The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

INTEGRATED ALGEBRA

Friday, June 19, 2009 — 1:15 to 4:15 p.m., only

Student Name: *Steve Watson*

School Name: *IHS @ PH*

Print your name and the name of your school on the lines above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 30 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [60]

Use this space for computations.

1 It takes Tammy 45 minutes to ride her bike 5 miles. At this rate, how long will it take her to ride 8 miles?

(1) 0.89 hour

(3) 48 minutes

(2) 1.125 hours

(4) 72 minutes

$$\frac{miles}{minutes} \quad \frac{5}{45} = \frac{8}{4}$$

$$5x = 45(8)$$
$$5x = 360$$
$$x = 72$$

2 What are the roots of the equation $x^2 - 7x + 6 = 0$?

(1) 1 and 7

(3) −1 and −6

(2) −1 and 7

(4) 1 and 6

$$ax^2 + bx + c = 0$$
$$a \cdot c = 6 \quad b = -7$$
factors of 6 that sum to −7 are $(-6)+(-1) = -7$
$$x^2 - 7x + 6 = 0$$
$$(x-6)(x-1) = 0$$
$$x = 6 \quad x = 1$$

3 Which expression represents $\dfrac{27x^{18}y^5}{9x^6y}$ in simplest form?

(1) $3x^{12}y^4$

(3) $18x^{12}y^4$

(2) $3x^3y^5$

(4) $18x^3y^5$

$$\frac{27}{9} \quad \frac{x^{18}}{x^6} \quad \frac{y^5}{y}$$
$$3 \quad x^{(18-6)} \quad y^{(5-1)}$$
$$3 \quad x^{12} \quad y^4$$

4 Marie currently has a collection of 58 stamps. If she buys s stamps each week for w weeks, which expression represents the total number of stamps she will have?

(1) $58sw$

(3) $58s + w$

(2) $58 + sw$

(4) $58 + s + w$

$+58$

s stamps each week for w weeks

$(s \cdot w)$
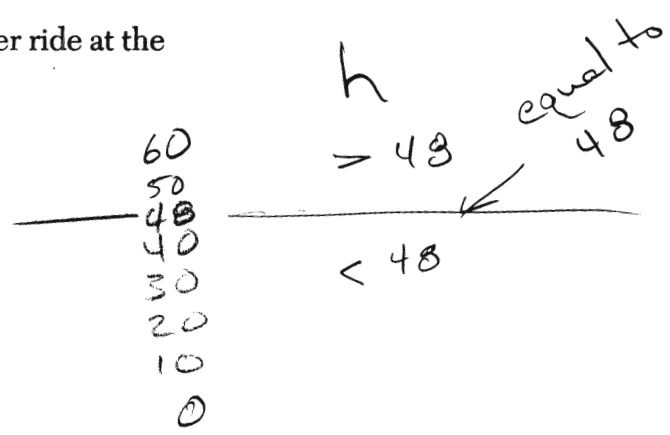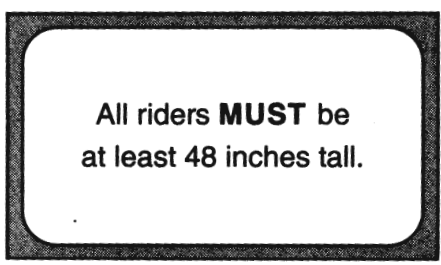
sw

$58 + sw$

qualitative means not numbers

5 Which data set describes a situation that could be classified as qualitative?

(1) the ages of the students in Ms. Marshall's Spanish class   *ages are numbers*

(2) the test scores of the students in Ms. Fitzgerald's class   *test scores are numbers*

(3) the favorite ice cream flavor of each of Mr. Hayden's students   *flavors are not numbers*

(4) the heights of the players on the East High School basketball team   *heights are numbers*

6 The sign shown below is posted in front of a roller coaster ride at the Wadsworth County Fairgrounds.

All riders **MUST** be at least 48 inches tall.

$$60$$
$$50$$
$$48$$
$$40$$
$$30$$
$$20$$
$$10$$
$$0$$

h ≥ 48 equal to 48
< 48

If h represents the height of a rider in inches, what is a correct translation of the statement on this sign?

(1) $h < 48$ (3) $h \leq 48$

(2) $h > 48$ (4) $h \geq 48$

7 Which value of x is the solution of the equation $\frac{2x}{3} + \frac{x}{6} = 5$?

(1) 6   (3) 15

(2) 10   (4) 30

$$\frac{2x}{3} + \frac{x}{6} = 5$$

$$\frac{12x + 3x}{18} = \frac{5}{1}$$

$$1(12x + 3x) = 18(5)$$

$$1(15x) = 90$$

$$15x = 90$$

$$x = 6$$

8 Students in Ms. Nazzeer's mathematics class tossed a six-sided number cube whose faces are numbered 1 to 6. The results are recorded in the table below.

Result	Frequency
1	3
2	6
3	4
4	6
5	4
6	7

← desired

30 total

$\text{Probability} = \dfrac{\text{desired outcomes}}{\text{total outcomes possible}}$

$P_{(4)} = \dfrac{6}{30}$

Based on these data, what is the empirical probability of tossing a 4?
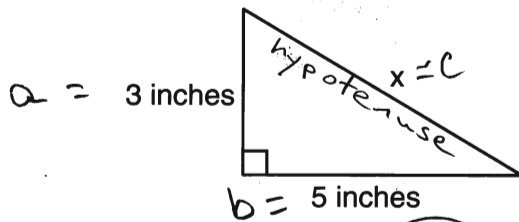
(1) $\dfrac{8}{30}$ (3) $\dfrac{5}{30}$

(2) $\dfrac{6}{30}$ (4) $\dfrac{1}{30}$

9 What is the value of x, in inches, in the right triangle below?

$a = 3$ inches   hypotenuse $x = c$

$b = 5$ inches

(1) $\sqrt{15}$ (3) $\sqrt{34}$

(2) 8   (4) 4

Pythagorean Theorem

$a^2 + b^2 = c^2$

$3^2 + 5^2 = c^2$

$9 + 25 = c^2$

$34 = c^2$

$\sqrt{34} = c$

10 What is $\sqrt{32}$ expressed in simplest radical form?

(1) $16\sqrt{2}$

(2) $4\sqrt{2}$

(3) $4\sqrt{8}$

(4) $2\sqrt{8}$

$\sqrt{32} = \sqrt{2}\,\sqrt{16}$

$\sqrt{32} = \sqrt{2}\,(4)$

$\sqrt{32} = 4\sqrt{2}$

11 If the speed of sound is 344 meters per second, what is the approximate speed of sound, in meters per hour?

| 60 seconds = 1 minute |
| 60 minutes = 1 hour |

$344\,(60)(60) = 1,238,400$

(1) 20,640

(2) 41,280

(3) 123,840

(4) 1,238,400

12 The sum of two numbers is 47, and their difference is 15. What is the larger number?

$x + y = 47 \qquad x - y = 15$

(1) 16

(2) 31

(3) 32

(4) 36

$$\begin{array}{r} x + y = 47 \\ x - y = 15 \\ \hline 2x \qquad = 62 \\ x \qquad = 31 \end{array}$$

$31 + y = 47$

$y = 47 - 31$

$y = 16$

13 If $a + ar = b + r$, the value of a in terms of b and r can be expressed as

(1) $\dfrac{b}{r} + 1$

(2) $\dfrac{1+b}{r}$

(3) $\dfrac{b+r}{1+r}$

(4) $\dfrac{1+b}{r+b}$

$a + ar = b + r$
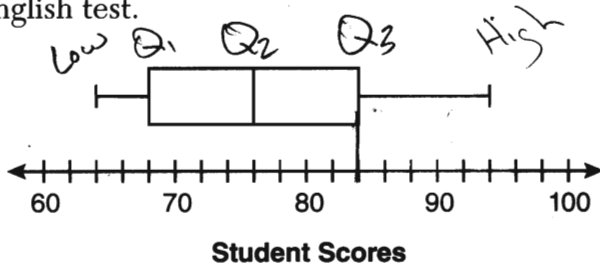
$a(1+r) = b + r$

$a = \dfrac{b+r}{1+r}$

14 Which value of x is in the solution set of $\frac{4}{3}x + 5 < 17$?

(1) 8

(2) 9

(3) 12

(4) 16

Handwritten work:

$\frac{4}{3}x + 5 < 17$

$\frac{4}{3}x < 17 - 5$

$\frac{4}{3}x < 12$

$\frac{4x}{3} < 12$

$4x < 3(12)$

$4x < 36$

$x < 9$

Careful-Don't pick 9

$x < 9$

15 The box-and-whisker plot below represents students' scores on a recent English test.

What is the value of the upper quartile?

(1) 68

(2) 76

(3) 84

(4) 94

16 Which value of n makes the expression $\frac{5n}{2n-1}$ undefined?

(1) 1

(2) 0

(3) $-\frac{1}{2}$

(4) $\frac{1}{2}$

Handwritten: undefined happens when you divide by zero

$2n - 1 = 0$

$2n = 1$

$n = \frac{1}{2}$

17 At Genesee High School, the sophomore class has 60 more students than the freshman class. The junior class has 50 fewer students than twice the students in the freshman class. The senior class is three times as large as the freshman class. If there are a total of 1,424 students at Genesee High School, how many students are in the freshman class?

(1) 202

(2) 205

(3) 235

(4) 236

Handwritten work:

$So = 60 + F$

$J = -50 + 2F$

$Se = 3F$

$F + So + J + Se = 1424$

Freshmen + Sophomores + Jrs + Seniors = 1424

$(F) + (60 + F) + (-50 + 2F) + (3F) = 1424$

$7F + 10 = 1424$

$7F = 1414$

$F = 202$

$$X = \frac{-b}{2a}$$

18 What are the vertex and axis of symmetry of the parabola $y = x^2 - 16x + 63$?

(1) vertex: $(8,-1)$; axis of symmetry: $x = 8$

(2) vertex: $(8,1)$; axis of symmetry: $x = 8$

(3) vertex: $(-8,-1)$; axis of symmetry: $x = -8$

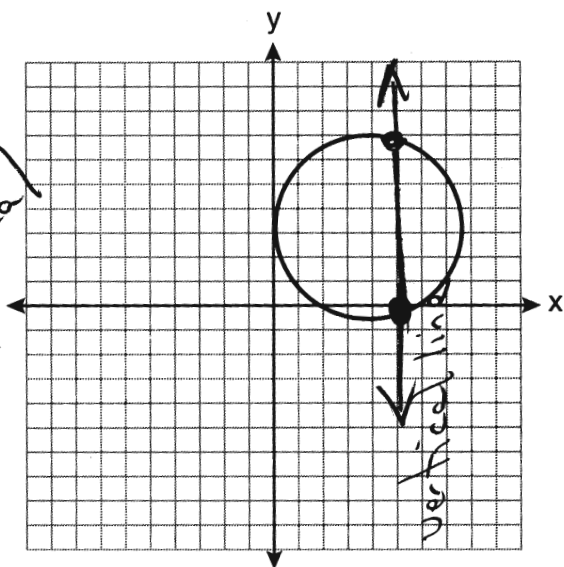(4) vertex: $(-8,1)$; axis of symmetry: $x = -8$

$$y = x^2 - 16x + 63$$
$$ax^2 + bx + c$$
$$a = 1 \quad b = -16 \quad c = 63$$
$$X = \frac{-b}{2a} = \frac{-(-16)}{2(1)} = \frac{16}{2} = 8$$

$$y = x^2 - 16x + 63$$
$$y = (8)^2 - 16(8) + 63$$
$$y = 64 - 128 + 63$$
$$y = 127 - 128$$
$$y = -1$$
$$(8, -1)$$

19 Which statement is true about the relation shown on the graph below?

Not a function. Fails vertical line test.


(1) It is a function because there exists one x-coordinate for each y-coordinate.

(2) It is a function because there exists one y-coordinate for each x-coordinate.

(3) It is *not* a function because there are multiple y-values for a given x-value.

(4) It is *not* a function because there are multiple x-values for a given y-value.

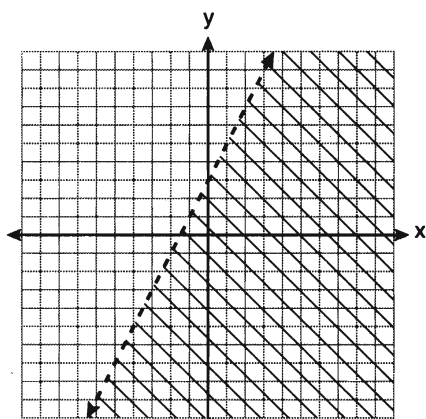**20** Which graph represents the solution of $3y - 9 \leq 6x$?

(1)

(3)

~~(2)~~

~~(4)~~

Handwritten work:
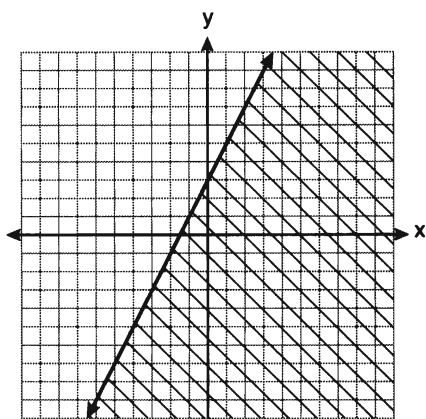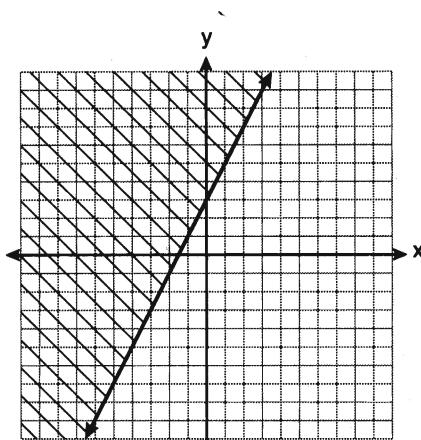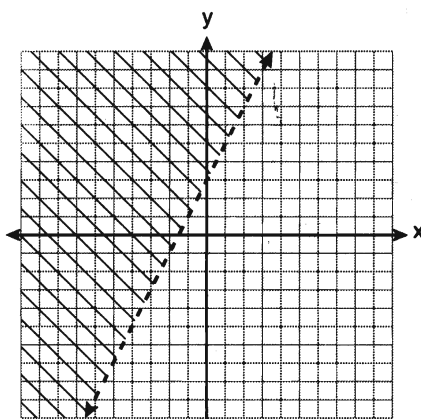
$3y - 9 \leq 6x$ ↑ solid line

$3y \boxed{-9} \leq 6x$ ✓

$3y \leq 6x + 9$

$y \leq \dfrac{6x}{3} + \dfrac{9}{3}$

$y \leq 2x + 3$

$y \leq mx + b$

$m = \text{slope} = 2 = \frac{2}{1}$

$b = y\text{-intercept} = +3$

$\leq$ shades below the line

21 Which expression represents $\dfrac{x^2 - 2x - 15}{x^2 + 3x}$ in simplest form?

(1) -5

(2) $\dfrac{x - 5}{x}$

(3) $\dfrac{-2x - 5}{x}$

(4) $\dfrac{-2x - 15}{3x}$

Handwritten work:

$\dfrac{x^2 - 2x - 15}{x^2 + 3x}$

$\dfrac{(x-5)(x+3)}{x(x+3)}$

22 What is an equation of the line that passes through the point (4,–6) and has a slope of –3?

(1) $y = -3x + 6$

(2) $y = -3x - 6$

(3) $y = -3x + 10$

(4) $y = -3x + 14$

Handwritten work:

$y = mx + b$

$y (-6)$	$-6 = -3(4) + b$
$m (-3)$	$-6 = -12 + b$
$x (4)$	$6 = b$
$b ?$	

$y = mx + b$

$y = -3x + 6$

Integrated Algebra – June '09   [8]

23 When $4x^2 + 7x - 5$ is subtracted from $9x^2 - 2x + 3$, the result is

(1) $5x^2 + 5x - 2$

(2) $5x^2 - 9x + 8$

(3) $-5x^2 + 5x - 2$

(4) $-5x^2 + 9x - 8$

$$9X^2 - 2x + 3$$
$$\text{Subtract} \quad 4X^2 + 7X - 5$$
$$\overline{5X^2 - 9X + 8}$$

24 The equation $y = x^2 + 3x - 18$ is graphed on the set of axes below.
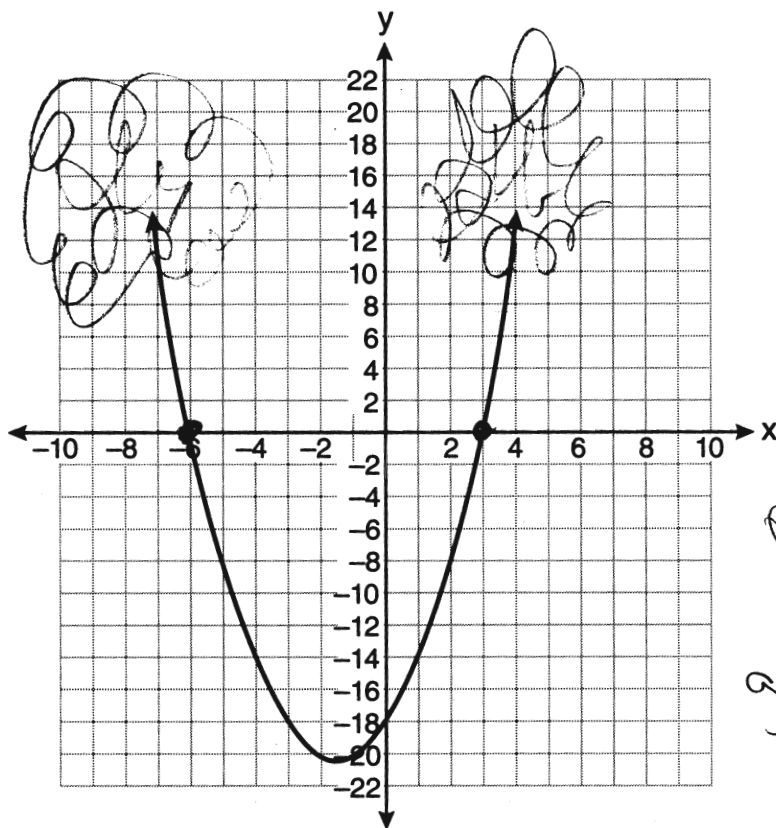
The roots of an equation are the point(s) where the graph crosses the x-axis.

Based on this graph, what are the roots of the equation $x^2 + 3x - 18 = 0$?

(1) -3 and 6

(2) 0 and -18

(3) 3 and -6

(4) 3 and -18

$$x^2 + 3X - 18 = 0$$
$$\text{Check } 3 \begin{cases} (3)^2 + 3(3) - 18 = 0 \\ 9 + 9 - 18 = 0 \checkmark \end{cases}$$
$$\text{Check } -6 \begin{cases} -6^2 + 3(-6) - 18 = 0 \\ 36 - 18 - 18 = 0 \checkmark \end{cases}$$

25 What is the value of the y-coordinate of the solution to the system of equations $x + 2y = 9$ and $x - y = 3$?

(1) 6

(2) 2

(3) 3

(4) 5

$$x + 2y = 9$$
$$x - y = 3 \quad \leftarrow \text{subtract this equation}$$
$$\overline{0 + 3y = 6}$$
$$y = 2$$

26 What is the additive inverse of the expression $a - b$?

(1) $a + b$
(3) $-a + b$
(2) $a - b$
(4) $-a - b$

Handwritten:
$$\begin{array}{c|c} a & -b \\ -a & +b \\ \hline 0 & 0 \end{array}$$

A number and its additive inverse sum to zero.

27 What is the product of 12 and 4.2×10^6 expressed in scientific notation?

(1) 50.4×10^6
(3) 5.04×10^6
(2) 50.4×10^7
(4) 5.04×10^7

Handwritten:
$$4.2 \times 10^6$$
$$12 \times 10^0$$
$$50.4 \times 10^{(6+0)}$$
$$50.4 \times 10^6$$
$$5.04 \times 10^7$$

28 To calculate the volume of a small wooden cube, Ezra measured an edge of the cube as 2 cm. The actual length of the edge of Ezra's cube is 2.1 cm. What is the relative error in his volume calculation to the nearest hundredth?

(1) 0.13
(3) 0.15
(2) 0.14
(4) 0.16

Handwritten:
$$\frac{Actual - Measured}{Actual}$$
$$\frac{2.1^{(3)} - 2^{(3)}}{2.1^{(3)}}$$
$$\frac{9.261 - 8}{9.261}$$
$$\frac{1.261}{9.261} = .13616242415 \quad Round$$
$$.14$$

29 What is $\dfrac{6}{4a} - \dfrac{2}{3a}$ expressed in simplest form?

(1) $\dfrac{4}{a}$
(3) $\dfrac{8}{7a}$
(2) $\dfrac{5}{6a}$
(4) $\dfrac{10}{12a}$

Handwritten:
$$\frac{6}{4a} - \frac{2}{3a}$$
$$\frac{18a - 8a}{12a^2} \Rightarrow \frac{10a}{12a^2} = \frac{5}{6a}$$
$$\frac{3}{2a} - \frac{2}{3a} = \frac{9}{6a} - \frac{4}{6a}$$

30 The set $\{11, 12\}$ is equivalent to

(1) $\{x | 11 < x < 12,$ where x is an integer$\}$ — no 11 or 12 in this set
(2) $\{x | 11 < x \le 12,$ where x is an integer$\}$ — no 11 in this set
(3) $\{x | 10 \le x < 12,$ where x is an integer$\}$ — no 12 in this set.
(4) $\{x | 10 < x \le 12,$ where x is an integer$\}$ — includes both 11 and 12

Part II

Answer all 3 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [6]

31 Determine how many three-letter arrangements are possible with the letters A, N, G, L, and E if no letter may be repeated.

5 letters

5 letters to choose from

3 letter arrangements

$$_5P_3 = \boxed{5} \cdot \boxed{4} \cdot \boxed{3} = \boxed{60}$$

↳ # of boxes

in first box

Answer

32 Factor completely: $4x^3 - 36x$

$$4x^3 - 36x$$

$$4(x^3 - 9x)$$

$$(4)(x)(x^2 - 9)$$

$$4x(x^2 - 9) \longleftarrow \text{difference of perfect squares}$$

$$\boxed{(4x)(x+3)(x-3)}$$

33 Some books are laid on a desk. Two are English, three are mathematics, one is French, and four are social studies. Theresa selects an English book and Isabelle then selects a social studies book. Both girls take their selections to the library to read. If Truman then selects a book at random, what is the probability that he selects an English book?

E E M M M F S S S S

↑
Theresa
takes E

↑
Isabelle
takes S
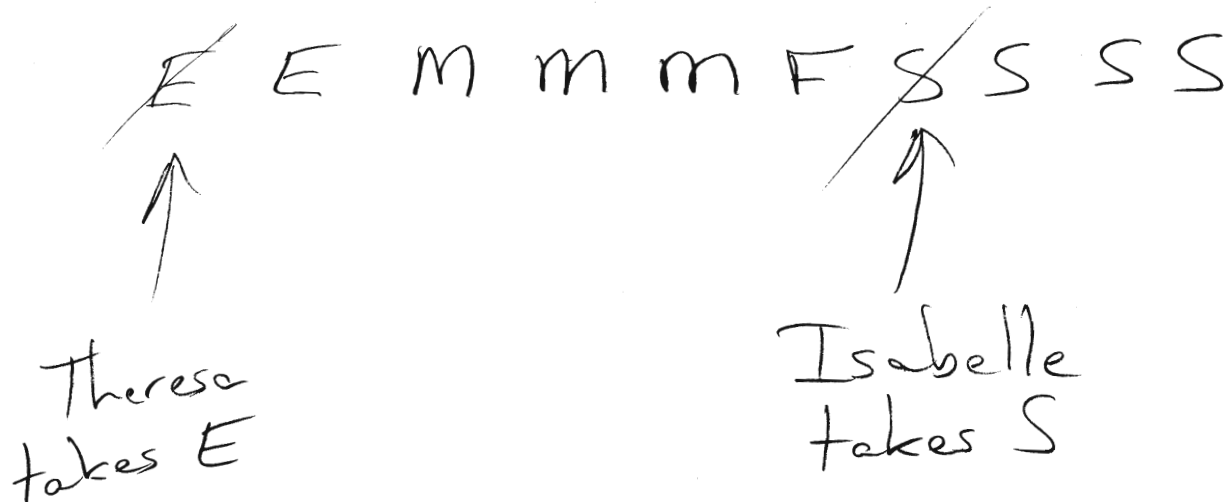
8 books are left
1 book is English

$$Probability = \frac{\# \text{ desired outcomes}}{\text{total possible outcomes}}$$

$$P\left(\begin{smallmatrix}English \\ Book\end{smallmatrix}\right) = \frac{1}{8}$$

Answer

Part III

Answer all 3 questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [9]
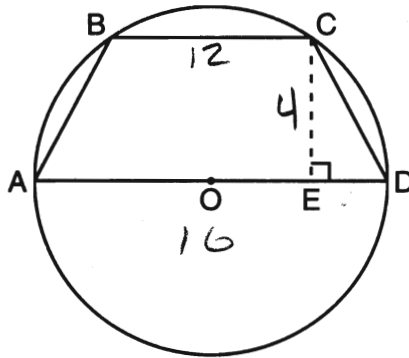
34 In the diagram below, the circumference of circle O is 16π inches. The length of $\overline{BC}$ is three-quarters of the length of diameter $\overline{AD}$ and $CE = 4$ inches. Calculate the area, in square inches, of trapezoid $ABCD$.

$$C = \pi d$$
$$C = 16\pi$$
$$C = \pi(16)$$
$$d = 16$$

$$\overline{AD} = 16$$
$$\overline{BC} = \frac{3}{4} \cdot 16$$
$$\overline{BC} = \frac{3}{4} \cdot \frac{16}{1} = \frac{48}{4} = 12$$

Area of Trapezoid $= \frac{1}{2}h(b_1 + b_2)$

Formula is from Reference Sheet

$$A = \frac{1}{2}(CE)(\overline{AD} + \overline{BC})$$
$$A = \frac{1}{2}(4)(16 + 12)$$
$$A = \frac{1}{2}(4)(28)$$
$$A = \frac{1}{2}(112)$$
$$A = 56 \text{ sq. inches}$$

35 A bank is advertising that new customers can open a savings account with a $3\frac{3}{4}\%$ interest rate compounded annually. Robert invests $5,000 in an account at this rate. If he makes no additional deposits or withdrawals on his account, find the amount of money he will have, to the *nearest cent*, after three years.

$$\left(5,000\right) \times \left(100\% + 3\frac{3}{4}\%\right)^3$$

$$\left(5,000\right) \times \left(103.75\%\right)\left(103.75\%\right)\left(103.75\%\right)$$

$$\left(5,000\right) \times \left(1.0375\right)\left(1.0375\right)\left(1.0375\right)$$

$$\left(5,000\right)\left(1.0375\right)^3$$
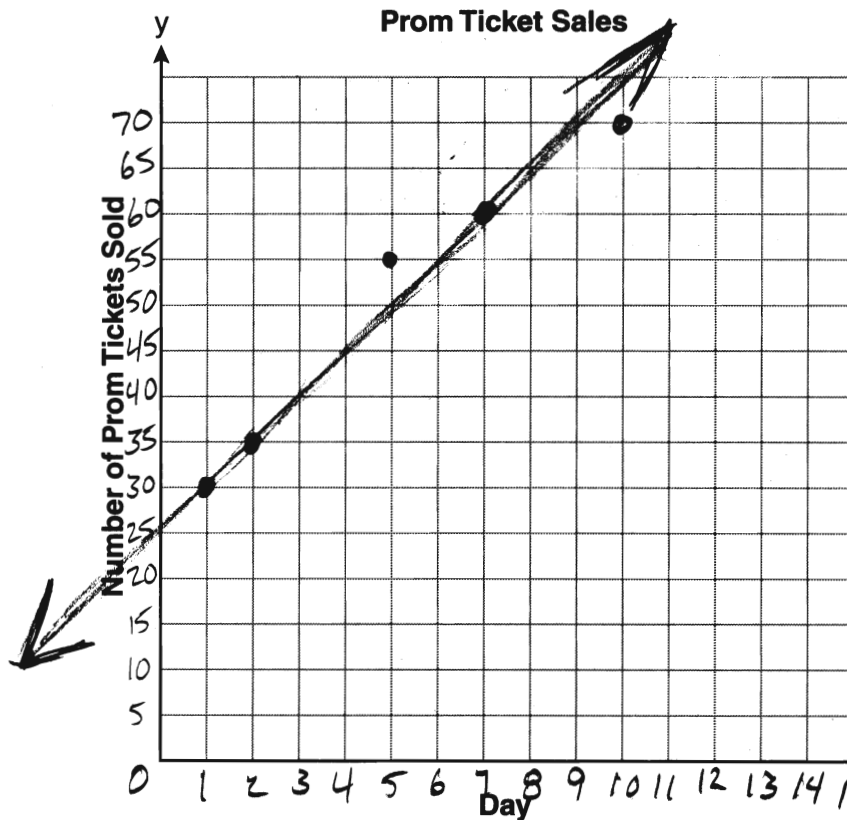
$$5583.85\overline{)7422}$$
round

$$\#\ 5583.86$$

36 The table below shows the number of prom tickets sold over a ten-day period.

Prom Ticket Sales

Day (x)	1	2	5	7	10
Number of Prom Tickets Sold (y)	30	35	55	60	70

Plot these data points on the coordinate grid below. Use a consistent and appropriate scale. Draw a reasonable line of best fit and write its equation.

Prom Ticket Sales

$$y = mx + b$$

$$\boxed{y = 5x + 25}$$
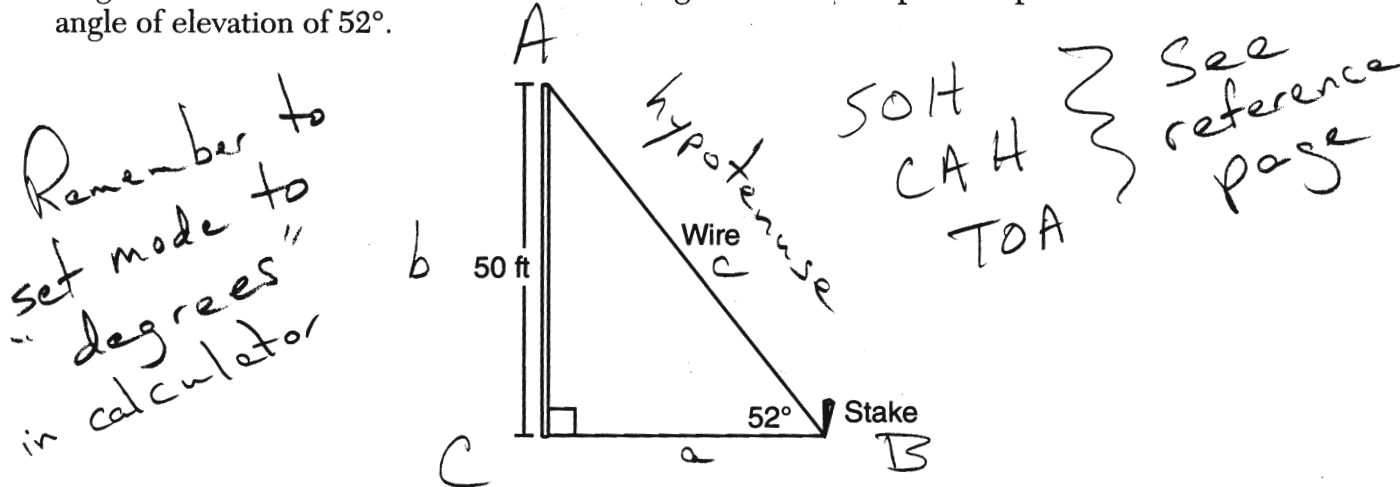
$$M = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{5}{1}$$

$$b = y\text{-intercept} \approx 25$$

Part IV

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

37 A stake is to be driven into the ground away from the base of a 50-foot pole, as shown in the diagram below. A wire from the stake on the ground to the top of the pole is to be installed at an angle of elevation of 52°.

Remember to set mode to "degrees" in calculator

SOH
CAH
TOA
} See reference page

A

b   50 ft

Wire   hypotenuse   c

52°   Stake

C   a   B

How far away from the base of the pole should the stake be driven in, to the *nearest foot*?

Find a

$$\tan = \frac{opposite}{adjacent}$$

$$\tan 52° = \frac{50}{a}$$

$$a = \frac{50}{\tan 52°} = 39.06428133$$

round

39 feet

What will be the length of the wire from the stake to the top of the pole, to the *nearest foot*?

$$\sin = \frac{opposite}{hypotenuse}$$

$$\sin 52° = \frac{50}{c}$$

$$c = \frac{50}{\sin 52°} = 63.45091075$$

round

63 feet

38 The Fahrenheit temperature readings on 30 April mornings in Stormville, New York, are shown below.

41°, 58°, 61°, 54°, 49°, 46°, 52°, 58°, 67°, 43°, 47°, 60°, 52°, 58°, 48°, ⎫ 30
44°, 59°, 66°, 62°, 55°, 44°, 49°, 62°, 61°, 59°, 54°, 57°, 58°, 63°, 60° ⎭ total

Using the data, complete the frequency table below.

Interval	Tally	Frequency
40–44	||||	4
45–49	⊬⊬	5
50–54	||||	4
55–59	⊬⊬ |||	8
60–64	⊬⊬ ||	7
65–69	||	2

30 total

On the grid on the next page, construct and label a frequency histogram based on the table.

Morning Fahrenheit Temperature Readings
Stormville, New York — April
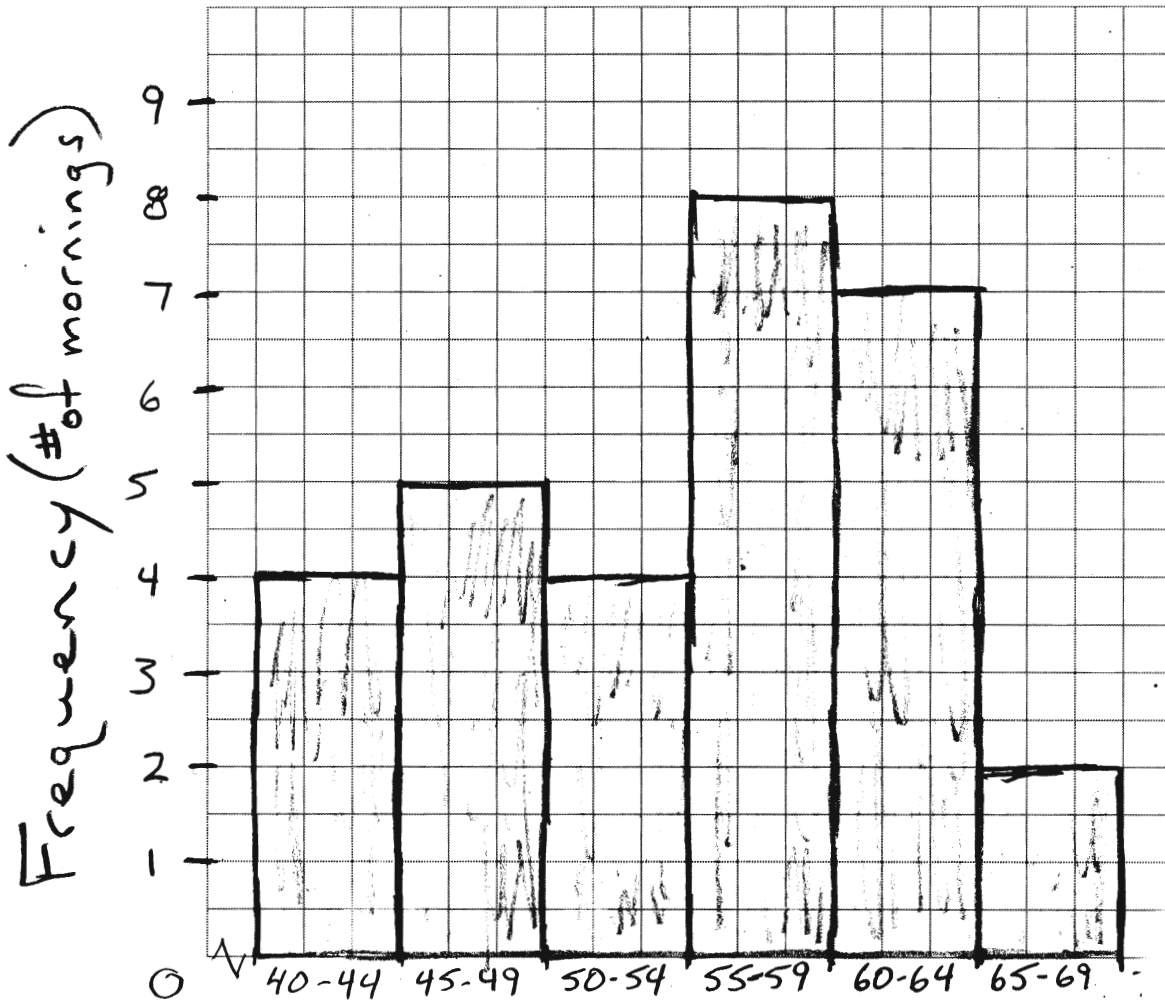
Morning Temperatures
(in degrees Fahrenheit)

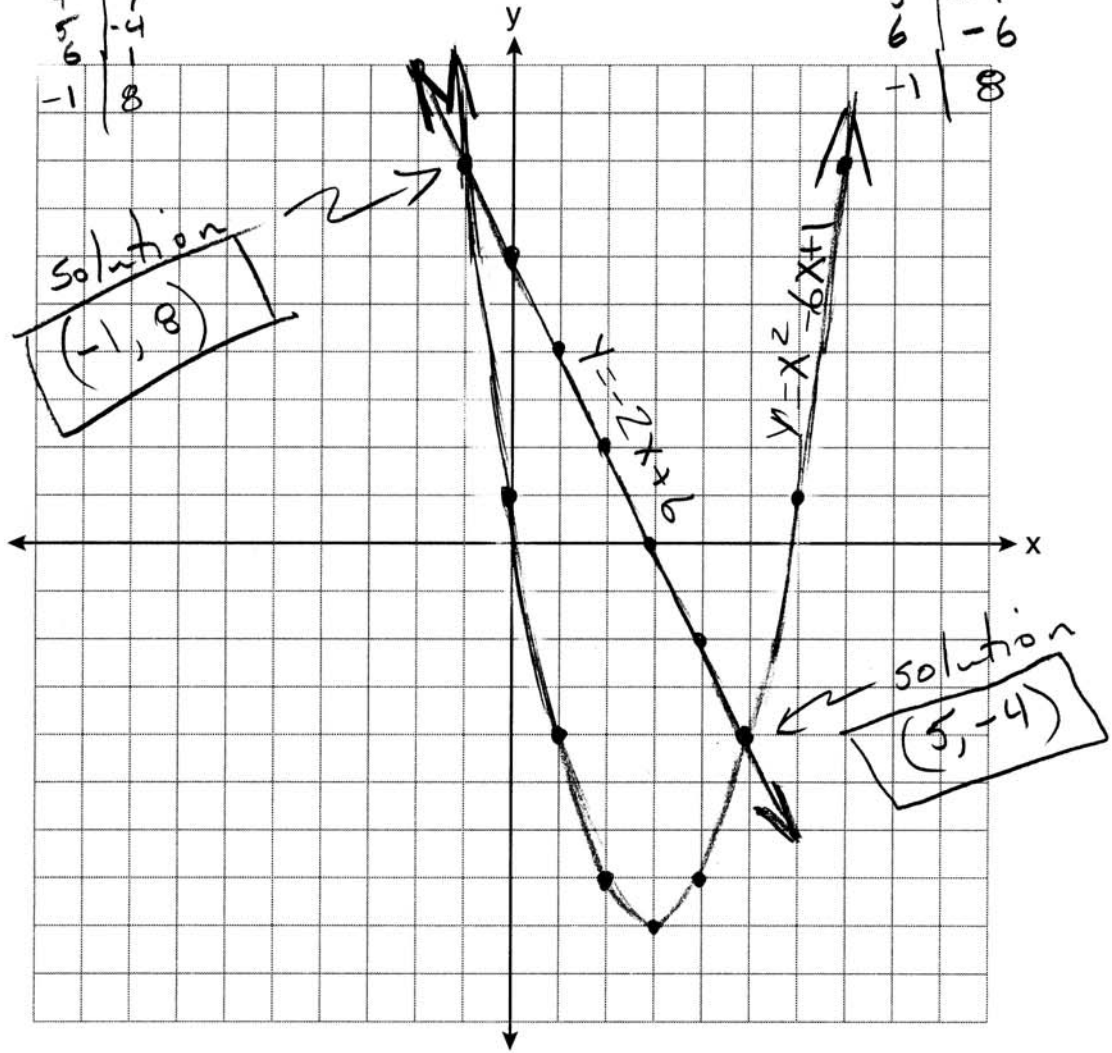**39** On the set of axes below, solve the following system of equations graphically for all values of x and y.

$$y = x^2 - 6x + 1$$
$$y + 2x = 6$$

$y = X^2 - 6X + 1$

X	Y
0	1
1	-4
2	-7
3	-8
4	-7
5	-4
6	1
-1	8

$y = -2X + 6$

X	Y
0	6
1	4
2	2
3	0
4	-2
5	-4
6	-6
-1	8

Solution
$(-1, 8)$

Solution
$(5, -4)$

$y = -2X + 6$

$y = X^2 - 6X + 1$

Note: Tables of values found using graphing calculator.

Reference Sheet

Trigonometric Ratios	$\sin A = \dfrac{opposite}{hypotenuse}$
	$\cos A = \dfrac{adjacent}{hypotenuse}$
	$\tan A = \dfrac{opposite}{adjacent}$

| Area | trapezoid | $A = \frac{1}{2}h(b_1 + b_2)$ |

| Volume | cylinder | $V = \pi r^2 h$ |

| Surface Area | rectangular prism | $SA = 2lw + 2hw + 2lh$ |
| | cylinder | $SA = 2\pi r^2 + 2\pi rh$ |

| Coordinate Geometry | $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$ |

Tear Here

Tear Here

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

INTEGRATED ALGEBRA

Friday, June 19, 2009 — 1:15 to 4:15 p.m., only

ANSWER SHEET

Student .....*Imaginary Student*..... Sex: ☐ Male ☐ Female Grade ....

Teacher ....*Mr. Steve*............. School *IHS @ PH*

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer all 30 questions in this part.

1 4	9 3	17 1	25 2
2 4	10 2	18 1	26 3
3 1	11 4	19 3	27 4
4 2	12 2	20 1	28 4
5 3	13 3	21 2	29 2
6 4	14 1	22 1	30 4
7 1	15 3	23 2	
8 2	16 4	24 3	

Your answers for Parts II, III, and IV should be written in the test booklet.

The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination and that I have neither given nor received assistance in answering any of the questions during the examination.

R S Watson
_____
Signature