The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

INTEGRATED ALGEBRA

Tuesday, January 24, 2012 — 9:15 a.m. to 12:15 p.m., only

Student Name: _____ Steve Watson _____

School Name: _____ JMAP _____

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice…

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 30 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [60]

Use this space for computations.

1 Which expression is equivalent to $64 - x^2$?

(1) $(8 - x)(8 - x)$ (3) $(x - 8)(x - 8)$

● $(8 - x)(8 + x)$ (4) $(x - 8)(x + 8)$

(1) $64 - 16x + x^2$ (3) $x^2 - 16x + 64$

(2) $64 + 8x - 8x - x^2$ (4) $x^2 - 64$

2 Mr. Smith invested \$2,500 in a savings account that earns 3% interest compounded annually. He made no additional deposits or withdrawals. Which expression can be used to determine the number of dollars in this account at the end of 4 years?

● $2500(1 + 0.03)^4$ (3) $2500(1 + 0.04)^3$ — 4% for 3 years

(2) $2500(1 + 0.3)^4$ (4) $2500(1 + 0.4)^3$ — 40% for 3 years

30% for 4 years

3 What is $2\sqrt{45}$ expressed in simplest radical form?

(1) $3\sqrt{5}$ ● $6\sqrt{5}$

(2) $5\sqrt{5}$ (4) $18\sqrt{5}$

$2\sqrt{45}$

$2\sqrt{9}\sqrt{5}$

$2 \cdot 3\sqrt{5}$

$6\sqrt{5}$

Check using calculator

$2\sqrt{45} = 13.41640786$

this works) $6\sqrt{5} = 13.41640786$

$3\sqrt{5} = 1.709975947$

These don't work $5\sqrt{5} = 1.37929661$

$18\sqrt{5} = 40.2492236$

4 Which graph does *not* represent a function?
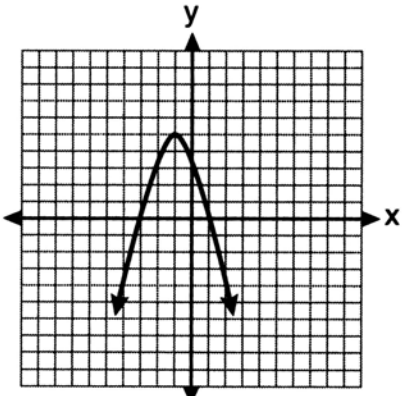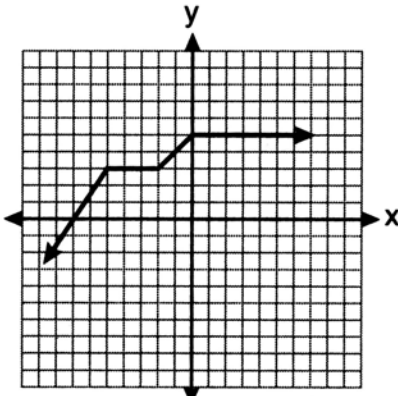
(1)

(2)

(4)

Fails the vertical line test

Function Definition
For every value of x, there is one and only one value of y

5 Timmy bought a <u>skateboard</u> and <u>two helmets</u> for a total of d dollars. If each helmet cost h dollars, the cost of the skateboard could be represented by

s + 2h = d
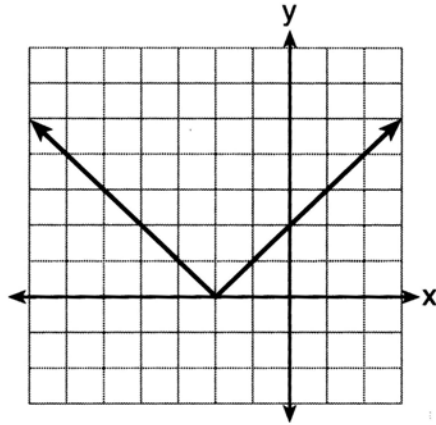
(1) $2dh$

● $d - 2h$

(2) $\dfrac{dh}{2}$

(4) $d - \dfrac{h}{2}$

s + 2h = d
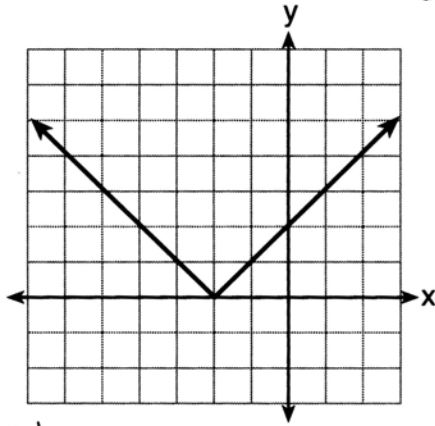*s = d - 2h*

6 The graph of $y = |x + 2|$ is shown below.

Every y value is positive
Touches x-axis at (-2,0).

Which graph represents $y = -|x + 2|$?

Makes the absolute value turn negative
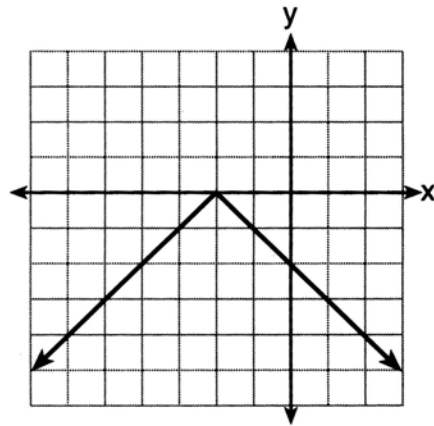
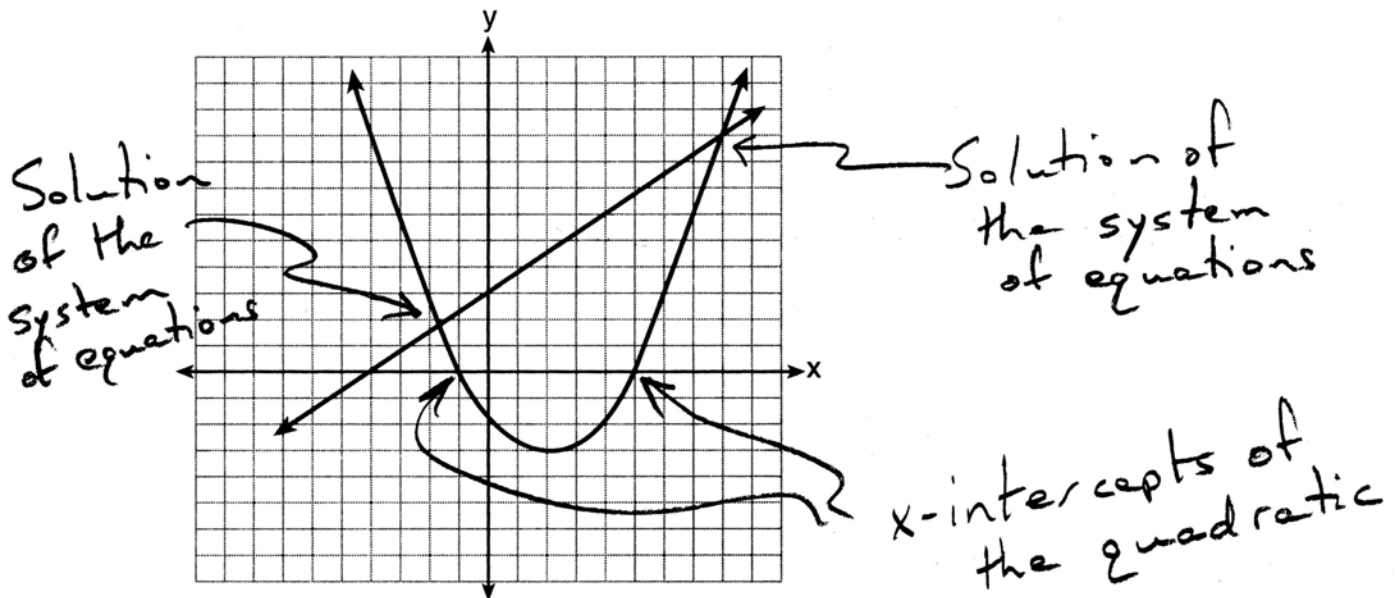*No Change* (1)
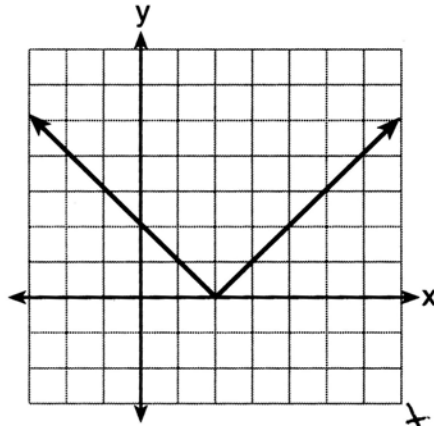
(3) *Does not touch x-axis at (-2,0)*
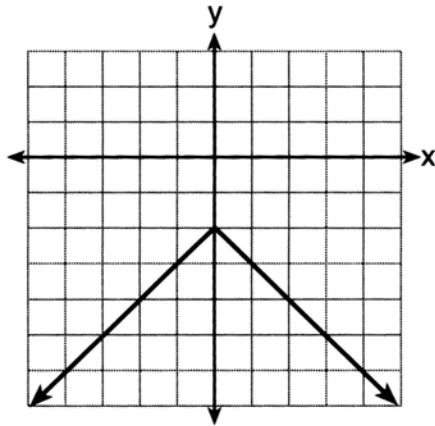
(2)

Does not touch x-axis at all.

● *This is the mirror image of |x+2| reflected on the negative side of the x-axis.*

[4]

7 Two equations were graphed on the set of axes below.

Solution of the system of equations

Solution of the system of equations

x-intercepts of the quadratic

Which point is a solution of the system of equations shown on the graph?

● (8,9)   (3) (0,3)

(2) (5,0)   (4) (2,−3)

Check
8
5                    $8 \times 5 = 40$

8 Byron is 3 years older than Doug. The product of their ages is 40. How old is Doug?

(1) 10   ● 5

(2) 8   (4) 4

$B = (3 + D)$

$BD = 40$

$(3+D)D = 40$

$3D + D^2 = 40$

$D^2 + 3D = 40$

$D^2 + 3D - 40 = 0$

$(D-5)(D+8) = 0$

$D - 5 = 0$ $D + 8 = 0$

$D = 5$ $D = -8$

Byron (is) 3 years older than Doug

$B = +3 \qquad + D$

The product of their ages (is) 40

Multiply BD $= 40$

Relative Error = $\dfrac{\text{measured} - \text{actual}}{\text{actual}}$

9 The actual dimensions of a rectangle are 2.6 cm by 6.9 cm. Andy measures the sides as 2.5 cm by 6.8 cm. In calculating the area, what is the <u>relative error</u>, to the *nearest thousandth*?

(1) 0.055

(3) 0.022

● 0.052

(4) 0.021

$$\frac{\overset{(17.94)}{(2.6 \times 6.9)} - \overset{(17)}{(2.5 \times 6.8)}}{\underset{(17.94)}{(2.6 \times 6.9)}} = \frac{.94}{17.94}$$

$$\frac{.94}{17.94} = .052\big|396878$$

$$.052\big|$$

10 Which graph represents the inequality $y > 3$?

●
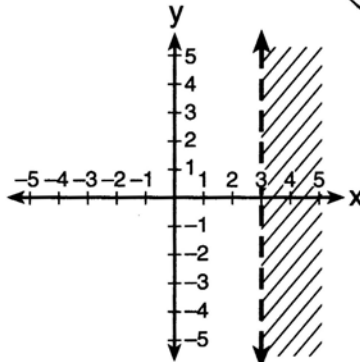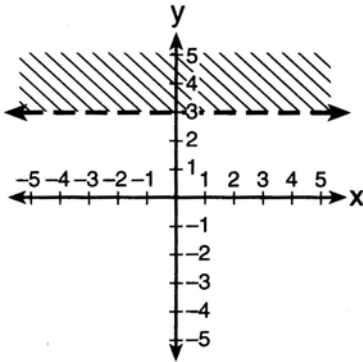
(3)   This graph shows $X > 3$

(2)   This graph shows $y \geq 3$

(4)   This graph shows $X \geq 3$

11 Which set of data can be classified as <u>quantitative</u>? ← numerical

   (1) first names of students in a chess club

   (2) ages of students in a government class   *ages are numerical*

   (3) hair colors of students in a debate club

   (4) favorite sports of students in a gym class

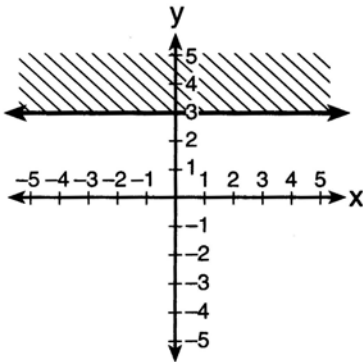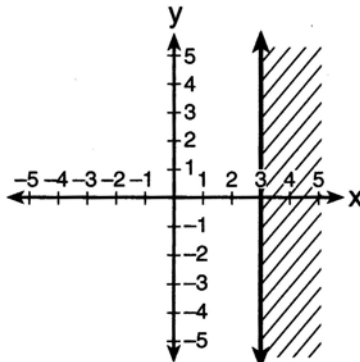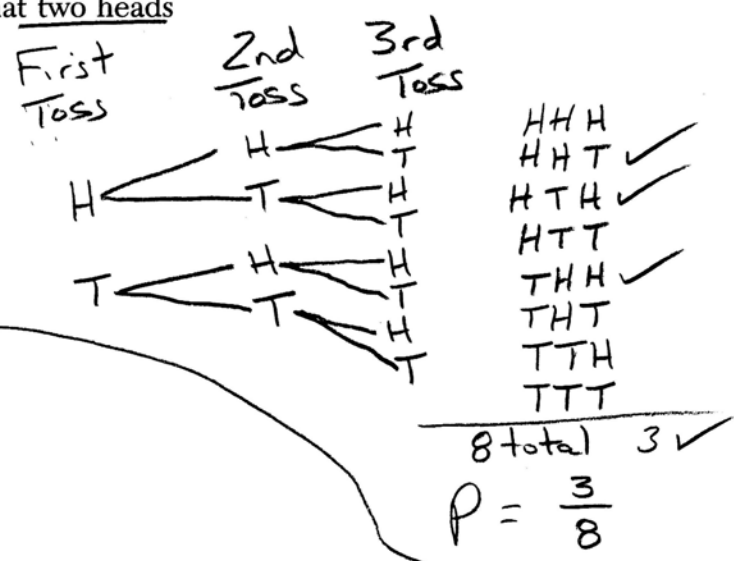**12** Three fair coins are tossed. What is the probability that <u>two heads</u> and <u>one tail</u> appear?

   (1) $\dfrac{1}{8}$ (3) $\dfrac{3}{6}$

   ● $\dfrac{3}{8}$ (4) $\dfrac{2}{3}$

First Toss 2nd Toss 3rd Toss

HHH
HHT ✓
HTH ✓
HTT
THH ✓
THT
TTH
TTT

8 total 3 ✓

$P = \dfrac{3}{8}$

13 What is the *sum* (add) of $-3x^2 - 7x + 9$ and $-5x^2 + 6x - 4$?

   ● $-8x^2 - x + 5$ (3) $-8x^2 - 13x + 13$

   (2) $-8x^4 - x + 5$ (4) $-8x^4 - 13x^2 + 13$

$$-3x^2 - 7x + 9$$
$$-5x^2 + 6x - 4$$
$$\overline{-8x^2 - x + 5}$$

14 For which values of x is the fraction $\dfrac{x^2 + x - 6}{x^2 + 5x - 6}$ undefined? → *denominator is zero*

   (1) 1 and −6   (3) 3 and −2

   (2) 2 and −3   (4) 6 and −1

$$x^2 + 5x - 6 = 0$$
$$(x+6)(x-1) = 0$$

$x + 6 = 0$ $x - 1 = 0$

$\boxed{x = -6}$ $\boxed{x = 1}$

Check
$$x^2 + 5x - 6 = 0$$
$$(-6)^2 + 5(-6) - 6 = 0$$
$$36 \quad - 30 \quad -6 = 0 \checkmark$$
$$36 - 36 = 0 \checkmark$$
$$(1)^2 + 5(1) - 6 = 0$$
$$1 + 5 - 6 = 0$$
$$6 - 6 = 0 \checkmark$$

15 What is the slope of the line that passes through the points $(2,-3)$ and $(5,1)$?

(1) $-\dfrac{2}{3}$

(3) $-\dfrac{4}{3}$

(2) $\dfrac{2}{3}$

● $\dfrac{4}{3}$

$slope = m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m = \dfrac{1-(-3)}{5-2} = \boxed{\dfrac{4}{3}}$

See other solution on graph paper.

16 The expression $\dfrac{(4x^3)^2}{2x}$ is equivalent to

(1) $4x^4$

(3) $8x^4$

(2) $4x^5$

● $8x^5$

$\dfrac{(4x^3)^2}{2x} = \dfrac{16\,x^6}{2\,x}$

$= 8\,x^{(6-1)} = 8x^5$

17 In the diagram below, circle O is inscribed in square $ABCD$. The square has an area of 36.

$Area_\square = S^2$

$S^2 = 36$

$S = \sqrt{36}$

$S = 6$
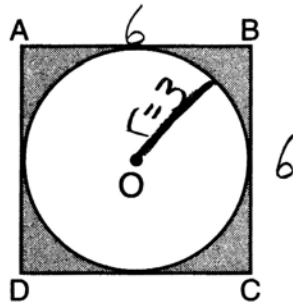
$Area_\bigcirc = \pi r^2$

$r = 3$

$Area_\bigcirc = \pi(3)^2$

$Area_O = 9\pi$

What is the area of the circle?

● 9π

(3) 3π

(2) 6π

(4) 36π

18 Which point lies on the graph represented by the equation $3y + 2x = 8$?

(1) $(-2,7)$

(3) $(2,4)$

(2) $(0,4)$

● $(7,-2)$

$3y + 2x = 8$

	$3y + 2x = 8$	
$(-2,7)$	$3(7) + 2(-2) = 8 \rightarrow$	$17 \neq 8$
$(0,4)$	$3(4) + 2(0) = 8 \rightarrow$	$12 \neq 8$
$(2,4)$	$3(4) + 2(2) = 8 \rightarrow$	$16 \neq 8$
$(7,-2)$	$3(-2) + 2(7) = 8$	$8 = 8$ ✓

19 The equation of the axis of symmetry of the graph of $y = 2x^2 - 3x + 7$ is

a   *b*   *c*

● (1) $x = \dfrac{3}{4}$

(3) $x = \dfrac{3}{2}$

(2) $y = \dfrac{3}{4}$

(4) $y = \dfrac{3}{2}$

axis of symmetry $= \dfrac{-b}{2a}$

$x = \dfrac{-(-3)}{2(2)}$

$\boxed{x = \dfrac{3}{4}}$

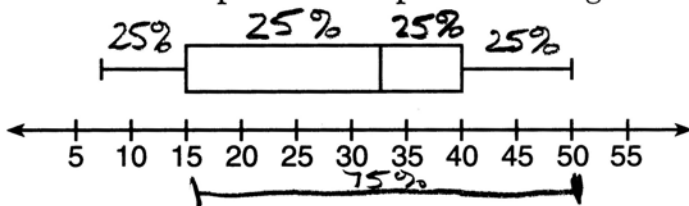**20** The box-and-whisker plot below represents the ages of 12 people.

25%   25%   25%   25%

5 10 15 20 25 30 35 40 45 50 55

75%

What percent of these people are age 15 or older?

(1) 25

● (3) 75

(2) 35

(4) 85

21 Campsite *A* and campsite *B* are located directly opposite each other on the shores of Lake Omega, as shown in the diagram below. The two campsites form a right triangle with Sam's position, *S*. The distance from campsite *B* to Sam's position is 1,300 yards, and campsite *A* is 1,700 yards from his position.

A

1,700 yards

a

C

Lake Omega

B   1,300 yards   S

b
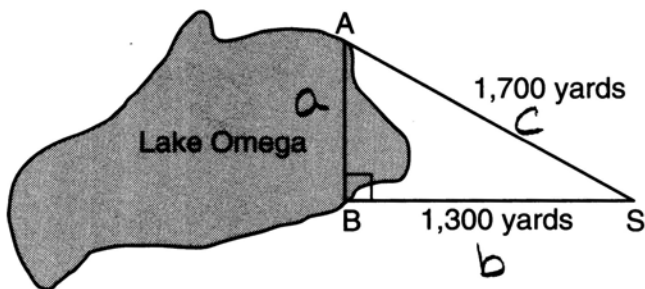
What is the distance from campsite *A* to campsite *B*, to the *nearest yard*?

● (1) 1,095

(3) 2,140

(2) 1,096

(4) 2,141

Pythagorean Theorem

$a^2 + b^2 = c^2$

$a^2 + (1300)^2 = (1700)^2$

$a^2 = (1700)^2 - (1300)^2$

$a = \sqrt{(1700)^2 - (1300)^2}$

$a = 1095.445115$

$a = 1095$

Check

$(1095)^2 + (1300)^2 = (1700)^2$

$1,199,025 + 1,690,000 = 2,890,000$

$2,889,025 \approx 2,890,000$

Difference is due to rounding

22 Which set builder notation represents {−2, −1, 0, 1, 2, 3}?

(1) {x|−3 ≤ x ≤ 3, where x is an integer} *includes −3*

(2) {x|−3 < x ≤ 4, where x is an integer} *includes 4*

(3) {x|−2 < x < 3, where x is an integer} *does not include −2*

(4) {x|−2 ≤ x < 4, where x is an integer}

23 The roots of the equation $3x^2 - 27x = 0$ are

● 0 and 9

(2) 0 and −9

(3) 0 and 3

(4) 0 and −3

$$3x^2 - 27x = 0$$
$$3x(x-9) = 0$$

$3x = 0$ $x - 9 = 0$

$x = \frac{0}{3}$ $\boxed{x = 9}$

$\boxed{x = 0}$

24 Which equation is an example of the use of the associative property of addition?

(1) $x + 7 = 7 + x$ *Commutative property of addition*

(2) $3(x + y) = 3x + 3y$ *distributive property*

(3) $(x + y) + 3 = x + (y + 3)$

(4) $3 + (x + y) = (x + y) + 3$ *commutative property*

25 Given:

$A = \{2, 4, 5, 7, 8\}$

$B = \{3, 5, 8, 9\}$

The symbol U means union, or joining together.

What is $A \cup B$?
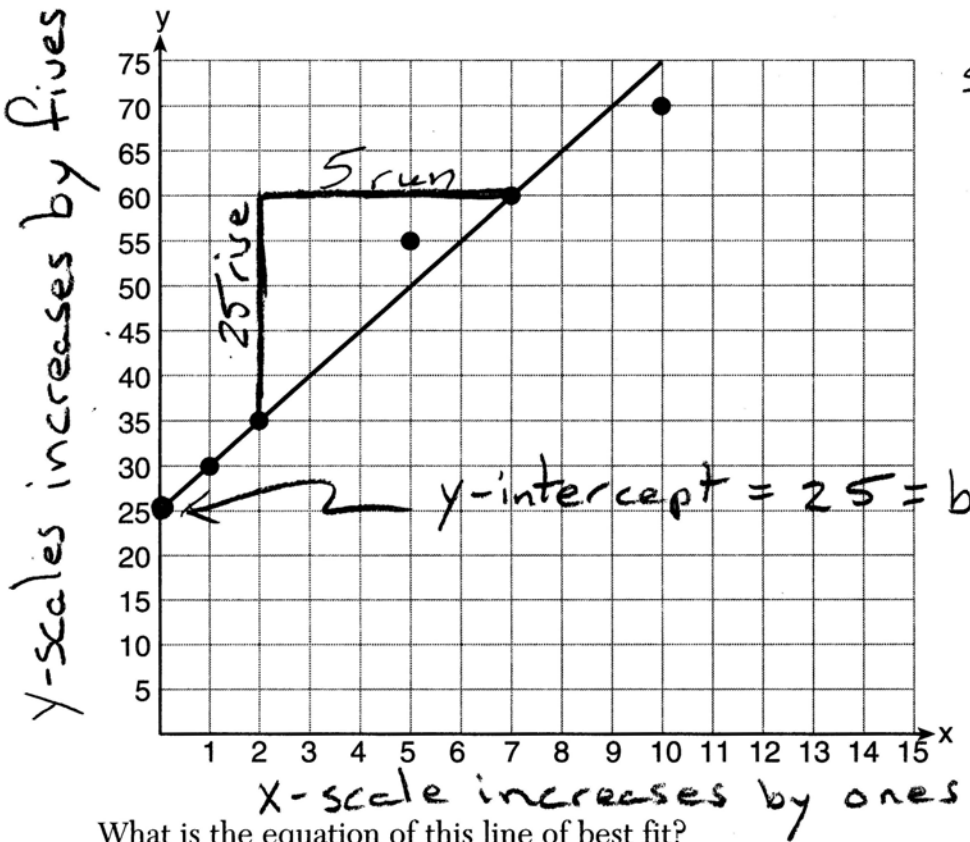
(1) {5}

(2) {5, 8}

(3) {2, 3, 4, 7, 9}

● {2, 3, 4, 5, 7, 8, 9}

[handwritten union computation]

2	−	4	5	−	7	8	
	3	−	5	−	−	8	9
2	3	4	5	−	7	8	9

26 The diagram below shows right triangle *LMP*.

SOH-CAH-TOA

$\tan = \dfrac{opposite}{adjacent} = \boxed{\dfrac{4}{3}}$

Which ratio represents the tangent of ∠*PLM*?

(1) $\dfrac{3}{4}$ ● $\dfrac{4}{3}$

(2) $\dfrac{3}{5}$ (4) $\dfrac{5}{4}$

27 Mr. Stanton asked his students to write an algebraic expression on a piece of paper. He chose four students to go to the board and write their expression.

Robert wrote: $4(2x + 5) \geq 17$ inequality

Meredith wrote: $3y - 7 + 11z$ expression

Steven wrote: $9w + 2 = 20$ equation

Cynthia wrote: $8 + 10 - 4 = 14$ equation

Which student wrote an algebraic expression?

(1) Robert   (3) Steven

● Meredith   (4) Cynthia

28 If $s = \dfrac{2x + t}{r}$, then x equals

● $\dfrac{rs - t}{2}$ (3) $2rs - t$

(2) $\dfrac{rs + 1}{2}$ (4) $rs - 2t$

$s = \dfrac{2x + t}{r}$

$(r)s = \left(\dfrac{2x + t}{r}\right)r$

$rs = 2x + t$

$rs - t = 2x$

$\dfrac{rs - t}{2} = x$

29 A scatter plot was constructed on the graph below and a line of best fit was drawn.

$y = mx + b$

$slope = m = \dfrac{rise}{run} = \dfrac{25}{5} = 5$

$m = 5$

$b = 25$

$\boxed{y = 5x + 25}$

y-scales increases by fives

25 rise   5 run

y-intercept $= 25 = b$

X-scale increases by ones

What is the equation of this line of best fit?

(1) $y = x + 5$ (3) $y = 5x + 5$

(2) $y = x + 25$ ● $y = 5x + 25$

30 What is the sum of $\dfrac{2y}{y + 5}$ and $\dfrac{10}{y + 5}$ expressed in simplest form?

$\dfrac{2y}{y+5} + \dfrac{10}{y+5}$

$\dfrac{2y+10}{y+5}$

$\dfrac{2(y+5)}{(y+5)}$

(1) 1   (3) $\dfrac{12y}{y + 5}$

● 2   (4) $\dfrac{2y + 10}{y + 5}$

$\boxed{2}$

Part II

Answer all 3 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

31 The length and width of the base of a rectangular prism are 5.5 cm and 3 cm. The height of the prism is 6.75 cm. Find the *exact* value of the surface area of the prism, in square centimeters.

rectangular prism

from the formula sheet

$$SA = 2\ell w + 2hw + 2\ell h$$

$$SA = 2(5.5)(3) + 2(6.75)(3) + 2(5.5)(6.75)$$

$$SA = 33 + 6(6.75) + 11(6.75)$$

$$SA = 33 + 40.5 + 74.25$$

$$SA = 147.75$$

$$\boxed{147.75 \text{ cm}^2}$$

32 Casey purchased a pack of assorted flower seeds and planted them in her garden. When the first 25 flowers bloomed, 11 were white, 5 were red, 3 were blue, and the rest were yellow. Find the empirical probability that a flower that blooms will be yellow.

$$\text{Probability} = \frac{\#\ observations\ of\ event}{total\ \#\ of\ observations}$$

White	11
Red	5
Blue	3
Yellow	? = 6
Total	25

$$P = \boxed{\frac{6}{25}}$$

33 Express in simplest form: $\dfrac{x^2 - 1}{x^2 + 3x + 2}$

$$\frac{x^2 - 1}{x^2 + 3x + 2}$$

$$\frac{(x+1)(x-1)}{(x+2)(x+1)}$$

$$\boxed{\frac{x-1}{x+2}}$$

Part III

Answer all 3 questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [9]

34 Solve algebraically for x: $2(x - 4) \geq \frac{1}{2}(5 - 3x)$

$$2(x-4) \geq \tfrac{1}{2}(5-3x)$$

Multiply everything by 2

$$4(x-4) \geq 5-3x$$

Distributive Property

$$4x - 16 \geq 5 - 3x$$

Add 3x $+3x \qquad\qquad +3x$

$$7x - 16 \geq 5$$

Add 16 $+16 \qquad +16$

$$7x \geq 21$$

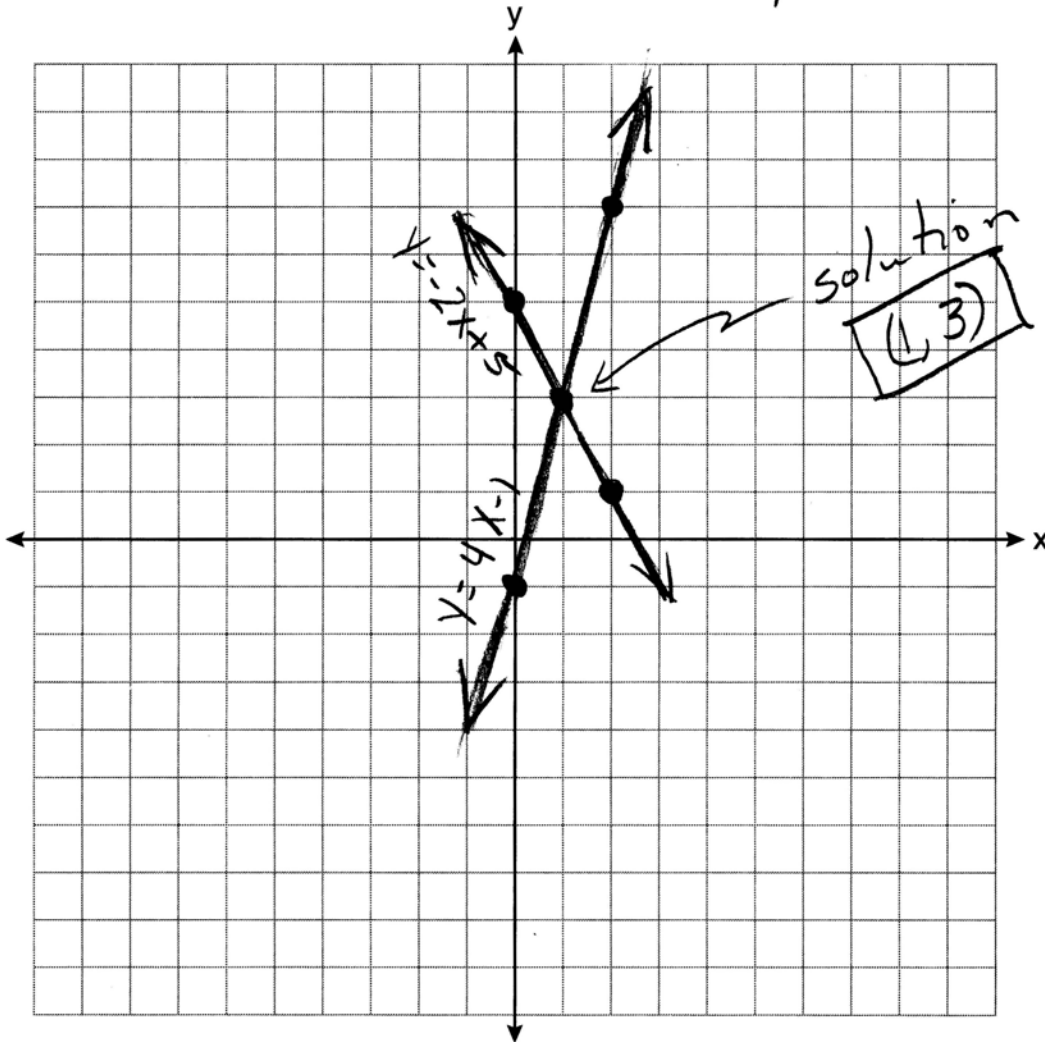Divide by 7 $\dfrac{7x}{7} \geq \dfrac{21}{7}$

$$\boxed{x \geq 3}$$

35 On the set of axes below, solve the following system of equations graphically. State the coordinates of the solution.

$$y = mx + b$$
$$y = 4x - 1$$
$$2x + y = 5 \quad \rightarrow \quad y = -2x + 5$$
$$y = mx + b$$

solution

(1, 3)

$y = -2x + 5$

$y = 4x - 1$

36 A turtle and a rabbit are in a race to see who is first to reach a point 100 feet away. The turtle travels at a constant speed of 20 feet per minute for the entire 100 feet. The rabbit travels at a constant speed of 40 feet per minute for the first 50 feet, stops for 3 minutes, and then continues at a constant speed of 40 feet per minute for the last 50 feet.

Determine which animal won the race and by how much time.
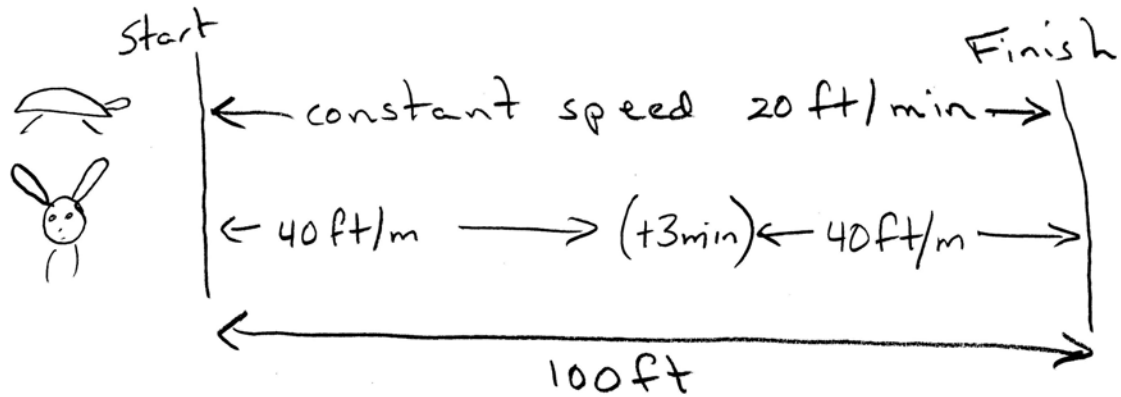
Turtle Time

$$\text{distance} = \text{speed} \times \text{time}$$

$$100 = 20t$$

$$\frac{100}{20} = t$$

5 minutes

The turtle won the race by $\frac{1}{2}$ minute.

Rabbit Time

$$\text{distance} = \text{speed} \times \text{time}$$

First 50 feet

$$50 = 40t$$

$$\frac{50}{40} = t$$

$$\frac{5}{4} = t$$

Second 50 feet   Same as 1st half.

Add 3 minutes for stopping

$$\frac{5}{4} + \frac{5}{4} + 3 = \text{total time}$$

$$\frac{10}{4} + 3 = \text{total time}$$

$$2\frac{1}{2} + 3 = \text{total time}$$

$$5\frac{1}{2} \text{ minutes}$$

Part IV

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

37 The sum of three consecutive odd integers is 18 less than five times the middle number. Find the three integers. [Only an algebraic solution can receive full credit.]

Let X represent the 1st odd integer

Let X+2 " " 2nd " "

Let X+4 " " 3rd " "

$$X + X + 2 + X + 4 = -18 + 5(X+2)$$

1st #   2nd #   3rd #   18 less   5 times middle #

$$3X + 6 = -18 + 5X + 10$$

$$3X + 6 = 5X - 8$$

$$\quad +8 \qquad\qquad +8$$

$$3X + 14 = 5X$$

$$-3X \qquad -3X$$

$$14 = 2X$$

$$7 = X$$

Check
7+9+11 = -18 + 5(9)
27 = -18 + 45
27 = 27 ✓

Answers

1st # 7
2nd # 9
3rd # 11

38 A sandwich consists of one type of bread, one type of meat, and one type of cheese. The possible choices are listed below.

Bread: white, rye

Meat: ham, turkey, beef

Cheese: American, Swiss

Draw a tree diagram or list a sample space of all the possible different sandwiches consisting of one type of bread, one type of meat, and one type of cheese.
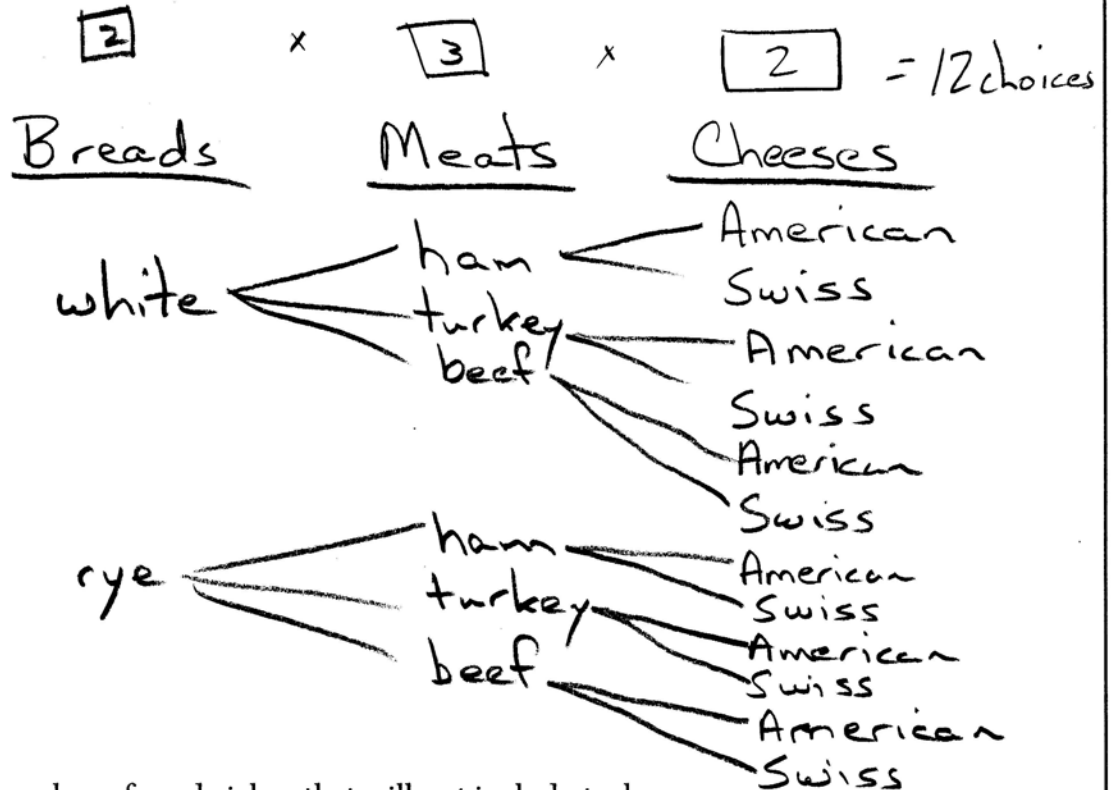
Determine the number of sandwiches that will *not* include turkey.

$$\boxed{2} \times \boxed{2} \times \boxed{2} = \boxed{8} \text{ answer}$$

Determine the number of sandwiches that will include rye bread and Swiss cheese.

$$\boxed{1} \times \boxed{3} \times \boxed{1} = \boxed{3} \text{ answer}$$

39 Shana wants to buy a new bicycle that has a retail price of $259.99. She knows that it will be on sale next week for 30% off the retail price. If the tax rate is 7%, find the total amount, to the *nearest cent*, that she will save by waiting until next week.

Buy Now

$259.99 (1 + .07)

$$259.99 (1.07) = \$278.1893$$

Wait

$$\$259.99 (1 - .30)$$

$$259.99 (.70) = \$181.993 \text{ before tax}$$

$$\$181.993 (1 + .07)$$

$$181.993 (1.07) = \$194.73251 \text{ with tax}$$

Savings

$$\begin{array}{r} 278.1893 \\ - 194.73251 \\ \hline 83.45679 \end{array}$$

$83.46

Shana will save $83.46 by waiting

Problem # 15

slope = $\dfrac{rise}{run} = \dfrac{4}{3}$

3

4

(5, 1)

(2, -3)