The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

INTEGRATED ALGEBRA

Thursday, June 14, 2012 — 1:15 to 4:15 p.m., only

Student Name: Steve Watson

School Name: JMAP

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

INTEGRATED ALGEBRA

Part I

Answer all 30 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [60]

Use this space for computations.

1 In a baseball game, the ball traveled 350.7 feet in 4.2 seconds. What was the average speed of the ball, in feet per second?

● 83.5
(3) 354.9
(2) 177.5
(4) 1,472.9

Average Speed = $\frac{\text{total distance}}{\text{total time}}$

$= \frac{350.7 \text{ feet}}{4.2 \text{ seconds}}$

$= 83.5 \text{ feet/second}$

2 A survey is being conducted to determine if a cable company should add another sports channel to their schedule. Which random survey would be the *least* biased?

(1) surveying 30 men at a gym → like sports (biased)
● surveying 45 people at a mall → might or might not like sports (less bias)
(3) surveying 50 fans at a football game → like sports (biased)
(4) surveying 20 members of a high school soccer team → like sports (biased)

3 The quotient of $\frac{8x^5 - 2x^4 + 4x^3 - 6x^2}{2x^2}$ is

(1) $16x^7 - 4x^6 + 8x^5 - 12x^4$
(2) $4x^7 - x^6 + 2x^5 - 3x^4$
(3) $4x^3 - x^2 + 2x - 3x$
● $4x^3 - x^2 + 2x - 3$

$\frac{8x^5}{2x^2} \left| \frac{-2x^4}{2x^2} \right| \frac{+4x^3}{2x^2} \left| \frac{-6x^2}{2x^2} \right.$

$4x^{(5-2)} - x^{(4-2)} + 2x^{(3-2)} - 3x^{(2-2)}$

$4x^3 - x^2 + 2x^1 - 3x^0$

$4x^3 - x^2 + 2x - 3$

4 Marcy determined that her father's age is four less than three times her age. If x represents Marcy's age, which expression represents her father's age?

● $3x - 4$
(3) $4x - 3$
(2) $3(x - 4)$
(4) $4 - 3x$

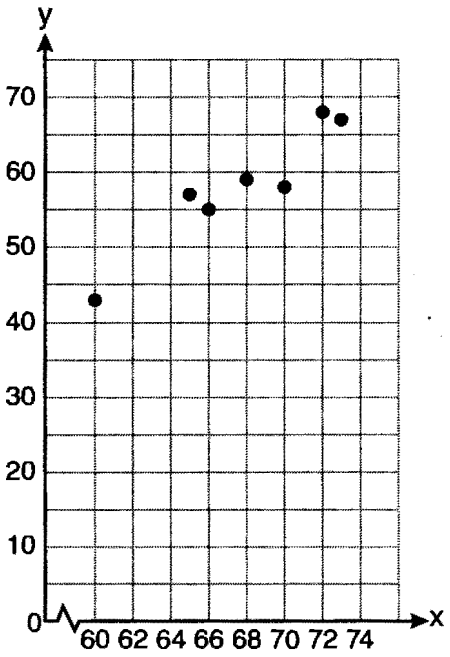
father's age = four less than 3 times X

father's age = $-4 + 3X$

father's age = $3X - 4$

Use this space for computations.

5 A set of data is graphed on the scatter plot below.



This scatter plot shows

(1) no correlation

(2) positive correlation

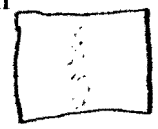
(3) negative correlation

(4) undefined correlation


No


negative


positive


undefined

6 Which situation is an example of bivariate data?

(1) the number of pizzas Tanya eats during her years in high school

(2) the number of times Ezra puts air in his bicycle tires during the summer

(3) the number of home runs Elias hits per game and the number of hours he practices baseball

(4) the number of hours Nellie studies for her mathematics tests during the first half of the school year

bivariate means two variables

bicycle → two wheels

biannual – two times each year

bimodal – two modes

Use this space for computations.

7 Brianna's score on a national math assessment exceeded the scores of 95,000 of the 125,000 students who took the assessment. What was her percentile rank?

(1) 6
(2) 24
(3) 31
● 76

$\frac{95,000}{125,000}(100) = 76$

8 If $A = \{0, 1, 3, 4, 6, 7\}$, $B = \{0, 2, 3, 5, 6\}$, and $C = \{0, 1, 4, 6, 7\}$, then $A \cap B \cap C$ is

(1) $\{0, 1, 2, 3, 4, 5, 6, 7\}$
● $\{0, 6\}$
(2) $\{0, 3, 6\}$
(4) $\{0\}$

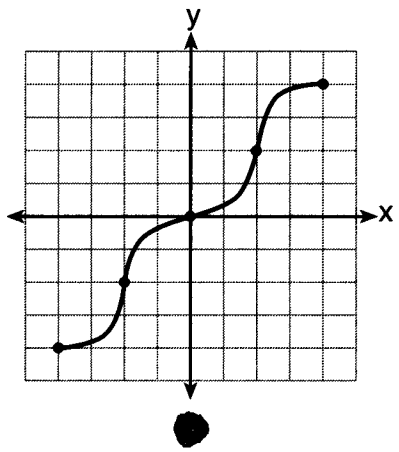
$A \cap B \cap C$ means the intersection of all three sets, so find each element that is common to all three sets.

0	yes
1	no
2	no

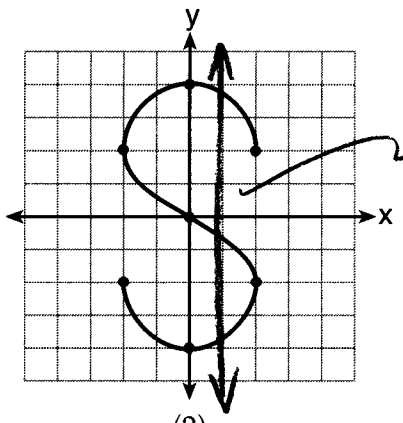
3	no
4	no
5	no

6	yes
7	no

9 Which graph represents a function?

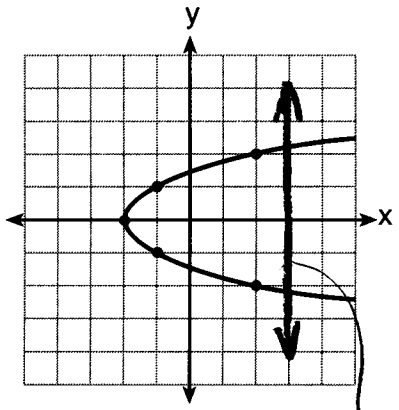


●



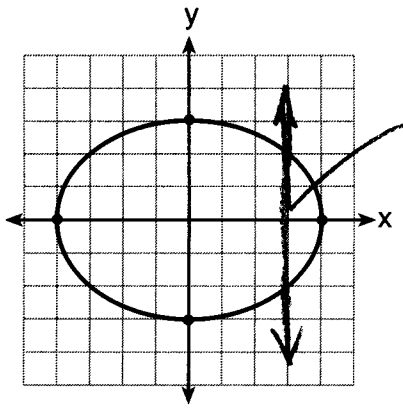
(3)

fails vertical line test.



(2)

fails vertical line test.



(4)

fails vertical line test

→ multiply

Use this space for computations.

10 What is the product of $(3x + 2)$ and $(x - 7)$?

(1) $3x^2 - 14$

● $3x^2 - 19x - 14$

(2) $3x^2 - 5x - 14$

(4) $3x^2 - 23x - 14$

Distributive Property

$(3x+2)(x-7)$

$3x^2 - 21x + 2x - 14$

$3x^2 - 19x - 14$

$5x < 55$

11 If five times a number is less than 55, what is the greatest possible integer value of the number?

(1) 12 $5(12) \not< 55$

● 10 $5(10) < 55$ true

(2) 11 $5(11) \not< 55$

(4) 9 $5(9) < 55$ true

10 has a greater integer value than 9, so 10 is the answer.

12 The line represented by the equation $2y - 3x = 4$ has a slope of

(1) $-\frac{3}{2}$

(3) 3

(2) 2

● $\frac{3}{2}$

$y = mx + b$

m → slope

$2y - 3x = 4$

$2y = 3x + 4$

$y = \frac{3}{2}x + 2$

$m = \frac{3}{2}$

13 What is the solution set of the system of equations $x + y = 5$ and $y = x^2 - 25$?

(1) {(0,5), (11,−6)}

(3) {(−5,0), (6,11)}

● {(5,0), (−6,11)}

(4) {(−5,10), (6,−1)}

$x + y = 5$ $\quad y = x^2 - 25$

$y = 5 - x$

$\therefore 5 - x = x^2 - 25$

$-5 + x \qquad -5 + x$

$0 = x^2 - 30 + x$

$0 = x^2 + x - 30$

$x^2 + x - 30 = 0$

$(x+6)(x-5) = 0$

$x + 6 = 0$

$x = -6$

$x + y = 5$

$-6 + y = 5$

$y = 11$

$x - 5 = 0$

$x = 5$

$x + y = 5$

$5 + y = 5$

$y = 0$

Use this space for computations.

14 What is the vertex of the parabola represented by the equation $y = -2x^2 + 24x - 100$?

(1) $x = -6$

● (6,−28)

(2) $x = 6$

(4) (−6,−316)

$x = \frac{-b}{2a}$ = axis of symmetry

$x = \frac{-24}{2(-2)} = \frac{-24}{-4} = \boxed{6}$

$y = -2x^2 + 24x - 100$

$y = -2(6)^2 + 24(6) - 100$

$y = -2(36) + 144 - 100 \longrightarrow y = -72 + 144 - 100 \rightarrow y = \boxed{-28}$

15 If $k = am + 3mx$, the value of m in terms of a, k, and x can be expressed as

● $\frac{k}{a + 3x}$

(3) $\frac{k - am}{3x}$

(2) $\frac{k - 3mx}{a}$

(4) $\frac{k - a}{3x}$

$k = am + 3mx$

$k = m(a + 3x)$

$\boxed{\frac{k}{a+3x} = m}$

16 Which expression represents $\frac{x^2 - 3x - 10}{x^2 - 25}$ in simplest form?

(1) $\frac{2}{5}$

(3) $\frac{x - 2}{x - 5}$

● $\frac{x + 2}{x + 5}$

(4) $\frac{-3x - 10}{-25}$

$\frac{x^2 - 3x - 10}{x^2 - 25}$

$\frac{(x-5)(x+2)}{(x+5)(x-5)}$

$\boxed{\frac{x+2}{x+5}}$

17 Which interval notation describes the set $S = \{x \mid 1 \leq x < 10\}$?

(1) [1,10]

● [1,10)

(2) (1,10]

(4) (1,10)

] or [can equal adjacent #

) or (can not equal adjacent #

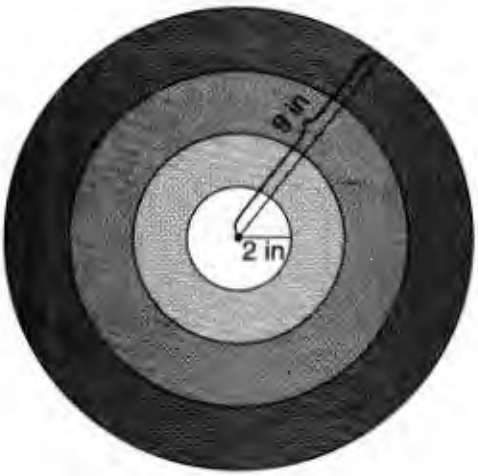
$1 \leq x < 10$

$[1, 10)$

Use this space for computations.

18 The bull's-eye of a dartboard has a radius of 2 inches and the entire board has a radius of 9 inches, as shown in the diagram below.

Geometric Probability



Area of Circle $= \pi r^2$

Area (entire board) $= \pi(9)^2 = 81\pi$

Area (bullseye) $= \pi(2)^2 = 4\pi$

$P_{(bullseye)} = \frac{\text{desired outcome (bullseye)}}{\text{total possible outcomes (entire board)}}$

If a dart is thrown and hits the board, what is the probability that the dart will land in the bull's-eye?

(1) $\frac{2}{9}$

● $\frac{4}{81}$

(2) $\frac{7}{9}$

(4) $\frac{49}{81}$

$P_{(bullseye)} = \frac{4\pi}{81\pi} = \boxed{\frac{4}{81}}$

19 What is one-third of 3^6?

(1) 1^2

● 3^5

(2) 3^2

(4) 9^6

$\frac{1}{3}(3^6) = \frac{1}{3} \times \frac{3^6}{1} = \frac{3^6}{3} = 3^{(6-1)}$

$= \boxed{3^5}$

20 The expression $\frac{2x+13}{2x+6} - \frac{3x-6}{2x+6}$ is equivalent to

Common denominator

● $\frac{-x+19}{2(x+3)}$

(3) $\frac{5x+19}{2(x+3)}$

(2) $\frac{-x+7}{2(x+3)}$

(4) $\frac{5x+7}{4x+12}$

$\frac{2x+13}{2x+6} - \frac{3x-6}{2x+6}$

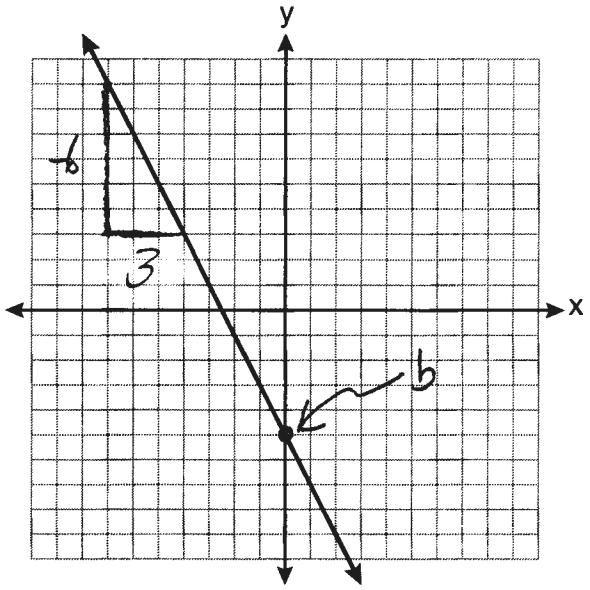
$\frac{2x+13-(3x-6)}{2x+6}$

$\frac{2x+13-3x+6}{2x+6}$

$\frac{-x+19}{2x+6} \rightarrow \boxed{\frac{-x+19}{2(x+3)}}$

21 Which equation is represented by the graph below?

Use this space for computations.



$y = mx + b$
↳ y-intercept
↳ slope

$b = -5$

$m = \frac{\text{rise}}{\text{run}} = \frac{-6}{3} = -2$

$y = -2x - 5$

$2y = -4x - 10$

(1) $2y + x = 10$

(2) $y - 2x = -5$

(3) $-2y = 10x - 4$

● $2y = -4x - 10$

22 Which coordinates represent a point in the solution set of the system of inequalities shown below?

See Scrap Graph Paper

$y \leq \frac{1}{2}x + 13$

$4x + 2y > 3$ →

$2y > -4x + 3$

$y > \frac{-4}{2}x + \frac{3}{2}$

$y > -2x + \frac{3}{2}$

(1) (−4,1)

(2) (−2,2)

(3) (1,−4)

● (2,−2)

Check

$y \leq \frac{1}{2}(x) + 13$

$-2 \leq \frac{1}{2}(2) + 13$

$-2 \leq 1 + 13$

$-2 \leq 14$ true ✓

$4x + 2y > 3$

$4(2) + 2(-2) > 3$

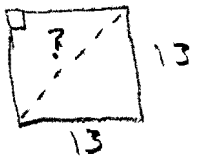
$8 - 4 > 3$

$4 > 3$ true ✓

Use this space for computations.

23 The length of one side of a square is 13 feet. What is the length, to the *nearest foot*, of a diagonal of the square?

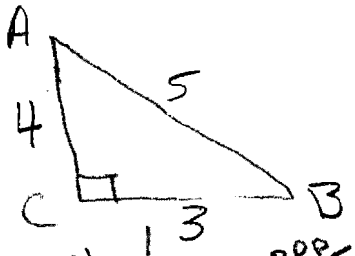
(1) 13
(3) 19
● 18
(4) 26



Pythagorean Theorem
$a^2 + b^2 = c^2$
$13^2 + 13^2 = c^2$
$169 + 169 = c^2$
$338 = c^2$
$\sqrt{338} = \sqrt{c^2}$
$18.38477631 = c$
$18 = c$

24 In $\triangle ABC$, $m\angle C = 90$. If $AB = 5$ and $AC = 4$, which statement is *not* true?

(1) $\cos A = \frac{4}{5}$ ✓
(3) $\sin B = \frac{4}{5}$ ✓
(2) $\tan A = \frac{3}{4}$ ✓
● $\tan B = \frac{5}{3}$ not true



SOH-CAH-TOA
$\sin = \frac{opp}{hyp} = \frac{4}{5}$ $\sin B = \frac{4}{5}$ | $\cos = \frac{adj}{hyp}$ $\cos A = \frac{4}{5}$ | $\tan = \frac{opp}{adj}$

$\tan A = \frac{?}{4} = \frac{3}{4}$
$\tan B = \frac{4}{?} = \frac{4}{3}$

25 If n is an odd integer, which equation can be used to find three consecutive odd integers whose sum is -3?

(1) $n + (n + 1) + (n + 3) = -3$
(2) $n + (n + 1) + (n + 2) = -3$
● $n + (n + 2) + (n + 4) = -3$
(4) $n + (n + 2) + (n + 3) = -3$

odd integers are always 2 apart
- never 1 or 3 apart

$? = BC$
$a^2 + b^2 = c^2$
$4^2 + ?^2 = 5^2$
$16 + ?^2 = 25$
$?^2 = 9$
$? = 3$
$BC = 3$

26 When $8x^2 + 3x + 2$ is subtracted from $9x^2 - 3x - 4$, the result is

(1) $x^2 - 2$
(3) $-x^2 + 6x + 6$
(2) $17x^2 - 2$
● $x^2 - 6x - 6$

$$9x^2 - 3x - 4$$
$$-\ (8x^2 + 3x + 2)$$
$$x^2 - 6x - 6$$

Use this space for computations.

27 Factored completely, the expression $3x^3 - 33x^2 + 90x$ is equivalent to

(1) $3x(x^2 - 33x + 90)$

(2) $3x(x^2 - 11x + 30)$

(3) $3x(x + 5)(x + 6)$

● $3x(x - 5)(x - 6)$

$3x^3 - 33x^2 + 90x$

$3(x^3 - 11x^2 + 30x)$

$3x(x^2 - 11x + 30)$

$3x(x-6)(x-5)$

28 Elizabeth is baking chocolate chip cookies. A single batch uses $\frac{3}{4}$ teaspoon of vanilla. If Elizabeth is mixing the ingredients for five batches at the same time, how many tablespoons of vanilla will she use?

$\frac{1}{3}$(# of teaspoons) = # tablespoons

3 teaspoons = 1 tablespoon

$\frac{3}{4}$ teaspoon times 5

$\frac{3}{4}\left(\frac{5}{1}\right) = \frac{15}{4}$ teaspoons

$\frac{15}{4} \times \frac{1}{3} = \frac{15}{12} = 1\frac{3}{12} = 1\frac{1}{4}$ tablespoons

↑ conversion factor

● $1\frac{1}{4}$

(2) $1\frac{3}{4}$

(3) $3\frac{3}{4}$

(4) $5\frac{3}{4}$

29 A car depreciates (loses value) at a rate of 4.5% annually. Greg purchased a car for \$12,500. Which equation can be used to determine the value of the car, V, after 5 years?

100% − 4.5% = 95.5%

(1) $V = 12{,}500(0.55)^5$

● $V = 12{,}500(0.955)^5$

(3) $V = 12{,}500(1.045)^5$

(4) $V = 12{,}500(1.45)^5$

The car retains (keeps) 95.5% of its value every year.

Convert 95.5% to a decimal → .955

Value = #12,500 (.955)(.955)(.955)(.955)(.955)

61st year, 62nd year, 63rd year, 64th year, 65th year

Value = #12,500 $(.955)^5$

Use this space for computations.

30 The cumulative frequency table below shows the length of time that 30 students spent text messaging on a weekend.

#	Interval	Minutes Used	Cumulative Frequency
2	31-40	31–40	2
3	41-50	31–50	5
5	51-60	31–60	10
9	61-70	31–70	19
11	71-80	31–80	30

n=30

There are 30 students. The first quartile of students would be $\frac{30 \text{ students}}{4 \text{ quartiles}} = 7.5$.

The interval that contains student #7.5 is 51-60.

Which 10-minute interval contains the first quartile?

(1) 31–40
(2) 41–50
(3) 51–60
(4) 61–70

Part II

Answer all 3 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

31 Solve the following system of equations algebraically for y:

$$2x + 2y = 9$$
$$2x - y = 3$$

Elimination Method

$$2x + 2y = 9$$
Subtract $$2x - y = 3$$
$$0x + 3y = 6$$

$$y = \frac{6}{3} = \boxed{2}$$

Substitution Method

$$2x + 2y = 9 \rightarrow 2x = -2y + 9$$
$$2x - y = 3 \rightarrow 2x = y + 3$$

$$\therefore -2y + 9 = y + 3$$
$$6 = 3y$$
$$\boxed{2} = y$$

32 Three storage bins contain colored blocks. Bin 1 contains 15 red and 14 blue blocks. Bin 2 contains 16 white and 15 blue blocks. Bin 3 contains 15 red and 15 white blocks. All of the blocks from the three bins are placed into one box.

If one block is randomly selected from the box, which color block would most likely be picked? Justify your answer.

	Bin 1	Bin 2	Bin 3	Total
Reds	15 red	0	15 red	30 red
Blues	14 blue	15 blue	0	29 blue
whites	0	16 white	15 white	31 white
Totals	29	31	30	90

A white block would most likely be picked. The probability of an event is based on the formula

$$P(\text{event}) = \frac{\#\text{ favorable outcomes}}{\text{total possible outcomes}}$$

$P(\text{white}) = \frac{31}{90}$ $P(\text{Blue}) = \frac{29}{90}$ $P(\text{Red}) = \frac{30}{90}$

$P(\text{white})$ is greater than $P(\text{Blue})$ or $P(\text{Red})$.

33 Students calculated the area of a playing field to be 8,100 square feet. The actual area of the field is 7,678.5 square feet. Find the relative error in the area, to the *nearest thousandth.*

$$\frac{\text{Actual} - \text{Measured}}{\text{Actual}}$$

$$\frac{8{,}100 - 7{,}678.5}{7{,}678.5} = \frac{421.5}{7{,}678.5} = 0.054\,893\,533\,9$$

tenths, hundredths, thousandths

$\uparrow$ bring this down

0.055

Part III

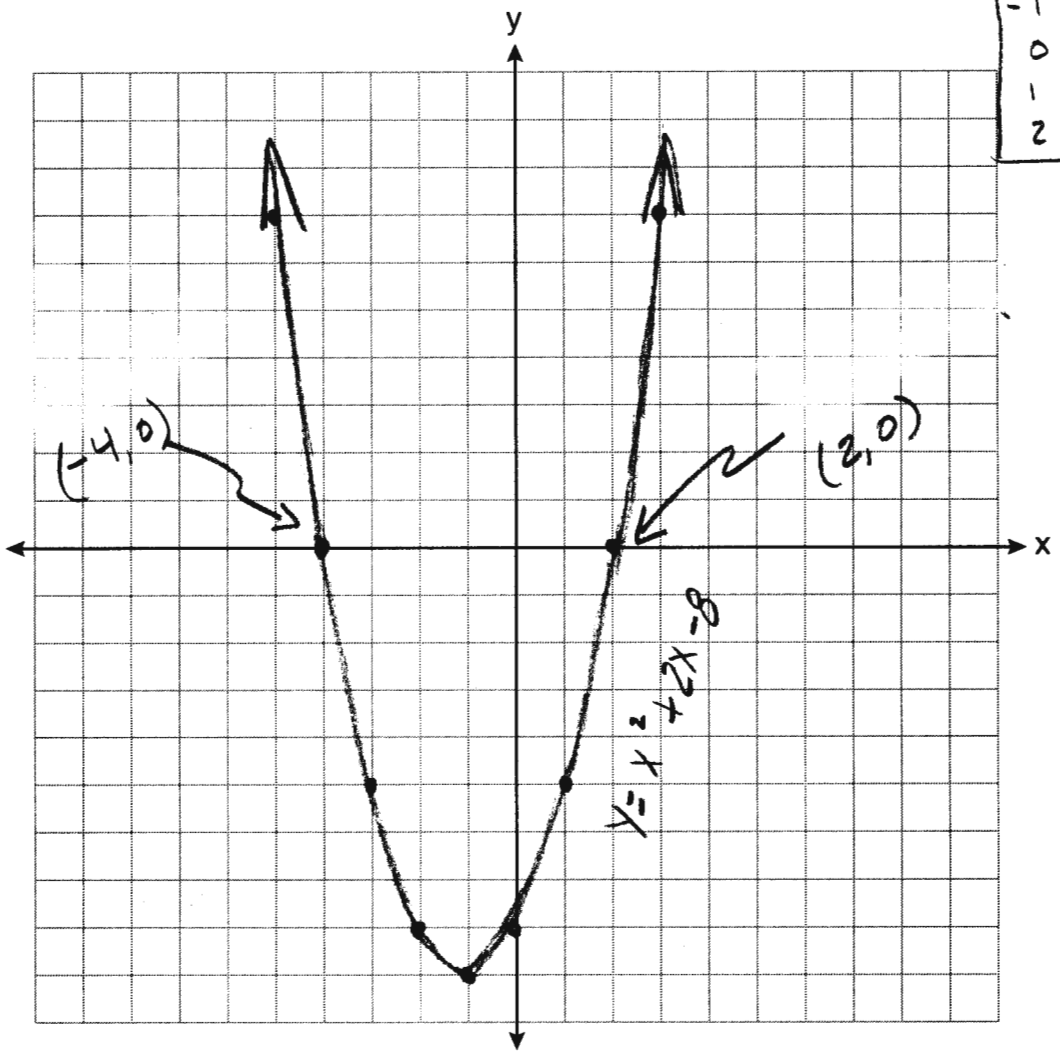
Answer all 3 questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [9]

34 On the set of axes below, graph the equation $y = x^2 + 2x - 8$.

Using the graph, determine and state the roots of the equation $x^2 + 2x - 8 = 0$.

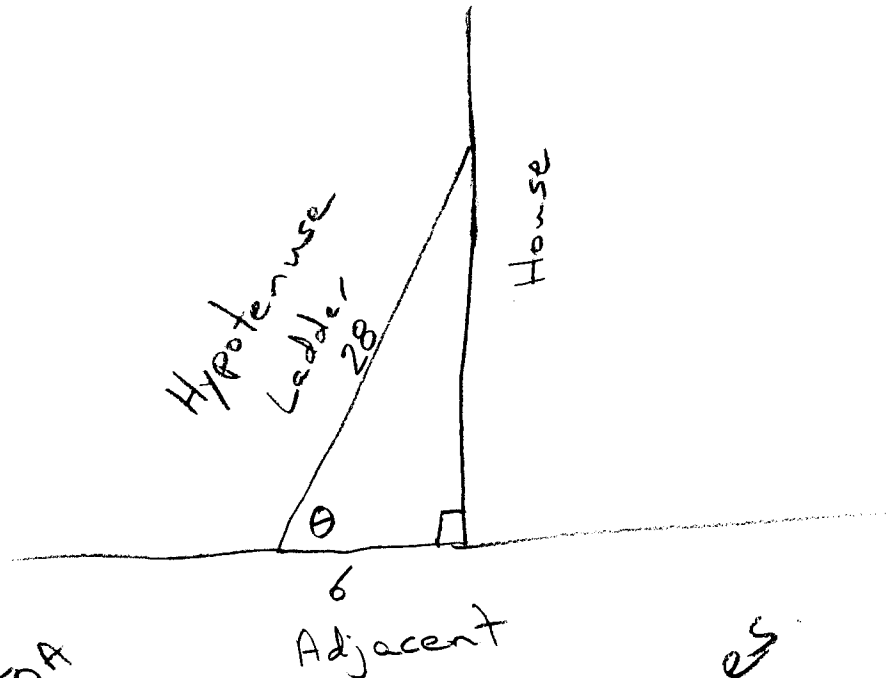
From graphing calculator →

x	y
-4	0
-3	-5
-2	-8
-1	-9
0	-8
1	-5
2	0



The roots are **-4 and 2**

35 A 28-foot ladder is leaning against a house. The bottom of the ladder is 6 feet from the base of the house. Find the measure of the angle formed by the ladder and the ground, to the *nearest degree.*



SOH-CAH-TOA

$\cos = \frac{adj}{hyp} = \frac{6}{28}$

$\arccos \frac{6}{28} = m\angle\theta$

Be sure to set calculator to degrees

$m\angle\theta = 77.62637488$

$m\angle\theta = \boxed{78°}$

36 Express $\frac{3\sqrt{75} + \sqrt{27}}{3}$ in simplest radical form.

$$\frac{3\sqrt{75} + \sqrt{27}}{3}$$

$$\frac{3\sqrt{25}\sqrt{3} + \sqrt{9}\sqrt{3}}{3}$$

$$\frac{3 \cdot 5\sqrt{3} + 3\sqrt{3}}{3}$$

$$\frac{15\sqrt{3} + 3\sqrt{3}}{3}$$

$$\frac{18\sqrt{3}}{3}$$

$$\boxed{6\sqrt{3}}$$

Part IV

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

37 Mike buys his ice cream packed in a rectangular prism-shaped carton, while Carol buys hers in a cylindrical-shaped carton. The dimensions of the prism are 5 inches by 3.5 inches by 7 inches. The cylinder has a diameter of 5 inches and a height of 7 inches.

Which container holds more ice cream? Justify your answer.

(from formula sheet)

Volume
Rectangular Prism

$V = lwh$

$V = (5)(3.5)(7)$

$V = 122.5$ cubic inches

Volume
Cylinder

$d = 5$

$r = \frac{5}{2} = 2.5$

$V = \pi r^2 h$

$V = \pi (2.5)^2 (7)$

$V = 137.44467 86$ cubic inches

Carol's cylinder holds about 15 cubic inches more ice cream than Mikes rectangular prism.

Determine, to the *nearest tenth of a cubic inch*, how much *more* ice cream the larger container holds.

$$\begin{array}{r} 137.44467 86 \\ -\ 122.5 \\ \hline 14.94467 859 \end{array}$$

14.9 cubic inches more

38 Solve algebraically for x: $3(x + 1) - 5x = 12 - (6x - 7)$

$3(x+1) - 5x = 12 - (6x - 7)$

$3x + 3 - 5x = 12 - 6x + 7$

$-2x + 3 = -6x + 19$

$4x = 16$

$x = 4$

Check

$3(4+1) - 5(4) = 12 - (6(4) - 7)$

$3(5) - 20 = 12 - (24 - 7)$

$15 - 20 = 12 - (17)$

$-5 = -5$ ✓

39 A large company must chose between two types of passwords to log on to a computer. The first type is a four-letter password using any of the 26 letters of the alphabet, without repetition of letters. The second type is a six-digit password using the digits 0 through 9, with repetition of digits allowed.

Determine the number of possible four-letter passwords.

Letter #1 Choices: 26 × Letter #2 Choices: 25 × Letter #3 Choices: 24 × Letter #4 Choices: 23 = 358,800 combinations

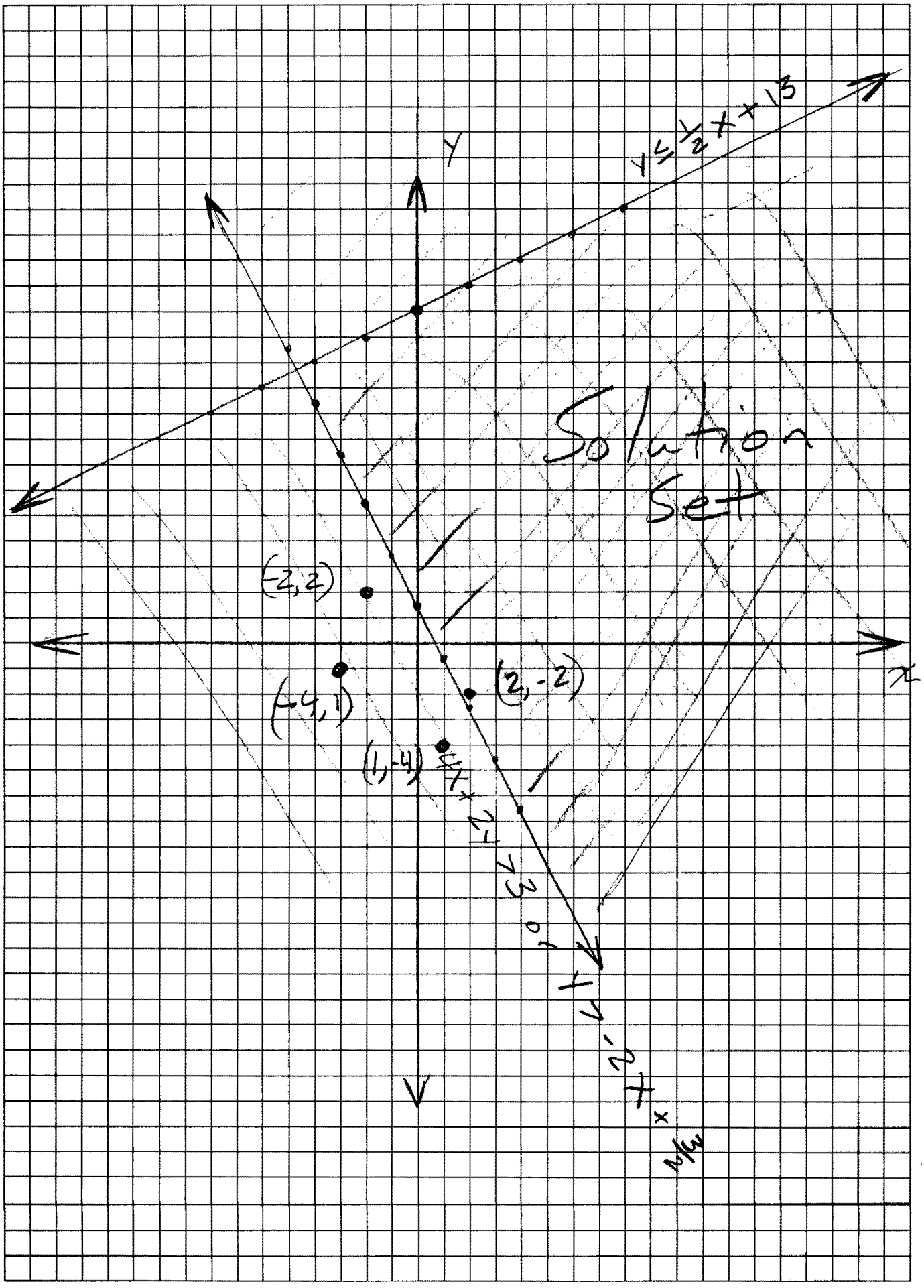
Determine the number of possible six-digit passwords.

Digit #1 Choices: 10 × Digit #2 Choices: 10 × Digit #3 Choices: 10 × Digit #4 Choices: 10 × Digit #5 Choices: 10 × Digit #6 Choices: 10 $= 10^6$ = 1,000,000 combinations

The company has 500,000 employees and needs a different password for each employee. State which type of password the company should choose. Explain your answer.

The company should choose 6-digit passwords. There are not enough 4-letter combinations for 500,000 employees to each have a different password.

Scrap Graph Paper — This sheet will *not* be scored.



Tear Here

Tear Here