2

undefined means dividing by zero. We only care to avoid zero in the denominator

$x+2=0$

$x=-2$

16 If $rx - st = r$, which expression represents x?

(1) $\frac{r+st}{r}$

(2) $\frac{r}{r+st}$

(3) $\frac{r}{r-st}$

(4) $\frac{r-st}{r}$

Isolate the "x" variable

$rx - st = r$

$rx = r + st$

$\frac{rx}{r} = \frac{r+st}{r}$

17 What is the solution of the equation $\frac{x+2}{2} = \frac{4}{x}$?

(1) 1 and -8

(2) 2 and -4

(3) -1 and 8

(4) -2 and 4

Cross Multiply

$\frac{x+2}{2} = \frac{4}{x}$

$(x+2)x = (2)(4)$

$x^2 + 2x = 8$

$x^2 + 2x - 8 = 0$

-8

$1 \quad 8$

$(2 \quad 4)$

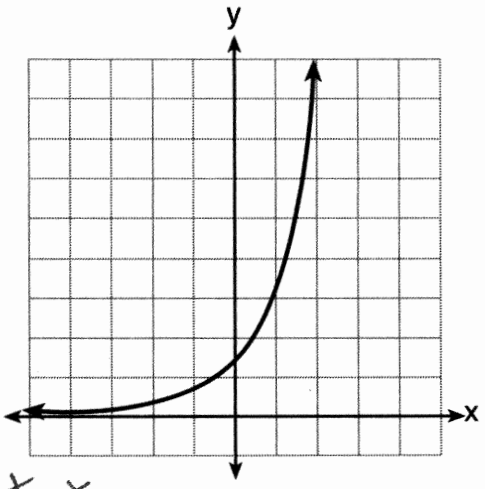
$(x-2)(x+4) = 0$

$x-2=0 \quad | \quad x+4=0$

$x=2 \quad | \quad x=-4$

18 Which type of function is graphed below?

Use this space for computations.



(1) linear not straight line

(2) quadratic not a parabola

● exponential

(4) absolute value not "V" shaped

19 What is the slope of the line represented by the equation $4x + 3y = 12$?

(1) $\frac{4}{3}$

(2) $\frac{3}{4}$

(3) $-\frac{3}{4}$

● $-\frac{4}{3}$

Strategy: Convert to slope intercept form.

$y = mx + b$

↳ y-intercept

↳ slope

$4x + 3y = 12$

$3y = -4x + 12$

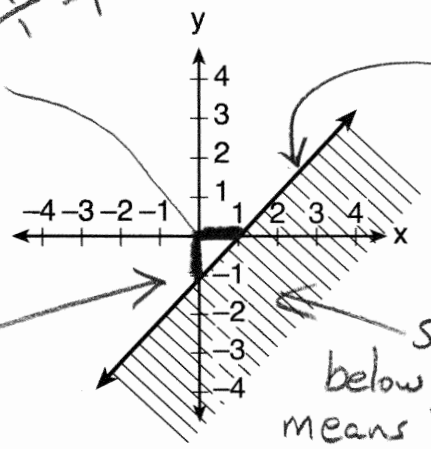
$y = \frac{-4}{3}x + 4$

$y = mx + b$

$m = -\frac{4}{3}$

20 The diagram below shows the graph of which inequality?

Use this space for computations.



1 = slope = $\frac{\text{rise}}{\text{run}} = \frac{1}{1} = 1$

solid lines are for ≤ and ≥
dotted lines are for < and >

y-intercept is −1

shaded below the line means "less than"

$y = mx + b$
$y \leq 1x - 1$

(1) $y > x - 1$
(3) $y < x - 1$
(2) $y \geq x - 1$
(4) $y \leq x - 1$

x = # sold
$C = 2x$
$J = x$
$2x + x \geq 90$

21 Carol plans to sell twice as many magazine subscriptions as Jennifer. If Carol and Jennifer need to sell at least 90 subscriptions in all, which inequality could be used to determine how many subscriptions, x, Jennifer needs to sell?

(1) $x \geq 45$
(3) $2x - x \geq 90$
(2) $2x \geq 90$
(4) $2x + x \geq 90$

22 When $2x^2 - 3x + 2$ is subtracted from $4x^2 - 5x + 2$, the result is

(1) $2x^2 - 2x$
(3) $-2x^2 - 8x + 4$
(2) $-2x^2 + 2x$
(4) $2x^2 - 8x + 4$

minus

$$\begin{array}{r} 4x^2 - 5x + 2 \\ 2x^2 - 3x + 2 \\ \hline 2x^2 - 2x + 0 \end{array}$$

23 Which expression represents the number of hours in w weeks and d days?

$7 \times 24 = 168$

24

(1) $7w + 12d$
(3) $168w + 24d$
(2) $84w + 24d$
(4) $168w + 60d$

168 (weeks) + 24 (days)

$168w + 24d$

Use this space for computations.

28 There are 18 students in a class. Each day, the teacher randomly selects three students to assist in a game: a leader, a recorder, and a timekeeper. In how many possible ways can the jobs be assigned?

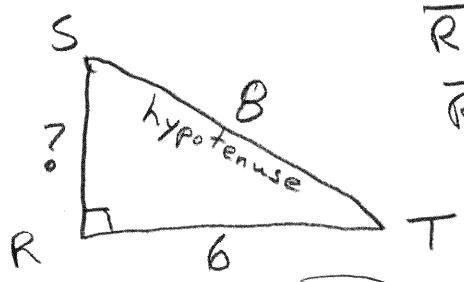
(1) 306
(2) 816
(3) 4896
(4) 5832

Leader Choices: 18 × Recorder Choices: 17 × Timekeeper Choices: 16 = 4896

29 In triangle RST, angle R is a right angle. If $TR = 6$ and $TS = 8$, what is the length of $\overline{RS}$?

(1) 10
(2) 2
(3) $2\sqrt{7}$
(4) $7\sqrt{2}$

Use Pythagorean Theorem



$a^2 + b^2 = c^2$
$\overline{RS}^2 + 6^2 = 8^2$
$\overline{RS}^2 = 8^2 - 6^2$
$\overline{RS}^2 = 64 - 36$
$\overline{RS}^2 = 28$
$RS = \sqrt{28}$
$RS = \sqrt{4}\sqrt{7}$
$RS = 2\sqrt{7}$

30 How many solutions are there for the following system of equations?

$y = x^2 - 5x + 3$
$y = x - 6$

Careful, this is the solution, not the number of solutions

(1) 1
(2) 2
(3) 3
(4) 0

$x^2 - 5x + 3 = x - 6$
$x^2 - 6x + 9 = 0$
$(x - 3)(x - 3) = 0$

$x - 3 = 0$	$x - 3 = 0$
$x = 3$	$x = 3$

9
1 9
3 3

24 Given:

$R = \{1, 2, 3, 4\}$
~~$A = \{0, 2, 4, 6\}$~~
$P = \{1, 3, 5, 7\}$

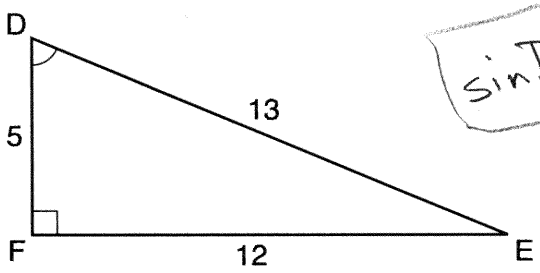
What is $R \cap P$?

(1) $\{0, 1, 2, 3, 4, 5, 6, 7\}$
● $\{1, 3\}$
(2) $\{1, 2, 3, 4, 5, 7\}$
(4) $\{2, 4\}$

Use this space for computations.

∩ means intersection
∪ means union
this set is not relevant to the problem.
intersection
these are the only elements common to Set R and Set P
This is the union of Set R and Set P

25 Which equation could be used to find the measure of angle D in the right triangle shown in the diagram below?



$\sin D = \frac{12}{13}$
$\cos D = \frac{5}{13}$
$\tan D = \frac{12}{5}$

See Formulas Page
$\sin = \frac{\text{opposite}}{\text{hypotenuse}}$
$\cos = \frac{\text{adjacent}}{\text{hypotenuse}}$
$\tan = \frac{\text{opposite}}{\text{adjacent}}$

(1) $\cos D = \frac{12}{13}$
(3) $\sin D = \frac{5}{13}$
(2) $\cos D = \frac{13}{12}$
● $\sin D = \frac{12}{13}$

26 If the roots of a quadratic equation are -2 and 3, the equation can be written as

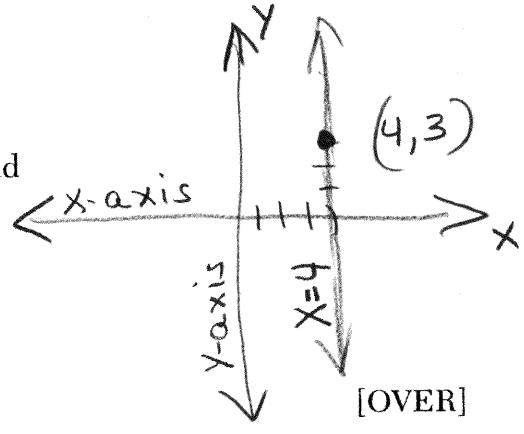
(1) $(x - 2)(x + 3) = 0$
(3) $(x + 2)(x + 3) = 0$
● $(x + 2)(x - 3) = 0$
(4) $(x - 2)(x - 3) = 0$

roots $x = -2$ | $x = 3$
factors $x + 2 = 0$ | $x - 3 = 0$
$(x+2)(x-3) = 0$

27 Which equation represents a line that is parallel to the y-axis and passes through the point (4,3)?

vertical

(1) $x = 3$
(3) $y = 3$
● $x = 4$
(4) $y = 4$



Part II

Answer all 3 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

31 Solve the inequality $-5(x - 7) < 15$ algebraically for x.

$-5(x-7) < 15$

$-5x + 35 < 15$

$-5x + 20 < 0$

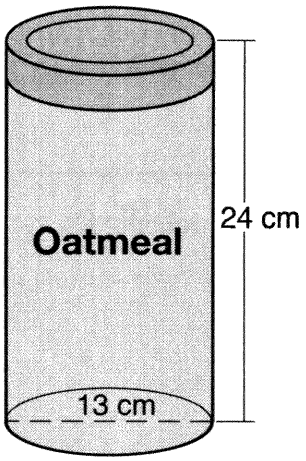
$20 < 5x$

$4 < x$

or

$x > 4$

32 Oatmeal is packaged in a cylindrical container, as shown in the diagram below.



diameter = 13

radius = $\frac{13}{2}$

Volume = Area × Height

$\text{Area}_{(\text{circle})} = \pi r^2$

The diameter of the container is 13 centimeters and its height is 24 centimeters. Determine, in terms of π, the volume of the cylinder, in cubic centimeters.

$A = \pi r^2$

$A = \pi \left(\frac{13}{2}\right)^2$

$\text{Volume} = \pi \left(\frac{13}{2}\right)^2 (24)$ ← height

$\text{Volume} = \pi (1014)$

$V = 1014\pi$

Part III

Answer all 3 questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [9]

34 The menu for the high school cafeteria is shown below.

Main Course	Vegetable	Dessert	Beverage
veggie burger	corn	gelatin	milk
pizza	green beans	fruit salad	juice
tuna sandwich	carrots	yogurt	bottled water
frankfurter		cookie	
chicken tenders		ice cream cup	

Determine the number of possible meals consisting of a main course, a vegetable, a dessert, and a beverage that can be selected from the menu.

Main Course Choices: 5 × Veggie Choices: 3 × Dessert Choices: 5 × Beverage Choices: 3 = Total Possible: 225 Answer

Determine how many of these meals will include chicken tenders.

Main Course Choices: 1 × Veggie Choices: 3 × Dessert Choices: 5 × Beverage Choices: 3 = Total Possible: 45 Answer

If a student chooses pizza, corn or carrots, a dessert, and a beverage from the menu, determine the number of possible meals that can be selected.

Main Course Choices: 1 × Veggie Choices: 2 × Dessert Choices: 5 × Beverage Choices: 3 = Total Possible: 30 Answer

33 The distance from Earth to Mars is 136,000,000 miles. A spaceship travels at 31,000 miles per hour. Determine, to the *nearest day*, how long it will take the spaceship to reach Mars.

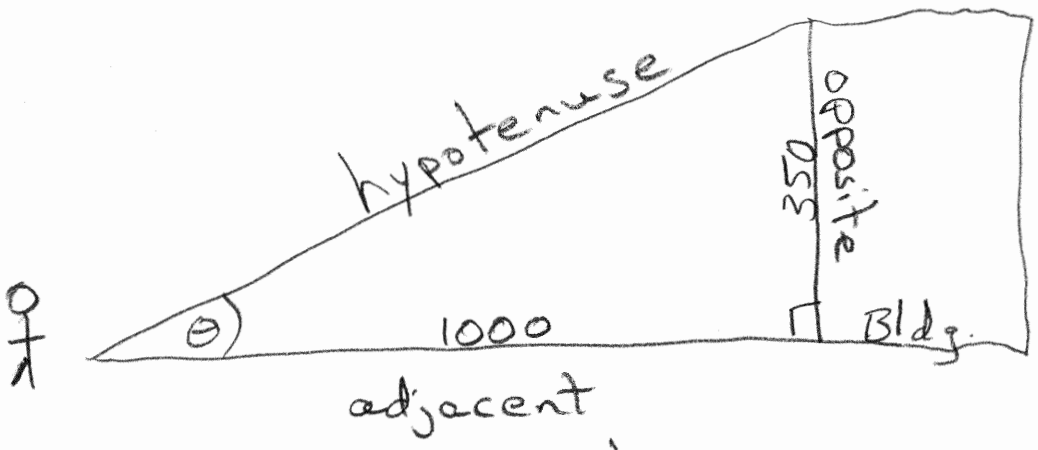
$$\frac{31{,}000 \text{ miles per hour}}{24 \text{ hours per day}}$$

$$744{,}000 \text{ miles per day}$$

$$\frac{136{,}000{,}000}{744{,}000} = 182.795699$$

183 days

35 A man standing on level ground is 1000 feet away from the base of a 350-foot-tall building. Find, to the *nearest degree*, the measure of the angle of elevation to the top of the building from the point on the ground where the man is standing.



adjacent

(See formula page)

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\theta = \frac{350}{1000}$$

$$\arctan\frac{350}{1000} = m\angle\theta$$

$$\boxed{19^\circ = m\angle\theta}$$

Note: Be sure to use degree mode in your graphing calculator.

36 Express $\sqrt{25} - 2\sqrt{3} + \sqrt{27} + 2\sqrt{9}$ in simplest radical form.

$5 - 2\sqrt{3} + \sqrt{27} + 2\sqrt{9}$

$5 - 2\sqrt{3} + \sqrt{9}\sqrt{3} + 2(3)$

$5 - 2\sqrt{3} + 3\sqrt{3} + 6$

$\boxed{11 + \sqrt{3}}$

Part IV

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

37 Solve algebraically: $\frac{2}{3x} + \frac{4}{x} = \frac{7}{x+1}$

[Only an algebraic solution can receive full credit.]

$$\frac{2}{3x} + \frac{4}{x} = \frac{7}{x+1}$$

$$\frac{(2)(x)}{(3x)(x)} + \frac{(3x)(4)}{(3x)(x)} = \frac{7}{x+1}$$

$$\frac{2x}{3x^2} + \frac{12x}{3x^2} = \frac{7}{x+1}$$

$$\frac{2}{3x} + \frac{12}{3x} = \frac{7}{x+1}$$

$$\frac{14}{3x} = \frac{7}{x+1}$$

$$14(x+1) = 3x(7)$$

$$14x + 14 = 21x$$

$$14 = 7x$$

$$\boxed{2 = x}$$

38 A jar contains five red marbles and three green marbles. A marble is drawn at random and not replaced. A second marble is then drawn from the jar.

Find the probability that the first marble is red and the second marble is green.

$P_{(A+B)} = P_{(A)} \times P_{(B)}$

5 R
3 G
8 total

$P_{(R)} = \frac{5}{8}$

4 R
3 G
7 total

$P_{G} = \frac{3}{7}$

$P_{(R+G)} = \left(\frac{5}{8}\right)\left(\frac{3}{7}\right) = \boxed{\frac{15}{56}}$

Find the probability that both marbles are red.

5 R
3 G
8 total

$P_{(R)} = \frac{5}{8}$

4 R
3 G
7 total

$P_{(R)} = \frac{4}{7}$

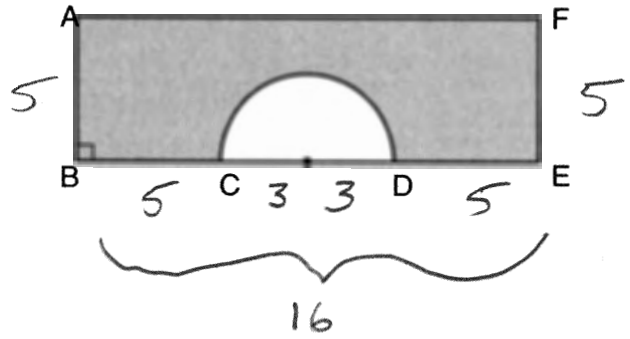
$P_{(R+R)} = \left(\frac{5}{8}\right)\left(\frac{4}{7}\right) = \boxed{\frac{20}{56}}$

Find the probability that both marbles are the same color.

$P_{(G+G)} = \left(\frac{3}{8}\right)\left(\frac{2}{7}\right) = \frac{6}{56}$

$P_{(R+R)} \text{ or } P_{(G+G)} = \left(\frac{20}{56}\right) + \left(\frac{6}{56}\right) = \boxed{\frac{26}{56}}$

39 In the diagram below of rectangle $AFEB$ and a semicircle with diameter $\overline{CD}$, $AB = 5$ inches, $AB = BC = DE = FE$, and $CD = 6$ inches. Find the area of the shaded region, to the *nearest hundredth of a square inch*.



Area Rectangle − Area Semicircle = Shaded Area

$(5 \times 16) - \frac{\pi r^2}{2} =$

$80 - \frac{9\pi}{2} =$

$80 - 4.5\pi$ = shaded area

65.86 in^2